Supplementary Material to "Monopoly Insurance and Endogenous Information"*

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Contents

1 Introduction **2**

1.1 Notation . . . 2

2 Neglecting menus with more than two contracts **2**

3 The reduced-form maximization problem of the insurer **2**

4 Limit results (proof of proposition 2) **4**

5 Numerical example **6**

6 Comparative statics results (proposition 3) **7**

7 Effect of effort on risk premium **7**

8 Perfect competition à la Rothschild and Stiglitz (1976) **9**

9 An oligopoly model à la Hotelling **11**

9.1 Relation oligopoly and monopoly . . . 13

10 Alternative setup with pecuniary cost of effort **15**

10.1 Introduction . . . 15

10.2 Contract choice . . . 16

10.3 Effort choice . . . 16

10.4 Contract design . . . 18

10.4.1 Constraints in contract design . . . 18

10.4.2 Optimal contract . . . 22

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1. Introduction

This supplementary material to our paper "Monopoly Insurance with Endogenous Information"(forthcoming in the *International Economic Review*) provides proofs that were omitted from the paper. It also explores three alternative setups — all discussed in the conclusions of the paper — in which: (i) there is perfect competition, (ii) there is an oligopoly, and (iii) the cost of effort is monetary.

1.1. Notation

We use the following notation (on top of the notation used in our paper):

$$\Delta_i = \overline{u}_i - \underline{u}_i.$$

2. Neglecting menus with more than two contracts

The following result was referred to in Section 2.1 of our paper.

Lemma S1. *For any equilibrium with more than two contracts in the menu, there is a corresponding equilibrium in which (i) the insurer offers at most two contracts and (ii) the expected profits of the insurer and the expected utility of the agent are the same as in the original equilibrium.*

Proof of lemma S1: Suppose that the insurer offers more than two contracts in equilibrium. We will show that there exists another equilibrium in which the insurer offers only two contracts and earns the same profits as in the original equilibrium.

As a first step, delete all contracts from the menu that are bought with zero probability under equilibrium play. Second, denote a pure strategy of the agent by (e, i, j), where e is the chosen effort, i is the contract in the menu that is chosen if the signal is high, and j is the contract chosen if the signal is low.[1] The pure strategy (e, i, j) induces some particular expected profits of the insurer. Let (e^*, i^*, j^*) denote the pure strategy that, among the pure strategies that the agent uses with positive probability in the original equilibrium, yields the highest expected profits. There must exist an equilibrium in which the insurer offers only the contracts i^* and j^*. This is because, by definition, offering only these contracts yields expected profits that are at least as high as the ones in the original equilibrium. Moreover, the agent will not have an incentive to deviate from such a new equilibrium, as he played (e^*, i^*, j^*) with positive probability in the original equilibrium (when his choice set was larger). We can also conclude that the insurer's expected profits cannot be strictly higher in the new equilibrium than in the original one. Because if they were, then the original situation would not have been an equilibrium (offering only i^* and j^* would have been a profitable deviation for the insurer). Finally, if in the original equilibrium the agent uses more than one pure strategy, then he must be indifferent between them. Therefore the expected utility of the agent is the same in the original and in the new equilibrium. □

3. The reduced-form maximization problem of the insurer

We here provide a detailed formulation of the insurer's profit maximization problem. By lemma 1 in the paper, the optimal menu of contracts solves the program stated in eq. (7) in our paper. The

[1] Possibly, i and j are the same contract.

constraints that are associated with the problem can be stated a follows:

$$\sum_{i\in\{l,h\}} \alpha_i\left[\beta_i(e)\underline{u}_i + (1-\beta_i(e))\overline{u}_i\right] - c(e) = \beta\underline{u}_l + (1-\beta)\overline{u}_l, \tag{1}$$

$$\begin{aligned}\alpha_h[(\beta_h(e)-\beta_h(e^h))\underline{u}_h + (\beta_h(e^h)-\beta_h(e))\overline{u}_h] + \alpha_l[\beta_l(e)\underline{u}_l + (1-\beta_l(e))\overline{u}_l] - c(e)\\ = \alpha_l\left[\beta^l(e^h)\underline{u}_0 + (1-\beta_l(e^h))\overline{u}_0\right] - c(e^h),\end{aligned} \tag{2}$$

$$c'(e) = \alpha\beta_h'(e)\left(\overline{u}_l - \underline{u}_l - \overline{u}_h + \underline{u}_h\right), \tag{3}$$

$$c'(e^h) = \alpha\beta_h'(e^h)\left(\overline{u}_0 - \underline{u}_0 - \overline{u}_h + \underline{u}_h\right), \tag{4}$$

$$e^h \geq e \geq 0. \tag{5}$$

The equalities in (1) and (2) are the binding (IG_l) and (EH) constraints, respectively. Since the constraints (1)–(4) are all linear in the agent's ex post utility levels, we can solve them for the four utility levels as functions of e and e^h. This leads to the following expressions:

$$\overline{u}_l(e,e^h) = \overline{u}_0 - \beta_l(e^h)\frac{c'(e^h)}{\alpha_h\beta_h'(e^h)} + \beta_l(e)\frac{c'(e)}{\alpha_h\beta_h'(e)} - \frac{c(e^h)-c(e)}{\alpha_l}, \tag{6a}$$

$$\underline{u}_l(e,e^h) = \underline{u}_0 + \left(1-\beta_l(e^h)\right)\frac{c'(e^h)}{\alpha_h\beta_h'(e^h)} - (1-\beta_l(e))\frac{c'(e)}{\alpha_h{\beta_h}'(e)} - \frac{c(e^h)-c(e)}{\alpha_l}, \tag{6b}$$

$$\overline{u}_h(e,e^h) = \overline{u}_0 - \beta_l(e^h)\frac{c'(e^h)}{\alpha_h\beta_h'(e^h)} - [\beta_h(e)-\beta_l(e)]\frac{c'(e)}{\alpha_h\beta_h'(e)} - \frac{c(e^h)-c(e)}{\alpha_l} + \frac{c(e)}{\alpha_h}, \tag{6c}$$

$$\underline{u}_h(e,e^h) = \underline{u}_0 + \left(1-\beta_l(e^h)\right)\frac{c'(e^h)}{\alpha_h\beta_h'(e^h)} - [\beta_h(e)-\beta_l(e)]\frac{c'(e)}{\alpha_h\beta_h'(e)} - \frac{c(e^h)-c(e)}{\alpha_l} + \frac{c(e)}{\alpha_h}. \tag{6d}$$

Plugging these expressions into the objective function (13) leads to a maximization problem over the two variables e and e^h, where the only constraints are $e^h \geq e \geq 0$. An equilibrium with pooling results if $e = 0$. An equilibrium with exclusion occurs if $e^h = e$. This is summarized in the following proposition.

Proposition S1. *The optimal contract menu is given by (6), where e and e^h solve the maximization problem stated in (8). The optimal contract menu satisfies the following first-order conditions:*

$$\begin{aligned}\frac{\partial\pi(e,e^h)}{\partial e} &= -\alpha_h\beta_h'(e)\left[h(\overline{u}_l) - h(\underline{u}_l) - h(\overline{u}_h) + h(\underline{u}_h)\right]\\ &\quad -\alpha_l\beta_l(e)(1-\beta_l(e))[h'(\overline{u}_l) - h'(\underline{u}_l)]\frac{c''(e)\beta_h'(e) - \beta_h''(e)c'(e)}{\alpha_h(\beta_h'(e))^2}\\ &\quad +\alpha_h\left[\beta_h(e)h'(\underline{u}_h) + (1-\beta_h(e))h'(\overline{u}_h)\right][\beta_h(e)-\beta_l(e)]\frac{c''(e)\beta_h'(e) - \beta_h''(e)c'(e)}{\alpha_h(\beta_h'(e))^2}\\ &\leq 0 \qquad \text{with “=” if } e > 0;\end{aligned} \tag{7}$$

$$\begin{aligned}\frac{\partial\pi(e,e^h)}{\partial e^h} &= \Big[\alpha_h\left(-\beta_h(e)h'(\underline{u}_h)(1-\beta_l(e^h)) + (1-\beta_h(e))h'(\overline{u}_h)\beta_l(e^h)\right)\\ &\quad +\alpha_l\left(-\beta_l(e)h'(\underline{u}_l)(1-\beta_l(e^h)) + (1-\beta_l(e))h'(\overline{u}_l)\beta_l(e^h)\right)\Big]\times\\ &\quad \frac{c''(e^h)\beta_h'(e^h) - \beta_h''(e^h)c'(e^h)}{\alpha_h(\beta_h'(e^h))^2}\\ &\leq 0 \qquad \text{with “=” if } e^h > e;\end{aligned} \tag{8}$$

$$e^h \;\geq\; e \;\geq\; 0, \tag{9}$$

where the $\overline{u}_i$ and $\underline{u}_i$ are given by (6).

To illustrate some of the insurer's incentives when designing the optimal menu, consider the effect on the profits of an increase in the effort level e, i.e., the left-hand side of (7). The first part of the derivative, on the first line, is strictly negative as long as the coverage of contract h is strictly higher than that of contract l. This part represents the missorting effect discussed in the text. The remaining part of the expression, on the second and third lines, represents the effect on the insurer's profit that goes via changes in the ex post utility levels in (6). For example, a higher e requires an increase in $(\overline{u}_l - \underline{u}_l) - (\overline{u}_h - \underline{u}_h)$ (see eq. (3)). And a larger $(\overline{u}_l - \underline{u}_l) - (\overline{u}_h - \underline{u}_h)$, in turn, relaxes ($\mathrm{IG_l}$), which enables the insurer to lower the utility levels in contract h and thus increase profits. This second part of (7) can therefore be positive, making it profitable for the insurer to induce a positive effort level.

4. Limit results (proof of proposition 2)

In proposition 2 of our paper we stated some limit results: What can we say about effort and the equilibrium menu as the information gathering costs become negligible or arbitrarily large? In particular, how do those limit menus relate to the menus in the insurance model in Stiglitz (1977)? In Stiglitz's classic model, the agent does not exert effort in order to learn about his accident risk. Instead, each agent type is assumed to know his risk from the outset of the game. We call the optimal contract menu in this model the *Stiglitz menu.* Proposition 2 also contains a limit result for α_l, which we will prove first.

Before proving the result, we restate the following equations which are used below:

$$U(e) = \sum_{i \in \{l,h\}} \alpha_i \left[\beta_i(e)\,\underline{u}_i + (1-\beta_i(e))\,\overline{u}_i\right] - c(e), \tag{10}$$

$$\begin{aligned} U'(e) &= \sum_{i \in \{l,h\}} \alpha_i \;\left[\beta_i'(e)\,\underline{u}_i - \beta_i'(e)\,\overline{u}_i\right] - c'(e) \\ &= \alpha_h \beta_h'(e)\left[(\overline{u}_l - \underline{u}_l) - (\overline{u}_h - \underline{u}_h)\right] - c'(e), \end{aligned} \tag{11}$$

$$\beta_l(e)\underline{u}_l + (1-\beta_l(e))\bar{u}_l \geq \beta_l(e)\underline{u}_0 + (1-\beta_l(e))\bar{u}_0, \tag{IR$_\mathrm{l}$}$$

$$\begin{aligned} &\alpha_h\left[\beta_h(e)\underline{u}_h + (1-\beta_h(e))\bar{u}_h\right] + \alpha_l\left[\beta_l(e)\underline{u}_l + (1-\beta_l(e))\bar{u}_l\right] - c(e) \\ &\quad\geq \alpha_h\left[\beta_h(e^h)\underline{u}_h + (1-\beta_h(e^h))\bar{u}_h\right] + \alpha_l\left[\beta_l(e^h)\underline{u}_0 + (1-\beta_l(e^h))\bar{u}_0\right] - c(e^h), \end{aligned} \tag{EH}$$

$$\alpha_h\beta_h'(e)(\Delta_l - \Delta_h) - c'(e) = 0, \tag{12}$$

$$\begin{aligned} \max_{\bar{u}_h,\underline{u}_h,\bar{u}_l,\underline{u}_l,e,e^h} \; & w - \beta D - \alpha_h\left[\beta_h(e)h(\underline{u}_h) + (1-\beta_h(e))h(\bar{u}_h)\right] \\ & -\alpha_l\left[\beta_l(e)h(\underline{u}_l) + (1-\beta_l(e))h(\bar{u}_l)\right] \end{aligned} \tag{13}$$

$$
\begin{aligned}
\frac{\partial \pi(e,e^h)}{\partial e} &= -\alpha_h \beta_h'(e)\left[h(\overline{u}_l) - h(\underline{u}_l) - h(\overline{u}_h) + h(\underline{u}_h)\right] \\
&\quad -\alpha_l \beta_l(e)(1-\beta_l(e))[h'(\overline{u}_l) - h'(\underline{u}_l)]\frac{c''(e)\beta_h'(e) - \beta_h''(e)c'(e)}{\alpha_h(\beta_h'(e))^2} \\
&\quad +\alpha_h\left[\beta_h(e)h'(\underline{u}_h) + (1-\beta_h(e))h'(\overline{u}_h)\right]\left[\beta_h(e) - \beta_l(e)\right]\frac{c''(e)\beta_h'(e) - \beta_h''(e)c'(e)}{\alpha_h(\beta_h'(e))^2} \\
&\leq 0 \quad \text{with “=” if } e > 0; && (14) \\
\frac{\partial \pi(e,e^h)}{\partial e^h} &= \Big[\alpha_h\left(-\beta_h(e)h'(\underline{u}_h)(1-\beta_l(e^h)) + (1-\beta_h(e))h'(\overline{u}_h)\beta_l(e^h)\right) \\
&\quad +\alpha_l\left(-\beta_l(e)h'(\underline{u}_l)(1-\beta_l(e^h)) + (1-\beta_l(e))h'(\overline{u}_l)\beta_l(e^h)\right)\Big] \times \\
&\quad \frac{c''(e^h)\beta_h'(e^h) - \beta_h''(e^h)c'(e^h)}{\alpha_h(\beta_h'(e^h))^2} \\
&\leq 0 \quad \text{with “=” if } e^h > e; && (15) \\
e^h &\geq e \geq 0, && (16)
\end{aligned}
$$

Proof of proposition 2: Note that, by proposition 1, the equilibrium utility levels $\underline{u}_l$, $\overline{u}_l$, $\underline{u}_h$, and $\overline{u}_h$ are between $\underline{u}_0$ and $\overline{u}_0$. This boundedness and (12) imply that the optimal effort is bounded from above by the e solving $\alpha_h\beta_h'(e)(\overline{u}_0 - \underline{u}_0) - c'(e) = 0$, which has a unique solution as $c'' > 0$ and $\beta_h'' \leq 0$. For $\alpha_h = 0$, the unique solution to this equation is 0. Since β_h' and c' are continuous, this implies that there exists a $\widetilde{\alpha}_h > 0$ such that the upper bound for the optimal effort is less than $\varepsilon > 0$ for all $\alpha_h < \widetilde{\alpha}_h$. Letting $\widetilde{\alpha}_l = 1 - \widetilde{\alpha}_h$ gives the result.

Next, we show that the game has a unique optimal contract for $\gamma = 0$ which is separating and induces infinite effort; that is, the Stiglitz contract menu $(\bar{u}_h^S, \underline{u}_h^S, \bar{u}_l^S, \underline{u}_l^S)$ results if $\gamma = 0$.

If the contract menu is separating (i.e., if strictly positive effort is better than zero effort), then $e = \infty$. To see this, note that the derivative of (10), stated in (11), is strictly positive if the menu is separating and costs are zero. This implies that the Stiglitz contract menu is the best separating (where “separating” here includes exclusion) contract menu if $\gamma = 0$. It remains to check that no pooling contract leads to higher profits.

With $\gamma = 0$, a pooling contract must satisfy ($\mathrm{IR_l}$) for $e = \infty$. Otherwise, the agent has an incentive to deviate to effort $e' = \infty$ and buy the contract only in case he receives a high signal. Hence, the most profitable pooling contract has full coverage and ($\mathrm{IR_l}$) binds. But then the standard proof (Stiglitz, 1977, see Property 3) showing that this pooling is not profit maximizing applies. This establishes that the Stiglitz contract menu is the unique optimal menu if $\gamma = 0$.

Second, we show that the set of optimal contract menus as a function of γ is closed. More precisely, let $(\gamma^m)_{m=1}^\infty$ be a sequence in $\mathbb{R}_+$ converging to γ. Let $u^m = (\bar{u}_h^m, \underline{u}_h^m, \bar{u}_l^m, \underline{u}_l^m)$ be an optimal contract under γ^m. Then the limit $u = \lim_{m\to\infty} u^m$ is an optimal contract menu under γ (whenever the limit exists). This follows from the continuity of profits (13) in utilities and the fact that the weak inequality constraints (EH) and ($\mathrm{IR_l}$) hold in the limit if they hold for every m.

Third, we show that the first two properties imply that the optimal contract menu converges to the Stiglitz contract menu as $\gamma \to 0$. The proof is by contradiction. Suppose, to the contrary, that there was a sequence of γ^m and u^m with $\lim_{m\to\infty}\gamma^m = 0$ such that $\max\{|\bar{u}_h^m - \bar{u}_h^S|, |\underline{u}_h^m - \underline{u}_h^S|, |\bar{u}_l^m - \bar{u}_l^S|, |\underline{u}_l^m - \underline{u}_l^S|\} \geq \varepsilon$ for all m and some $\varepsilon > 0$. Since $u^m \in [\underline{u}_0, \bar{u}_0]^4$, the Bolzano-Weierstrass theorem implies that u^m has a convergent subsequence. By the previous two properties, the limit of this subsequence must be the Stiglitz contract menu. But then $\max\{|\bar{u}_h^m - \bar{u}_h^S|, |\underline{u}_h^m - \underline{u}_h^S|, |\bar{u}_l^m - \bar{u}_l^S|, |\underline{u}_l^m - \underline{u}_l^S|\} \geq \varepsilon$ cannot hold for all m: It will be violated for elements of the convergent subsequence with m high enough.

This is the desired contradiction.

Hence, we have shown that equilibrium menus are close to the Stiglitz menu for γ close enough to 0. As the Stiglitz menu is separating, the equilibrium menu for small enough γ cannot be pooling.

Now we turn to the case $\gamma \to \infty$.

First, it is shown that equilibrium effort e is less than some level $\tilde{e}(\gamma)$ and that $\lim_{\gamma\to\infty} \tilde{e}(\gamma) = 0$. As equilibrium contracts (by proposition 1) have partial coverage, (12) implies $e < \tilde{e}(\gamma)$, where $\tilde{e}(\gamma)$ is the effort level solving $\alpha\beta_h'(\tilde{e})\Delta_0 = \gamma c'(\tilde{e})$. By the implicit function theorem, $\tilde{e}'(\gamma) = \frac{\gamma c'(\tilde{e})}{\alpha\beta_h''(\tilde{e})\Delta_0 - \gamma c''(e)} < 0$ and therefore $\tilde{e}$ is uniquely determined by the defining equation. $\tilde{e}$ exists for all $\gamma \in \mathbb{R}_+$ by the assumption $c'(0) = 0$. From $c'' > 0$, it follows that $c'(e) > 0$ for every $e > 0$. Therefore, $\lim_{\gamma\to\infty} \tilde{e}(\gamma) = 0$ (otherwise, the defining equation cannot be satisfied for γ high enough as β_h' is bounded).

Second, we show that $\lim_{\gamma\to\infty} \tilde{e}(\gamma) = 0$ implies $\lim_{\gamma\to\infty} \Delta_l = 0$. We divide this step into two cases. In case 1, we concentrate on separating (including exclusion) equilibria. Case 2 covers pooling equilibria.

In case 1, (14) holds with equality. Note that the first two terms in (14) are negative and the third is positive. Therefore, (14) can only hold with equality if

$$(1-\alpha)\beta_l(e)(1-\beta_l(e))(h'(\bar{u}_l) - h'(\underline{u}_l)) < \alpha\left(\beta_h(e)h'(\underline{u}_h) + (1-\beta_h(e))h'(\bar{u}_h)\right)(\beta_h(e) - \beta_l(e)).$$

Note $\lim_{\gamma\to\infty} \beta_h(e) - \beta_l(e) = 0$ because $\lim_{\gamma\to\infty} \tilde{e}(\gamma) = 0$ and $e < \tilde{e}(\gamma)$. Consequently, the right hand side of the inequality above tends to zero as $\gamma \to \infty$. Therefore, the left-hand side also must converge to zero. As $\lim_{\gamma\to\infty} \beta_l(e(\gamma)) = \beta$, this implies that $\lim_{\gamma\to\infty} h'(\bar{u}_l) - h'\underline{u}_l) = 0$ which implies $\lim_{\gamma\to\infty} \Delta_l = 0$. As $0 \le \Delta_h \le \Delta_l$, $\lim_{\gamma\to\infty} \Delta_h = 0$ has to hold as well. This completes the proof for case 1.

In case 2, we look at pooling contracts. This implies $0 = e < e^h$ and therefore (15) holds with equality. (15) can then be rearranged to (assuming pooling, i.e. $e = 0$)

$$h'(\bar{u}) = \frac{\beta}{1-\beta}\frac{1-\beta_l(e^h)}{\beta_l(e^h)}h'(\bar{u}),$$

where $(\bar{u}, \underline{u})$ denotes the equilibrium pooling contract. Note that also $e^h \le \tilde{e}(\gamma)$ and therefore $\lim_{\gamma\to\infty} \beta_l(e^h(\gamma)) = \beta$. Therefore, the equation above can hold for high γ only if $\lim_{\gamma\to\infty} \bar{u}(\gamma) - \underline{u}(\gamma) = 0$; that is, the contract converges to full coverage as γ goes to infinity. □

5. Numerical example

This section explains some details about the numerical example presented in table 1 in our paper. In this example, we use the linear signal technology $g(e) = \min\{e, 1\}$.[2] This gives us, for $e < 1$,

$$\beta_h(e) = \alpha_h\theta^h + \alpha_l\theta^l + \alpha_l(\theta^h - \theta^l)e, \qquad \beta_h'(e) = \alpha_l(\theta^h - \theta^l).$$

As the insurer's maximization program is not necessarily quasiconcave, we use a grid maximization where we search for the highest profits on a grid of 500 equally spaced effort levels between 0 and 1 (and an equally spaced grid for e^h). The result of this grid search is then given as starting value to a maximization algorithm. We use the "ralg" algorithm of the openopt package (`http://openopt.org`); see our websites for the Python code.

[2] As g is non-differentiable at $e = 1$ and flat thereafter, our assumptions are strictly speaking only satisfied if $e < 1$. From the first-order condition (3), we can—with the here assumed functions—conclude that $e^3 = \frac{\alpha(1-\alpha)(\theta^h - \theta^l)(\Delta_l - \Delta_h)}{4\gamma}$. As $\Delta_l - \Delta_h \le \Delta_0 = 1$ with our parameter values, $e^3 \le \frac{0.007875}{\gamma}$ and therefore $e < 1$ if $\gamma > 0.007875$.

6. Comparative statics results (proposition 3)

In this section, we prove proposition 3 from the main text. Part (i) is already proven in the main text.

Part (ii): Consider first the claim about the profits in the pooling case (where $e = 0$). Note that the only binding constraint (EH) is relaxed if γ is increased: Using the envelope theorem, the derivative with respect to γ of the left-hand side is $-c(e)$ and the derivative of the right hand side is $-c(e^h)$. This means that a contract satisfying (EH) for γ' will also satisfy (EH) for all $\gamma'' > \gamma'$. Hence, profits are non-decreasing in γ as the set of feasible pooling contracts expands as γ increases. Since (EH) is binding, profits are in fact increasing. □

7. Effect of effort on risk premium

In this section, we briefly discuss how information acquisition affects classification risk as measured by the risk premium. It turns out that the effect is ambiguous.

The extent of classification risk is usually described by the risk premium ρ. This is implicitly defined by

$$\alpha_h \left[(1-\beta_h(e))u(w-p_h) + \beta_h(e)u(w-p_h-D+R_h) - c(e)\right] \\ + \alpha_l \left[(1-\beta_l(e))u(w-p_l) + \beta_l(e)u(w-p_l-D+R_l) - c(e)\right] = u(w-\bar{p}-\bar{D}+\bar{R}-\rho) - c(e),$$

where a bar indicates an expected value. In particular, $\bar{D} = \beta D$, $\bar{p} = \alpha_l p_l + \alpha_h p_h$, and $\bar{R} = \alpha_h \beta_h(e) R_h + \alpha_l \beta_l(e) R_l$. Hence, $\partial \bar{R}/\partial e = \alpha_h \beta_h'(e)(R_h - R_l)$ where we use $\alpha_h \beta_h'(e) = -\alpha_l \beta_l'(e)$. Note that $\bar{p}$ and $\bar{D}$ do not depend on e, as the share of high signals is α_h (for every $e \geq 0$) in our signal technology.

Using the implicit function theorem (after canceling out $c(e)$ on both sides), yields[3]

$$\rho'(e) = \frac{\alpha_h \beta_h'(e)\left[u'(w-\bar{p}-\bar{D}+\bar{R}(e)-\rho)(R_h-R_l) - \Delta\right]}{u'(w-\bar{p}-\bar{D}+\bar{R}-\rho)}$$

where $\Delta = \Delta_l - \Delta_h > 0$. We want to point out two straightforward implications:

First, if the contracts are pooling, then $\rho'(e)$ equals zero as $\Delta = 0$ and $R_h - R_l = 0$. Intuitively, this makes sense: If the consumer buys the same contract regardless of the signal, then improving the signal does not have any impact on the level of risk the consumer faces.

Second, if $\bar{R}$ did not depend on e, the effect of e on the risk premium would be straightforwardly negative as the positive term in the numerator would not appear. Intuitively, a larger e means that the consumer learns more about his true type, which lowers the risk premium.[4]

As $\bar{R}$ does depend on e, however, there is another effect present; moreover, this effect goes in the opposite direction. Intuitively, higher effort leads to improved sorting and therefore higher expected indemnity payments. This "income effect" means that the consumer requires a higher risk premium to be willing to be exposed to the risk, so ρ goes up. (Arguably, this effect is not related to classification

[3] We concentrate on marginal effects for which the additional effort does not affect the contract choice strategy of the consumer; that is, the consumer buys the high (low) coverage contract after observing the high (low) signal.

[4] One could – at first sight – conjecture the opposite, i.e. as β_l and β_h are further appart when effort increases, it seems as there is more classification risk. However, this reasoning does not take into account that the true risk types are θ^h and θ^l which do not change in effort. The fact that β_h and β_l are further apart does not mean that there is more classification risk (this would be the case if θ^h and θ^l were for exogenous reasons further apart); it simply means that the sorting improved, i.e. the consumer learned more about his risk type and therefore the risk of getting the "wrong signal" and consequently buying the "wrong" contract is reduced (where "wrong" means low coverage contract for θ^h types and high coverage for θ^l).

risk, which suggests that the risk premium is not a perfect measure of risk classification in our setup with endogenous information.)

It turns out that the overall effect is, in general, ambiguous. To verify this claim we will now study a numerical example which shows that $\rho'(e)$ can indeed have both signs. Let the offered contracts be such that

$$\underline{u}_h = u(w - p_h - D + R_h) = 2 \qquad \bar{u}_h = u(w - p_h) = 4$$
$$\underline{u}_l = u(w - p_l - D + R_l) = 1 \qquad \bar{u}_l = u(w - p_l) = 5.$$

Furthermore we assume that the share of high signals is $\alpha_h = 1/2$ and that the signal technology leads to $\beta_h(e) = 1/2 + e$ and $\beta_l(e) = 1/2 - e$. In this case, the expected utility when exerting effort e is

$$U(e) = 1/2[(1/2 + e) * 2 + (1/2 - e) * 4] + 1/2[(1/2 - e) * 1 + (1/2 + e) * 5] - c(e) = 3 + e - c(e).$$

Now assume that $u(z) = z^\alpha$ where $\alpha \in (0, 1)$. We can then solve for the risk premium ρ as defined above as

$$\rho(e) = (w - \bar{p} - \bar{D} + \bar{R}) - (3 + e)^{1/\alpha}.$$

As $\bar{R} = 1/2(1/2 + e)R_h + 1/2(1/2 - e)R_l$, we get for the derivative of ρ with respect to e

$$\rho'(e) = (R_h - R_l)/2 - \frac{1}{\alpha}(3 + e)^{(1-\alpha)/\alpha}.$$

To obtain the utility values above with the utility function $u(z) = z^\alpha$, the variables p_h, R_h, p_l and R_l have to satisfy $w - p_h = 4^{1/\alpha}$, $w - p_h - D + R_h = 2^{1/\alpha}$, $w - p_l = 5^{1/\alpha}$ and $w - p_l - D + R_l = 1$. Combining these conditions gives $R_h - R_l = 2^{1/\alpha} + 5^{1/\alpha} - 4^{1/\alpha} - 1$. If we choose $\alpha = 0.4$, then $\partial\rho/\partial e$ is positive at first and turns negative later; see figure 1.[5]

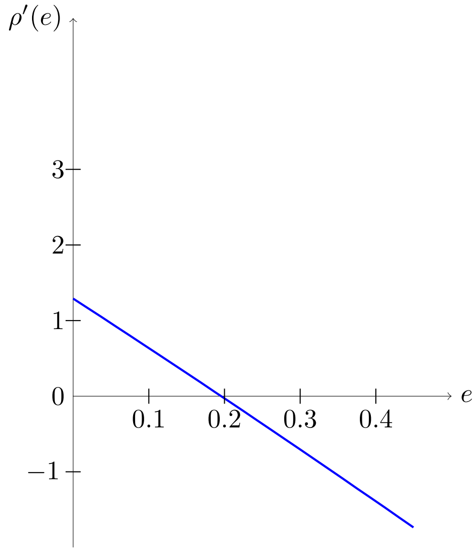

Figure 1: $\rho'(e)$ with $\alpha = 0.4$

Note that we did not specify the cost function c yet. If $c(e) = \gamma e^2$, then for γ sufficiently high only those e for which ρ' is positive will be "realistic." Hence, for different values of the parameter γ we have that ρ' is either changing sign or that it is positive in the relevant range.

[5] Note that we chose the example such that effort is immediately decision relevant; that is, for every $e > 0$ consumers with a high (low) signal will strictly prefer the high (low) coverage contract. Hence, all levels of e depicted in the graph are relevant.

Consequently, we cannot draw any general conclusion on the effect of information acquisition on the risk premium.

8. Perfect competition à la Rothschild and Stiglitz (1976)

This section illustrates that the distortion at the top result from proposition 1 holds also in a Rothschild-Stiglitz (RS) equilibrium of a perfectly competitive market. The result is, therefore, likely to be a general result in insurance models with endogenous information acquisition and not be specific to a particular market structure. We consider a perfectly competitive market in the following sense: There are many insurance companies offering insurance contracts at the first stage of the model. The consumer side of the model does not change. We look at a RS equilibrium, that is, a menu of contracts such that (i) each contract in the menu makes non-negative profits and (ii) no insurer can make positive profits by offering an additional contract given that this menu is offered. The consumer side of the model is the same as in our monopoly model in the paper. We will again concentrate on pure strategy equilibria. That is, all consumers choose the same effort level and all consumers with a given signal choose the same insurance contract.

Proposition S2. *The high-coverage contract bought in a separating or exclusion RS equilibrium is downward distorted.*

Proof. Suppose otherwise — that is, suppose consumers with a high signal buy contract (p_h, R_h) with $R_h \geq D$. Consider a deviation contract (p_d, R_d) such that (p_d, R_d) has slightly less coverage than (p_h, R_h) (in the sense that $R_h - R_d = \varepsilon > 0$) and that a consumer with risk $\beta_h(e^*)$ is indifferent between (p_d, R_d) and (p_h, R_h). Clearly, this implies $\Delta_h - \Delta_d > 0$. We will first argue that a consumer faced with this deviation contract (and the original equilibrium contracts) will exert effort $e^d < e^*$ and buy the contract (p_d, R_d) (at least if he receives a certain signal).

The strategy of exerting effort e^* and buying (p_d, R_d) when receiving the high signal while buying (p_l, R_l) when receiving the low signal gives the same expected payoff to the consumer as the strategy of exerting effort e^* and buying the contract (p_h, R_h) when receiving a high signal while buying (p_l, R_l) when receiving a low signal. This follows from the fact that a type with risk $\beta_h(e^*)$ is indifferent between (p_d, R_d) and (p_h, R_h). Since (p_d, R_d) has a lower coverage than (p_h, R_h), the consumer can achieve an even higher expected payoff by exerting effort $e' < e^*$ and buying (p_d, R_d) when receiving a high signal and (p_l, R_l) when receiving a low signal.

This implies that the deviation contract will be bought by some consumers. If it were not bought by any consumer, the optimal consumer behavior from the initial equilibrium would still be optimal. However, we just showed that there is a strategy using the deviation contract with which the consumer can achieve a higher payoff than in the initial equilibrium.

If the consumer exerts no effort and simply buys the deviation contract, then the deviation is profitable. Since we started from a RS separating/exclusion equilibrium, $p_h \geq \beta_h(e^*) R_h$. Consequently, $p_d > \beta R_d$ for $\varepsilon > 0$ small enough.

Therefore, we will for the remainder of the proof consider the case where the optimal strategy of the consumer is to exert some effort $e^d \neq e^*$ where $e^d > 0$ and then to buy the deviation contract either when receiving a high or a low signal. If he buys the deviation contract upon receiving a low signal, then the deviation is clearly profitable: In the initial equilibrium, we had $p_h \geq \beta_h(e^*) R_h$ and therefore we will for sure have $p_d > \beta_l(e^d) R_d$ (for ε small enough). If the consumer buys the deviation contract upon receiving a high signal, the deviation is still profitable: Since a consumer with risk $\beta_h(e^*)$ is indifferent between (p_d, R_d) and (p_h, R_h) (and a consumer with a higher risk will prefer (p_h, R_h) which has the higher coverage), we can conclude that $\beta_h(e^d) < \beta_h(e^*)$. Otherwise, it would not be optimal for the consumer to buy (p_d, R_d) after exerting effort e^d and receiving a high signal. This implies $e^d < e^*$ — that is, reducing the coverage for the high signal consumers leads to less effort.

We will now show that the deviation contract is profitable. First consider the case of overinsurance, i.e. $R_h > D$. In this case, the deviation contract would be profitable in the hypothetical case where consumers exert effort e^*: Since a $\beta_h(e^*)$ type is indifferent between (p_d, R_d) and (p_h, R_h) and since reducing coverage (starting from (p_h, R_h)) is efficient, the deviating insurer has to make more profits than an insurer selling (p_h, R_h) to $\beta_h(e^*)$ types. Since $e^d < e^*$, the deviation profits are even higher since the consumers buying the deviation contract have a lower risk than $\beta_h(e^*)$.

Next we turn to the case of full coverage, that is, where $p_h = R_h$. The slope of the indifference curve of a high-signal consumer at a full coverage contract is $\beta_h(e^*)$ in the (R_h, p_h)-space; that is, $dp_h/dR_h|_{U_h=const} = \beta_h(e^*)$. Consequently (at a first order approximation), $p_d = p_h - \beta_h(e^*)\varepsilon$. Per consumer deviation profits are (using a first order approximation)

$$\begin{aligned} p_d - \beta_h(e^d)R_d &= p_h - \beta_h(e^*)\varepsilon - \big(\beta_h(e^d)R_d - \beta_h(e^*)R_d + \beta_h(e^*)R_d - \beta_h(e^*)R_h + \beta_h(e^*)R_h\big) \\ &= p_h - \beta_h(e^*)\varepsilon - \big(\beta_h'(e)(e^d - e^*)R_d - \beta_h(e^*)\varepsilon + \beta_h(e^*)R_h\big) \\ &= p_h - \beta_h(e^*)R_h - \beta_h'(e)(e^d - e^*)R_d \geq -\beta_h'(e)(e^d - e^*)R_d > 0 \end{aligned}$$

where the first inequality follows from the fact that the (p_h, R_h) contract must yield non-negative profits in the initial RS equilibrium.

Consequently, the deviation is profitable which contradicts that the original set of contracts was an RS equilibrium. □

It is well known that in the original RS model (with exogenous types) no pooling equilibria exist. The standard proof of this result does, however, not go through in our extension with endogenous information. To be more precise, the standard argument why pooling cannot emerge in RS is the following: In a hypothetical pooling equilibrium there is a profitable deviation that offers slightly less coverage than the pooling contract and attracts only the low risk types. As the premium is almost the same but expected indemnities are much lower (as only low risk types buy the deviation contract), the deviation is profitable. The point is that the risk of those buying the deviation contract is θ^l and therefore discretely lower than the average risk in the pooling contract. This is no longer true with endogenous information: If the coverage of the deviation contract is only marginally lower than in the pooling contract, then consumers will exert only very little effort. This means that β_l, i.e. the average risk of those buying the deviation contract, is almost the same as the average risk β. Hence, the deviation does not create the discrete jump in risk (and therefore does not reduce expected indemnity payments discretely). The deviation might, therefore, not be profitable (this will depend on the specifics of the pooling contract, signal technology and costs of information acquisition).

For this reason, pooling equilibria cannot be ruled out when information is endogenous. The following proposition, however, shows that even in a pooling equilibrium (with endogenous information acquisition) the equilibrium contract has less than full coverage.

Proposition S3. *In a pooling RS equilibrium, the bought contract has less than full coverage.*

Proof. Suppose the pooling equilibrium contract has full coverage, that is, $R = D$. We will show that a profitable deviation contract exists. Let (p_d, R_d) be such that (a) $R_d = D - \varepsilon$ for some small $\varepsilon > 0$ and (b) a consumer with risk β is indifferent between (p, R) and (p_d, R_d).

The deviation contract is bought (by some consumers). The consumer can achieve a higher payoff than in the initial equilibrium by exerting some effort and buying the deviation contract when receiving a low signal while buying the (p, R) contract when receiving a high signal (recall the assumption $c'(0) = 0$). Consequently, the deviation contract will be optimally bought by some consumers as otherwise the maximum payoff a consumer can achieve is the one of the initial equilibrium. Note that — by construction — those consumers buying the deviation contract have a risk strictly below β as they would otherwise prefer (p, R) over (p_d, R_d).

At a full coverage pooling contract, the slope of a consumer's indifference curve is β in the (R, p)-space (i.e., $dp/dR|_{U=const} = \beta$). Consequently (at a first order approximation), $p_d = p - \beta\varepsilon$. Per consumer deviation profits are $p_d - \beta_l(e^d)R_d = p - \beta\varepsilon - \big(\beta_l(e^d)R_d - \beta_l(0)R_d + \beta_l(0)R_d - \beta R + \beta R\big) = p - \beta\varepsilon - \big(\beta_l'(e)e^d R_d - \beta\varepsilon + \beta R\big) = p - \beta R - \beta_l'(e)e^d R_d \geq -\beta_l'(e)e^d R_d > 0$ where the first inequality follows from the fact that equilibrium profits in the initial RS equilibrium have to be non-negative. Consequently, the deviation profits are positive for $\varepsilon > 0$ small enough which contradicts that the original situation was an RS equilibrium. □

9. An oligopoly model à la Hotelling

This section shows that our "distortion at the top" result carries over to a simple oligopoly setup. The timing, information structure, and action sets of the players are the same as in the monopoly setup. Now, however, we analyze a model with two insurers that are located at the endpoints of a Hotelling line of length one. Consumers are uniformly distributed on the Hotelling line and the share of high-risk consumers is α_h at every location; that is, consumers and insurers cannot infer anything about risk from the location of a consumer. An agent located at $x \in [0, 1]$ incurs transportation costs xt when buying from insurer A and transportation costs $(1 - x)t$ when buying from insurer B, where $t > 0$ is a parameter. Transportation costs enter additively in consumer utility; that is, consumers maximize the utility from insurance (which is the same as in the monopoly setup) minus transportation costs. The two insurers simultaneously offer a menu of contracts $\{(p^i_h, R^i_h), (p^i_l, R^i_l), (0, 0)\}$ at stage one of the game (where $p^i_h \geq p^i_l$ and $R^i_h \geq R^i_l$ and $i \in \{A, B\}$). Consequently, the (p^i_h, R^i_h) contract is the contract of insurer i targeted at consumers receiving a high signal while the contract (p^i_l, R^i_l) is targeted at consumers receiving a low signal. The contract $(0, 0)$ denotes the outside option. For simplicity, we assume that the consumers must incur the transportation costs also when choosing the outside option (the transportation costs associated with contract $(0, 0)$ equal xt or $(1 - x)t$, whichever is smallest). This implies that transportation costs do not matter for the participation decision. Consequently, firms will still compete even if t is high. Given that we already analyzed a monopoly setup in the paper, this allows us to here concentrate on the case where firms actually compete.[6]

We will now concentrate on symmetric separating equilibria of this model, that is, equilibria where $R^A_j = R^B_j = R_h$ and $p^A_j = p^B_j = p_j$, for $j \in \{h, l\}$, and $R_h > R_l > 0$.[7] Our goal here is not to give a full derivation of a symmetric separating equilibrium but merely to show that in such an equilibrium the high-coverage contract is distorted. Note that the third stage of our model (the consumer's contract choice) does not change. In a symmetric equilibrium, consumers located in the interval $[0, 0.5)$ will buy

[6] Boone and Schottmüller (forthcoming) use a similar oligopoly model in a different context (in particular there is no endogenous information acquisition in their model).

[7] Similar arguments apply to potential exclusion and pooling equilibria but we want to avoid cluttering the exposition with case distinctions.

from insurer A and consumers located in the interval $(0.5, 1]$ will buy from insurer B. Given that the insurers offer the same contracts (by symmetry of the equilibrium), each consumer knows from which insurer he will buy later when he is choosing his effort in the second stage of the model. Consequently, the consumer's effort choice problem is the same as in the monopoly setup (given that the insurers offer the same menu of contracts). Clearly, this does not necessarily hold off the equilibrium path where insurers offer different menus of contracts. However, an analysis of this complex case turns out to be unnecessary for showing the point we want to make.

We now turn to the first stage (menu design by the insurers). We will assume that $R_h = D$ and then show that an insurer has a profitable deviation where he uses $R_d < D$. That is, we cannot have full coverage in a symmetric separating equilibrium. The argument follows the proof of proposition 1 in the paper. This reasoning still applies because of the following: To prove proposition 1, we assumed that the monopolist offered full coverage for high-signal consumers and showed a profitable deviation. This deviation respected all constraints, left the low-coverage contract unchanged, left the utility of a high-signal consumer unchanged while increasing the ex ante expected utility of a consumer — that is, through lower effort costs the low-signal consumer was made better off. If we use the same deviation in the duopoly setting, the same effects as in the monopoly setting work but the insurer will in addition increase his demand. Put differently, the deviation becomes even more profitable! The rest of this section develops this idea in greater detail.

The insurer maximizes profits subject to the same constraints as in the monopoly setup. The only difference is that the level of demand — that is, how many of the consumers with a high/low risk signal buy from an insurer — will depend on the contracts offered. Note that the first part of lemma 1 in the paper still applies, as its proof did not require any equilibrium analysis: (EH) and ($\mathrm{IG_l}$) are simply stricter constraints than ($\mathrm{IC_h}$) and ($\mathrm{IR_l}$) and therefore the latter two are implied by the former. Consequently, we only have to track the (EH), ($\mathrm{IG_l}$) and ($\mathrm{IG_h}$) constraints when showing a profitable deviation.

Take a symmetric separating equilibrium and suppose that the high-coverage contract had full coverage ($R_h = D$). We will now show that firm A has a profitable deviation. To do so, we have to show first that ($\mathrm{IG_h}$) cannot hold with equality in this situation. Suppose it did. Since ($\mathrm{IC_l}$) is slack, we know that $\beta_l(e^*)\underline{u}_l + (1 - \beta_l(e^*))\overline{u}_l > u_h$ where u_h is the utility level a consumer gets when buying the full-coverage contract. This implies that $\beta_l(e^*)h(\underline{u}_l) + (1 - \beta_l(e^*))h(\overline{u}_l) > h(u_h)$, because h is convex (recall that h is the inverse function of u). This implies that

$$2\pi_A^* = w - \beta D - \alpha_h h(u_h) - \alpha_l[\beta_l(e^*)h(\underline{u}_l) + (1 - \beta_l(e^*))h(\overline{u}_l)] < w - \beta D - h(u_h) = 2\pi_A^{pool}$$

where π_A^{pool} are the profits firm A would make if it offered only the full-coverage contract (i.e., if A did not offer the low-coverage contract): Since ($\mathrm{IG_h}$) was binding, consumers in the interval $[0, 0.5)$ could still achieve the same utility as before by exerting zero effort and simply buy the high-coverage contract from firm A. They cannot do better than this because this would contradict the optimality of their strategy in the initial equilibrium. Given that A could earn higher profits by simply not offering the low-coverage contract, we can conclude that ($\mathrm{IG_h}$) cannot bind in a separating equilibrium in which the high-coverage contract offers full coverage.[8]

[8] The argument goes through with slight modification if we consider an exclusion equilibrium: As the inequality in the profit comparison was strict, A could increase his profits by marginally lowering the premium as the consumers best response is then to exert zero effort and buy the full-coverage contract directly; that is, A can gain $\alpha_l/2$ consumers if he decreases the premium by an infinitesimal amount and these consumers will then only have average risk β instead of $\beta_h(e^*)$.

After establishing that ($\mathrm{IG_h}$) is slack, we turn to showing that insurer A has a profitable deviation. First, suppose demand is fixed: All consumers located on $[0, 0.5)$ buy from insurer A (and cannot buy from insurer B), while no consumer located on $(0.5, 1]$ ever buys from insurer A. Then the analysis in the proof of proposition 1 applies: The indifference curve of a high-signal type through the full-coverage contract (p_h, R_h) is flatter than the isoprofit curve (in the $(\underline{u}_h, \overline{u}_h)$-space). Furthermore, the indifference curve of an ex ante consumer (i.e., a consumer just after stage 1 and just before he invests in effort) has the same slope $-\beta_h(\cdot)/(1 - \beta_h(\cdot))$ as the indifference curve of the high-signal type (see footnote 26 in the paper). Consequently, there is a partial-coverage contract (p_d, R_d) with $R_h = D > R_d > R_l$ such that (i) insurer A has higher profit than in the original equilibrium, (ii) a high-signal consumer is indifferent between the original contract and the deviation contract and (iii) the ex ante utility of a consumer is the same when offered the menu $\{(p_d, R_d), (p_l, R_l)\}$.[9]

If we can show that the deviation menu $\{(p_d, R_d), (p_l, R_l)\}$ satisfies all constraints, it will follow that the deviation is profitable even if we do not fix the demand to be all consumers located on $[0, 0.5)$: Consider first consumers receiving a low signal. These consumers will prefer the low-coverage over the high-coverage contracts. As the low-coverage contract offered by the insurers is the same, the low-signal consumers will simply buy their insurance from the insurer closest to their location; that is, exactly half of the low-signal consumers will still buy from insurer A. Since the deviation contract (p_d, R_d) was chosen such that high-signal consumers are indifferent between this contract and the original (p_h, R_h) contract, the same applies to high-signal consumers. Consequently, the deviation profit with fixed demand is the same as the one with non-fixed demand and the deviation is profitable. (Note that this is not necessarily the "optimal deviation". Other deviations that gain additional demand could very well be even more profitable.)

It remains to show that the deviation menu satisfies the constraints (EH), ($\mathrm{IG_l}$) and ($\mathrm{IG_h}$). The constraint (EH) is relaxed by the change. The constraint ($\mathrm{IG_l}$) is not affected by the change since the low-coverage contract is the same and the ex ante utility stays the same. The constraint ($\mathrm{IG_h}$) was slack from the start and therefore it will still be slack as long as the deviation contract does not differ too much from the original contract.

In conclusion, we have shown that the high-coverage contract cannot give full coverage in a symmetric separating (or exclusion) equilibrium of this duopoly game.

9.1. Relation oligopoly and monopoly

In this subsection, we will show two ways in which the monopoly model of the paper can be viewed as a limit case of the oligopoly model above. These constructions will allow us to conclude that our most interesting welfare result ("consumer surplus can be increasing in γ") is not due to the (very special) monopoly assumption but will also hold in some oligopoly settings.

Note that the exact distribution of consumers — we assumed it was uniform — did not really play a role in the arguments above. For now, let us assume consumers are distributed according to the

[9] Conditions (ii) and (iii) might hold simultaneously only at a first order approximation. However, this is enough to show existence of a profitable deviation. One can imagine that the (p_l, R_l) contract is changed "at the second order" to have (ii) and (iii). Since this change is only at the second order, the demand effect of this will be negligible and the argument below carries through.

following density (where the parameter a is in $[0,1]$)

$$f_a(x) = \begin{cases} a & \text{if } x \in [1/4 + a/4, 3/4 - a/4] \\ a + 16\frac{1-a}{(1+a)^2}(1/4 + a/4 - x) & \text{if } x < 1/4 + a/4 \\ a + 16\frac{1-a}{(1+a)^2}(x - 3/4 + a/4) & \text{if } x > 3/4 - a/4. \end{cases}$$

For $a = 1$, we obtain the uniform distribution. For $a = 0$ the density is linearly decreasing on $[0, 1/4)$; then zero on $[1/4, 3/4]$ and finally linearly increasing on $(3/4, 1]$. For a in between zero and one, the density is linearly decreasing on $[0, 1/4 + a/4)$; then constant and equal to a on $[1/4 + a/4, 3/4 - a/4]$ and finally linearly increasing on $(3/4 - a/4, 1]$. The density is clearly symmetric around 1/2 for any given $a \in [0, 1]$.

Now let the transportation cost parameter t be fixed at a sufficiently high value, that is, a value such that a consumer located at 1/4 would never buy from firm B.[10] Note that due to the assumption that transportation costs have to be incurred even when remaining uninsured, a high t does not lead to local monopolies! That is, consumers around $x = 1/2$ still buy insurance coverage. We will now argue that the oligopoly model converges to the monopoly model as $a \to 0$. Intuitively, $a \to 0$ implies that there are less and less consumers for which firms compete: By the assumption of a sufficiently high t, consumers located in $[0, 1/4]$ and $[3/4, 1]$ are "captured" in the sense that they will always buy from the closest insurer (or remain uninsured). Hence, the mass of consumers for which insurers compete is bounded form above by $F_a(3/4) - F_a(1/4) = \int_{1/4}^{3/4} f_a(x)\, dx = a/2 + 2\frac{1-a}{2(1+a)^2}a^2 = a/2 + \frac{1-a}{(1+a)^2}a^2$, which continuously approaches zero as $a \to 0$.

We can split up the profits of insurer A in profits from customers located in $[0, 1/4]$ and customers located further away. Note that the profit maximization problem for consumers in $[0, 1/4]$ (i.e. if we ignore all other consumers) is the same as our monopoly problem in the paper. Due to the assumption that transportation costs are also incurred when remaining uninsured, these consumers' contract and effort choice is the same as in the monopoly setup. Consequently, the insurer's profit maximization problem is a convex combination of a monopoly problem and a competitive problem where the weight on the monopoly problem is continuous in a and approaching 1 as $a \to 0$. In the generic case where the monopoly problem has a unique solution, this implies that the optimal duopoly menu must converge to the monopoly menu as $a \to 0$.[11]

The previous discussion also gives a second limit result. Now, let the distribution of consumers be uniform (or any other fixed distribution with positive density and no masspoint at 1/2). We can show that the duopoly problem converges to the monopoly problem as the transportation cost parameter t grows. Note that a consumer located at $x < 1/2$ will never buy from insurer B if $2t(1/2 - x) > u(w - \theta^l D) - [(1 - \theta^h)u(w) + \theta^h u(w - D)]$. The left-hand side of this inequality denotes the additional transportation cost incurred when buying from insurer B instead of buying from A. The right-hand side is an upper bound on the value of insurance a consumer might buy (see footnote 10). Consequently,

[10] A sufficient condition for this would be $t/2 > u(w - \theta^l D) - [(1 - \theta^h)u(w) + \theta^h u(w - D)]$ as the left hand side denotes the additional transportation cost of a consumer at $x = 1/$ when buying from B instead of A and the right hand side of this inequality is an upper bound on the equilibrium value of insurance: A high risk type has the highest value of insurance and the best contract he could get would be full coverage at a price that just breaks even for a low risk type.

[11] A proof by contradiction of this claim is standard and sketched here: Suppose otherwise and take a sequence of equilibrium menus (each menu corresponding to some a) not converging to the monopoly menu as $a \to 0$. As the space of menus is bounded (i.e. $R_i \in [0, D]$ and $p_i \in [0, R_i]$), it is compact and therefore this sequence must have a converging subsequence. At the limit of this subsequence, profits are discretely lower than in the optimal monopoly menu. But this implies that the monopoly menu would yield higher profits than the menus in the subsequence for a low enough (by continuity of the profit function in a). This contradicts the optimality of the menus in the (sub-)sequence.

all consumers located at $x < 1/2 - \{u(w - \theta^l D) - [(1-\theta^h)u(w) + \theta^h u(w-D)]\}/(2t)$ are "captured" by insurer A. Similarly, all consumers located at $x > 1/2 + \{u(w-\theta^l D) - [(1-\theta^h)u(w) + \theta^h u(w-D)]\}/(2t)$ are captured by insurer B. This implies that more and more consumers are captured as t grows large and the mass of consumers for which insurers compete shrinks to zero as $t \to \infty$. The same argument as above implies that the duopoly equilibrium menu converges to the optimal monopoly menu as $t \to \infty$. Hence, for large but finite t, the equilibrium menu of the oligopoly model is close to the optimal menu in the monopoly model. Recall again that in this model consumers do not refrain from buying as t grows large, i.e. consumers at all locations will still buy, since transportation costs have to be incurred even when remaining uninsured by assumption.

The previous results show that for a certain set of parameters — e.g., for large but finite t — the equilibrium menu is close to the optimal monopoly menu. Consequently, consumer surplus will also be close to consumer surplus in the monopoly model.[12] This implies that a figure as figure 4 in the paper which plots consumer surplus against γ would look similar (given that t is sufficiently large). In particular, consumer surplus has to jump up at some critical γ level where the equilibrium contract switches from separating to pooling (for those utility, cost functions and parameters used in this example). Consumers might therefore lose from a reduction in information gathering costs also in the oligopoly model; that is, we obtain the same possibility result for the oligopoly model as in the paper: For some parameter values consumer surplus is lower if information gathering costs are reduced through γ.

10. Alternative setup with pecuniary cost of effort

10.1. Introduction

This section explores an alternative setup in which the cost of effort is monetary. More precisely, the expected utility of an agent with perceived risk $\tilde{\beta}$ who exerts effort e and buys an insurance contract (p, R) is given by

$$\tilde{\beta}u(w - p - D + R - e) + (1 - \tilde{\beta})u(w - p - e).$$

That is, here the cost of effort enters in the argument of the utility function as a monetary cost, instead of being additively separable as in the paper. The purpose of this section is to argue that the results in the paper are not driven by the additive structure we assume there. In particular, we will show that with CARA preferences and given certain technical assumptions (to be specified in condition S1 below),[13] a result that is the equivalent of proposition 1 in the paper holds. We will also outline proofs for why also the other propositions and lemmas stated in the paper hold in the alternative setting studied here.

The ex ante utility of an agent exerting effort e and buying insurance contract (p_h, R_h) when receiving signal σ^h and buying insurance contract (p_l, R_l) when receiving signal σ^l is

$$\begin{aligned} U(e) &= \alpha_h \left[\beta_h(e)u(w - p_h - D + R_h - e) + (1 - \beta_h(e))u(w - p_h - e)\right] \\ &\quad + \alpha_l \left[\beta_l(e)u(w - p_l - D + R_l - e) + (1 - \beta_l(e))u(w - p_l - e)\right] \\ &= \alpha_h \left[\beta^h(e)\underline{u}_h(e) + (1 - \beta_h(e))\bar{u}_h(e)\right] + \alpha_l \left[\beta_l(e)\underline{u}_l(e) + (1 - \beta_l(e))\bar{u}_l(e)\right] \qquad (17) \end{aligned}$$

[12] We can abstract from the equilibrium transportation costs here as they will — in symmetric equilibria — not vary in γ anyway.

[13] The technical assumptions are satisfied, for example, if we assume either one of the signaling technologies described in the paper (in the model description and in Appendix B, respectively).

where we define $\bar{u}_i$ and $\underline{u}_i$ analogously to the paper. However, these variables are now functions of e and are no longer fixed by the offered contract. The rest of the setup — e.g., timing of events and assumptions on u — remains unchanged.

We will solve this new model by backwards induction.

10.2. Contract choice

The contract choice problem of the agent does not really change, relative to the analysis in the paper, because the effort choice was already determined previously. Hence, $\bar{u}_i(e)$ and $\underline{u}_i(e)$ are constant with respect to e at this point in time. Denoting by $\tilde{\beta}$ the agents perceived probability of suffering the loss D, we get the following.

The agent will remain uninsured if

$$\tilde{\beta} \leq \min\left(\frac{\bar{u}_0(e) - \bar{u}_l(e)}{\Delta_0(e) - \Delta_l(e)}, \frac{\bar{u}_0(e) - \bar{u}_h(e)}{\Delta_0(e) - \Delta_h(e)}\right).$$

The agent buys contract l if

$$\frac{\bar{u}_0(e) - \bar{u}_l(e)}{\Delta_0(e) - \Delta_l(e)} \leq \tilde{\beta} \leq \frac{\bar{u}_l(e) - \bar{u}_h(e)}{\Delta_l(e) - \Delta_h(e)}.$$

In the remaining cases, that is, when $\tilde{\beta}$ is high enough, the agent will buy the high-coverage contract h.

The notation is as in the paper: $\Delta_i(e) = \bar{u}_i(e) - \underline{u}_i(e)$ and a subscript 0 denotes the outside option of remaining uninsured (formally, $p_0 = R_0 = 0$). Furthermore, as in the paper, we let contract h be the contract with more coverage, i.e. $p_l \leq p_h$ and $R_l \leq R_h$. This implies $\Delta_h(e) \leq \Delta_l(e)$ for any $e \geq 0$.

10.3. Effort choice

We write utility as function of effort as

$$U(e) = \alpha_h\left[\beta_h(e)\underline{u}(\sigma^h, e) + (1 - \beta_h(e))\bar{u}(\sigma^h, e)\right] + \alpha_l\left[\beta_l(e)\underline{u}(\sigma^l, e) + (1 - \beta_l(e))\bar{u}(\sigma^l, e)\right], \qquad (18)$$

where $\underline{u}(\sigma^i, e)$ is the utility an agent gets in the case of an accident and when buying the contract an agent with signal σ^i optimally buys after exerting effort e. Similarly, $\bar{u}(\sigma^i, e)$ is the utility an agent will get in the no accident case when he bought the contract an agent should optimally buy after exerting effort e and receiving signal σ^i.

If an agent buys contract h after receiving a high signal and contract l after receiving a low signal, his first-order condition in the effort choice problem is

$$\begin{aligned}\alpha_h\beta_h{}'(e)\left(\Delta_l(e) - \Delta_h(e)\right) &+ \alpha_h\left(\beta_h(e)\underline{u}_h'(e) + (1 - \beta_h(e))\bar{u}_h'(e)\right)\\ &+ \alpha_l\left(\beta_l(e)\underline{u}_l'(e) + (1 - \beta_l(e))\bar{u}_l'(e)\right) \leq 0 \quad \text{with “ = ” if } e > 0. \qquad (19)\end{aligned}$$

Note that $\underline{u}_i'(e) < 0$ and $\bar{u}_i'(e) < 0$ as higher effort costs reduce disposable income. Consequently, the first term is positive while the second and third terms are negative. The second-order condition of this maximization problem is

$$\begin{aligned}\alpha_h\beta_h{}''(e)\left(\Delta_l(e) - \Delta_h(e)\right) &+ 2\alpha_h\beta_h{}'(e)\left(\Delta_l'(e) - \Delta_h'(e)\right)\\ &+ \alpha_h\left(\beta_h(e)\underline{u}_h''(e) + (1 - \beta_h(e))\bar{u}_h''(e)\right) + \alpha_l\left(\beta_l(e)\underline{u}_l''(e) + (1 - \beta_l(e))\bar{u}_l''(e)\right) \leq 0. \qquad (20)\end{aligned}$$

Because of the concavity of u, the second line is negative. The first term is negative by $\beta_h{}'' \leq 0$, which we used in the paper. But the second term is positive by the strict concavity of u. This implies that

— in contrast to the setup in the paper — there could be multiple local maxima with $e > 0$. Whether this complication occurs depends on the properties of the signal technology and the utility function.

The possible existence of multiple interior local maxima complicates the analysis considerably. Some of the proofs in the paper still go through but for some we need more structure. The goal of this section is to illustrate that similar effects and results as the one described in the paper also exist if the effort cost is monetary. Therefore, we do not attempt a full-fledged analysis but restrict ourselves to a more structured environment in which all our results will go through. Nevertheless, we will point out in the remainder where additional complications would arise if we analyzed a more general framework. The more structured environment is given by the following condition.

Condition S1. *The utility function u has constant absolute risk aversion, i.e. $u(x) = -\mathbf{e}^{-\eta x}$ for some $\eta > 0$ (where $\mathbf{e}$ is the Euler number 2.718...). The signal technology is such that ${\beta_h}'(e) + \eta\beta_h(e)$ is strictly quasiconvex.*

The following lemma states that condition S1 solves the problem of multiple local interior maxima. It also shows that reasonable signal technologies satisfy the requirement in condition S1.

Lemma S2. *Under condition S1, there exists at most one $e > 0$ that satisfies (19) and (20). For both example technologies used in the paper, ${\beta_h}'(e) + \eta\beta_h(e)$ is strictly quasiconvex.*

Proof. With CARA preferences the first-order condition (19) can be written as (after multiplying through with $\mathbf{e}^{-\eta(-w+e)}$)

$$\alpha_h {\beta_h}'(e)\left(-\mathbf{e}^{-\eta(-p_l)} + \mathbf{e}^{-\eta(-p_l-D+R_l)} + \mathbf{e}^{-\eta(-p_h)} - \mathbf{e}^{-\eta(-p_h-D+R_h)}\right) - \eta\alpha_h\beta_h(e)\mathbf{e}^{-\eta(-p_h-D+R_h)}$$
$$- \eta\alpha_h(1-\beta_h(e))\mathbf{e}^{-\eta(-p_h)} - \eta\alpha_l\beta_l(e)\mathbf{e}^{-\eta(-p_l-D+R_l)} - \eta\alpha_l(1-\beta_l(e))\mathbf{e}^{-\eta(-p_l)} = 0.$$

Using $\beta = \alpha_h\beta_h(e) + \alpha_l\beta_l(e)$ and $1-\beta = \alpha_h(1-\beta_h(e)) + \alpha_l(1-\beta_l(e))$, this can be rewritten as

$$\alpha_h\left({\beta_h}'(e) + \eta\beta_h(e)\right)\left(-\mathbf{e}^{-\eta(-p_l)} + \mathbf{e}^{-\eta(-p_l-D+R_l)} + \mathbf{e}^{-\eta(-p_h)} - \mathbf{e}^{-\eta(-p_h-D+R_h)}\right)$$
$$= \eta\alpha_h\left(\mathbf{e}^{-\eta(-p_h)} - \mathbf{e}^{-\eta(-p_l)}\right) + \eta\beta\mathbf{e}^{-\eta(-p_l-D+R_l)} + \eta(1-\beta)\mathbf{e}^{-\eta(-p_l)}. \quad (21)$$

The only term including effort in the last equation is $\left({\beta_h}'(e) + \eta\beta_h(e)\right)$. All other terms are constant in the effort choice stage. By assumption, $\left({\beta_h}'(e) + \eta\beta_h(e)\right)$ is strictly quasiconvex. Hence, the first-order condition is satisfied with equality at at most two positive effort levels. Since $U(e)$ is differentiable at every $e > 0$, this implies that there can be at most one maximum with $e > 0$.[14]

For the linear signal technology used in the numerical example of the paper, we have ${\beta_h}'(e) + \eta\beta_h(e) = e(\theta^h - \theta^l)\alpha_l\eta + (1-\alpha_h(1-\eta))\theta^h - \alpha_l(1-\eta)\theta^l$, which is linear and therefore strictly quasiconvex in e.

For the signal technology given as an example in the model section of the paper, we have

$$\eta{\beta_h}'(e) + {\beta_h}''(e) = \frac{\alpha_l(\theta^h - \theta^l)}{(e+1)^2}\left(\eta - \frac{2}{e+1}\right),$$

which is strictly negative for $e < \frac{2-\eta}{\eta}$ and strictly positive for $e > \frac{2-\eta}{\eta}$. Hence, ${\beta_h}'(e) + \eta\beta_h(e)$ is strictly quasiconvex. □

[14] The possible second solution of the first-order condition has to be a local minimum because between any two local maxima, there has to be a local minimum. Also note that $\lim_{e\to\infty} U(e) = -\infty$ which implies that U has a maximum.

10.4. Contract design

10.4.1. Constraints in contract design

The constraints in the principal's profit maximization problem are the same as in the paper. With monetary disutility of effort these constraints read

$$\alpha_h(\beta_h(e)\underline{u}_h(e) + (1-\beta_h(e))\bar{u}_h(e)) + \alpha_l(\beta_l(e)\underline{u}_l(e) + (1-\beta_l(e))\bar{u}_l(e)) \\ \geq \beta\underline{u}_l(0) + (1-\beta)\bar{u}_l(0), \quad (\mathrm{IG_l})$$

$$\alpha_h(\beta_h(e)\underline{u}_h(e) + (1-\beta_h(e))\bar{u}_h(e)) + \alpha_l(\beta_l(e)\underline{u}_l(e) + (1-\beta_l(e))\bar{u}_l(e)) \\ \geq \beta\underline{u}_h(0) + (1-\beta)\bar{u}_h(0), \quad (\mathrm{IG_h})$$

$$\alpha_h\left(\beta_h(e)\underline{u}_h(e) + (1-\beta_h(e))\bar{u}_h(e)\right) + \alpha_l\left(\beta_l(e)\underline{u}_l(e) + (1-\beta_l(e))\bar{u}_l(e)\right) \\ \geq \alpha_h\left(\beta_h(e^h)\underline{u}_h(e^h) + (1-\beta_h(e^h))\bar{u}_h(e^h)\right) + \left(\alpha_l\left(\beta_l(e^h)\underline{u}_0(e^h) + (1-\beta_l(e^h))\bar{u}_0(e^h)\right)\right), \quad (\mathrm{EH})$$

$$\beta_l(e)\underline{u}_l(e) + (1-\beta_l(e))\bar{u}_l(e) \geq \beta_l(e)\underline{u}_0(e) + (1-\beta_l(e))\bar{u}_0(e), \quad (\mathrm{IR_l})$$

$$\beta_h(e)\underline{u}_h(e) + (1-\beta_h(e))\bar{u}_h(e) \geq \beta_h(e)\underline{u}_l(e) + (1-\beta_h(e))\bar{u}_l(e), \quad (\mathrm{IC_h})$$

where e denotes the optimal effort given that the agent buys contract h in case of signal σ^h and contract l in case of signal σ^l. Effort e^h denotes the optimal effort when the agent buys contract h in case of signal σ^h and no insurance in case of signal σ^l.

In the paper, we could neglect the following deviation of the agent: Exert effort e^l and buy contract l if the signal is σ^h and do not buy insurance if the signal is σ^l. To illustrate, let us assume $e^l < e$. Then, this deviation was ruled out by ($\mathrm{IR_l}$). Because $\beta^l(e) < \beta^l(e^l)$ an agent with a low signal would find it optimal to buy contract l instead of remaining uninsured after exerting effort e^l. In the monetary cost framework, this is no longer straightforward. The problem is that the utilities depend on e. If the utility function exhibits strongly decreasing absolute risk aversion and e^l is much lower than e, a σ^l agent might find it better to stay uninsured after exerting effort e^l. The reason is that exerting an effort much below e makes him richer and therefore less risk averse. For this reason, we have to introduce the additional constraint

$$\alpha_h\left(\beta_h(e)\underline{u}_h(e) + (1-\beta_h(e))\bar{u}_h(e)\right) + \alpha_l\left(\beta_l(e)\underline{u}_l(e) + (1-\beta_l(e))\bar{u}_l(e)\right) \\ \geq \alpha_h\left(\beta_h(e^l)\underline{u}_l(e^l) + (1-\beta_h(e^l))\bar{u}_l(e^l)\right) + \alpha_l\left(\beta_l(e^l)\underline{u}_0(e^l) + (1-\beta_l(e^l))\bar{u}_0(e^l)\right) \quad (\mathrm{EL})$$

when we want to deal with the general framework. However, it is straightforward that condition S1 allows us to use the same reasoning as in the paper to rule out that (EL) binds. Wealth effects can be neglected with CARA preferences.

For similar reasons, we also have to track the ex ante individual rationality constraint

$$\alpha_h(\beta_h(e)\underline{u}_h(e) + (1-\beta_h(e))\bar{u}_h(e)) + \alpha_l(\beta_l(e)\underline{u}_l(e) + (1-\beta_l(e))\bar{u}_l(e)) \\ \geq \beta\underline{u}_0(0) + (1-\beta)\bar{u}_0(0). \quad (\mathrm{EAIR})$$

In the setting of the paper, this constraint was implied by ($\mathrm{IR_l}$) and ($\mathrm{IG_l}$): Given that a low-signal agent preferred contract l to the outside option, also an agent exerting zero effort (and therefore having a risk higher than the low-signal agent) preferred contract l to the outside option. Again this is no longer clear in the monetary effort cost setup: Risk aversion after exerting effort $e > 0$ could be different from risk aversion when not exerting effort. Therefore, the preferences over contract l and contract 0 could depend on the level of exerted effort. With CARA preferences the reasoning from the paper goes through as risk aversion does not depend on exerted effort.

As in the paper, some constraints are implied by others. First we present an intermediate result for which we do not have to use condition S1.

Lemma S3. *(IC_h) is implied by (IG_l). (IR_l) is generically implied by (EH) (and it is always implied under condition S1).*

Proof. First consider the claim that ($\mathrm{IC_h}$) is implied by ($\mathrm{IG_l}$). Using $\beta = \alpha_h\beta_h(e) + \alpha_l\beta_l(e)$ and $1-\beta = \alpha_h(1-\beta_h(e)) + \alpha_l(1-\beta_l(e))$, ($\mathrm{IG_l}$) can be written as

$$\alpha_h\beta_h(e)\left(\underline{u}_h(e) - \underline{u}_l(0)\right) + \alpha_h(1-\beta_h(e))\left(\bar{u}_h(e) - \bar{u}_l(0)\right) \\ + \alpha_l\beta_l(e)\left(\underline{u}_l(e) - \underline{u}_l(0)\right) + \alpha_l(1-\beta_l(e))\left(\bar{u}_l(e) - \bar{u}_l(0)\right) \geq 0.$$

Since the terms on the second line are negative ($\underline{u}_l(e) < \underline{u}_l(0)$ and $\bar{u}_l(e) < \bar{u}_l(0)$), the terms on the first line must be positive:

$$\alpha_h\beta_h(e)\left(\underline{u}_h(e) - \underline{u}_l(0)\right) + \alpha_h(1-\beta_h(e))\left(\bar{u}_h(e) - \bar{u}_l(0)\right) > 0.$$

Canceling α_h and using $\underline{u}_l(e) < \underline{u}_l(0)$ as well as $\bar{u}_l(e) < \bar{u}_l(0)$ gives

$$\beta_h(e)\left(\underline{u}_h(e) - \underline{u}_l(e)\right) + (1-\beta_h(e))\left(\bar{u}_h(e) - \bar{u}_l(e)\right) > 0,$$

which implies ($\mathrm{IC_h}$).

The second part of the proof is by contradiction. Suppose ($\mathrm{IR_l}$) was binding. Then (EH) could be written as $f(e) \geq f(e^h)$, where

$$f(e) \equiv\ \alpha_h\left(\beta_h(e)\underline{u}_h(e) + (1-\beta_h(e))\bar{u}_h(e)\right) + \alpha_l\left(\beta_l(e)\underline{u}_0(e) + (1-\beta_l(e))\bar{u}_0(e)\right).$$

Note that e^h is defined as the maximizer of f and therefore $f(e^h) \geq f(e)$. Hence, we would get that e and e^h are both global maximizers of f. Since e is derived from the effort choice problem with contracts h and l, i.e. from (19) and (20), in which $\bar{u}_0$ and $\underline{u}_0$ do not play a role, e will generically not be a global maximizer of f (contradiction in a generic sense!). With the structure imposed by condition S1 an even stronger result is obtained: It follows from condition S1 that f can only have one interior maximizer which is e^h. Hence, we get a contradiction. □

Lemma S4. *Assume condition S1 holds. Then (EAIR) and (EL) is implied by other constraints and (IG_h) is slack in the optimal contract menu. (IG_l) and (EH) are binding in the optimal contract menu.*

Proof. First consider the claim that (EAIR) is implied by other constraints. ($\mathrm{IR_l}$) and $\beta > \beta_l(e)$ imply

$$\beta(\underline{u}_l(e) - \underline{u}_0(e)) + (1-\beta)(\bar{u}_l(e) - \bar{u}_0(e)) \geq 0.$$

Using CARA preferences and multiplying through by $\mathbf{e}^{-\eta e}$, gives

$$\beta(-\mathbf{e}^{-\eta(w-p_l-D+R_l)} + \mathbf{e}^{-\eta(w-D)}) + (1-\beta)(-\mathbf{e}^{-\eta(w-p_l)} + \mathbf{e}^{-\eta w}) \geq 0.$$

This is equivalent to

$$\beta\underline{u}_l(0) + (1-\beta)\bar{u}_l(0) \geq \beta\underline{u}_0(0) + (1-\beta)\bar{u}_0(0).$$

Together with ($\mathrm{IG_l}$), the last equation implies (EAIR).

To show that (EL) is slack, we distinguish two cases. First, $e^l \leq e$. Then multiplying ($\mathrm{IR_l}$) through with $\mathbf{e}^{-\eta e}$ yields

$$\beta_l(e)(-\mathbf{e}^{-\eta(w-p_l-D+R_l)} + \mathbf{e}^{-\eta(w-D)}) + (1-\beta_l(e))(-\mathbf{e}^{-\eta(w-p_l)} + \mathbf{e}^{-\eta w}) \geq 0.$$

Using $\beta_l(e) \leq \beta_l(e^l)$ and multiplying through with $\mathbf{e}^{\eta e^l}$ gives

$$\beta_l(e^l)(-\mathbf{e}^{-\eta(w-p_l-D+R_l-e^l)} + \mathbf{e}^{-\eta(w-D-e^l)}) + (1-\beta_l(e^l))(-\mathbf{e}^{-\eta(w-p_l-e^l)} + \mathbf{e}^{-\eta(w-e^l)}) \geq 0.$$

Therefore,

$$\beta_l(e^l)\underline{u}_l(e^l) + (1-\beta_l(e^l))\bar{u}_l(e^l) \geq \beta_l(e^l)\underline{u}_0(e^l) + (1-\beta_l(e^l))\bar{u}_0(e^l).$$

Hence, the right-hand side of (EL) satisfies the following:

$$\begin{aligned}
& \alpha_h\left(\beta_h(e^l)\underline{u}_l(e^l) + (1-\beta_h(e^l))\bar{u}_l(e^l)\right) + \alpha_l\left(\beta_l(e^l)\underline{u}_0(e^l) + (1-\beta_l(e^l))\bar{u}_0(e^l)\right) \\
\leq\; & \alpha_h\left(\beta_h(e^l)\underline{u}_l(e^l) + (1-\beta_h(e^l))\bar{u}_l(e^l)\right) + \alpha_l\left(\beta_l(e^l)\underline{u}_l(e^l) + (1-\beta_l(e^l))\bar{u}_l(e^l)\right) \\
\leq\; & \beta\underline{u}_l(0) + (1-\beta)\bar{u}_l(0),
\end{aligned}$$

where the last inequality is strict for $e^l > 0$. Therefore (EL) is implied by ($\mathrm{IG_l}$).

Second, $e^l > e$. In this case, ($\mathrm{IC_h}$) multiplied through by $\mathbf{e}^{-\eta e}$ yields

$$\beta_h(e)(-\mathbf{e}^{-\eta(w-p_h-D+R_h)} + \mathbf{e}^{-\eta(w-p_l-D+R_l)}) + (1-\beta_h(e))(-\mathbf{e}^{-\eta(w-p_h)} + \mathbf{e}^{-\eta(w-p_l)}) \geq 0.$$

Using $\beta_h(e^l) > \beta_h(e)$ and multiplying through with $\mathbf{e}^{\eta e^l}$, gives

$$\beta_h(e^l)(-\mathbf{e}^{-\eta(w-p_h-D+R_h-e^l)} + \mathbf{e}^{-\eta(w-p_l-D+R_l-e^l)}) + (1-\beta_h(e^l))(-\mathbf{e}^{-\eta(w-p_h-e^l)} + \mathbf{e}^{-\eta(w-p_l-e^l)}) \geq 0.$$

Therefore, the right-hand side of (EL) satisfies the following:

$$\begin{aligned}
& \alpha_h\left(\beta_h(e^l)\underline{u}_l(e^l) + (1-\beta_h(e^l))\bar{u}_l(e^l)\right) + \alpha_l\left(\beta_l(e^l)\underline{u}_0(e^l) + (1-\beta_l(e^l))\bar{u}_0(e^l)\right) \\
\leq\; & \alpha_h\left(\beta_h(e^l)\underline{u}_h(e^l) + (1-\beta_h(e^l))\bar{u}_h(e^l)\right) + \alpha_l\left(\beta_l(e^l)\underline{u}_0(e^l) + (1-\beta_l(e^l))\bar{u}_0(e^l)\right) \\
\leq\; & \alpha_h\left(\beta_h(e^h)\underline{u}_h(e^h) + (1-\beta_h(e^h))\bar{u}_h(e^h)\right) + \alpha_l\left(\beta_l(e^h)\underline{u}_0(e^h) + (1-\beta_l(e^h))\bar{u}_0(e^h)\right),
\end{aligned}$$

where the last inequality follows from the definition of e^h as the effort level maximizing expected utility when deciding between contracts h and 0. Consequently, (EH) implies (EL).

Now we turn to the claim about ($\mathrm{IG_h}$). Suppose ($\mathrm{IG_h}$) was binding. We will show that the principal has a deviation pooling contract that satisfies all constraints and increases profits. Hence, the initial menu cannot be optimal. We distinguish two cases.

First, suppose $\bar{u}_h(e) > \underline{u}_h(e)$. In this case, we claim that profits can be increased by only offering contract h, i.e. by dropping contract l from the menu. Since ($\mathrm{IG_h}$) was binding, the agent can achieve the same utility as before by buying contract h without exerting effort. This must be his optimal choice as restricting his choice set cannot result in higher ex ante utility. In particular, (EAIR) and (EH) are not affected and ($\mathrm{IG_l}$) is irrelevant in the pooling situation. Now we have to show that profits are increased:

$$\begin{aligned}
\pi^{deviation} &= w - \beta D - \beta h(\underline{u}_h(0)) - (1-\beta)h(\bar{u}_h(0)) \\
&= w - \beta D - \alpha_h \left(\beta_h(e)h(\underline{u}_h(0)) + (1-\beta_h(e))h(\bar{u}_h(0))\right) \\
&\quad -\alpha_l \left(\beta_l(e)h(\underline{u}_h(0)) + (1-\beta_l(e))h(\bar{u}_h(0))\right) \\
&= w - \beta D - e - \alpha_h \left(\beta_h(e)h(\underline{u}_h(e)) + (1-\beta_h(e))h(\bar{u}_h(e))\right) \\
&\quad -\alpha_l \left(\beta_l(e)h(\underline{u}_h(e)) + (1-\beta_l(e))h(\bar{u}_h(e))\right) \\
&> w - \beta D - e - \alpha_h \left(\beta_h(e)h(\underline{u}_h(e)) + (1-\beta_h(e))h(\bar{u}_h(e))\right) \\
&\quad -\alpha_l \left(\beta_l(e)h(\underline{u}_l(e)) + (1-\beta_l(e))h(\bar{u}_l(e))\right) \\
&= \pi^{initial},
\end{aligned}$$

where the inequality follows from the following observation: We know that $\beta_l(e)\underline{u}_l(e)+(1-\beta_l(e))\bar{u}_l(e) \geq \beta_l(e)\underline{u}_h(e)+(1-\beta_l(e))\bar{u}_h(e)$. Let us assume for now that this inequality held with equality. Because h is strictly convex and $\underline{u}_l(e) < \underline{u}_h(e) < \bar{u}_h(e) < \bar{u}_l(e)$, the line connecting the points $(\underline{u}_h(e), h(\underline{u}_h(e)))$ and $(\bar{u}_h(e), h(\bar{u}_h(e)))$ is strictly below the line connecting the points $(\underline{u}_l(e), h(\underline{u}_l(e)))$ and $(\bar{u}_l(e), h(\bar{u}_l(e)))$. If — as we assumed for now — $\beta_l(e)\underline{u}_l(e) + (1-\beta_l(e))\bar{u}_l(e) = \beta_l(e)\underline{u}_h(e) + (1-\beta_l(e))\bar{u}_h(e)$, this implies that $\beta_l(e)h(\underline{u}_l(e)) + (1-\beta_l(e))h(\bar{u}_l(e)) > \beta_l(e)h(\underline{u}_h(e)) + (1-\beta_l(e))h(\bar{u}_h(e))$. If the inequality $\beta_l(e)\underline{u}_l(e)+(1-\beta_l(e))\bar{u}_l(e) \geq \beta_l(e)\underline{u}_h(e)+(1-\beta_l(e))\bar{u}_h(e)$ holds strictly, the inequality $\beta_l(e)h(\underline{u}_l(e))+(1-\beta_l(e))h(\bar{u}_l(e)) > \beta_l(e)h(\underline{u}_h(e)) + (1-\beta_l(e))h(\bar{u}_h(e))$ is satisfied even clearer.[15]

Second, suppose $\bar{u}_h(e) \leq \underline{u}_h(e)$. Then a full-coverage pooling contract that gives the same ex ante utility to the agent increases profits and is feasible. Let $u_p(0) = \alpha_h(\beta_h(e)\underline{u}_h(e) + (1-\beta_h(e))\bar{u}_h(e)) + \alpha_l(\beta_l(e)\underline{u}_l(e) + (1-\beta_l(e))\bar{u}_l(e))$ be the utility level of the full coverage deviation contract. The agent can achieve the same utility level as before by buying the pooling contract without exerting effort. Hence, (EAIR) is satisfied (and ($\mathrm{IG_l}$) is irrelevant in pooling). Because ($\mathrm{IG_h}$) was binding initially by assumption, we have $u_p(0) = \beta\underline{u}_h(0) + (1-\beta)\bar{u}_h(0)$. As $\beta_h(e') > \beta$ for any $e' > 0$ and $\bar{u}_h(0) \leq \underline{u}_h(0)$ by the assumption $\bar{u}_h(e) \leq \underline{u}_h(e)$ and CARA, it follows that $u_p(0) \leq \beta_h(e')\underline{u}_h(0) + (1-\beta_h(e'))\bar{u}_h(0)$. Using the CARA preferences and multiplying this inequality through with $\mathbf{e}^{\eta e'}$, leads to the inequality $u_p(e') \leq \beta_h(e')\underline{u}_h(e') + (1-\beta_h(e'))\bar{u}_h(e')$. Therefore, the right-hand side of (EH) is (weakly) lower in the deviation menu than in the initial menu for any effort level. Since the left-hand side of (EH) is unchanged by the definition of u_p, (EH) is (weakly) relaxed by the deviation. Finally, we show that profits are higher under the deviation contract:

$$\begin{aligned}
\pi^{deviation} &= w - \beta D - h(u_p(0)) \\
&= w - \beta D \\
&\quad -h\left(\alpha_h(\beta_h(e)\underline{u}_h(e) + (1-\beta_h(e))\bar{u}_h(e)) + \alpha_l(\beta_l(e)\underline{u}_l(e) + (1-\beta_l(e))\bar{u}_l(e))\right) \\
&> w - \beta D - \alpha_h \left(\beta_h(e)h(\underline{u}_h(e)) + (1-\beta_h(e))h(\bar{u}_h(e))\right) \\
&\quad -\alpha_l \left(\beta_l(e)h(\underline{u}_l(e)) + (1-\beta_l(e))\bar{u}_l(e)\right) \\
&\geq w - \beta D - e - \alpha_h \left(\beta_h(e)h(\underline{u}_h(e)) + (1-\beta_h(e))h(\bar{u}_h(e))\right) \\
&\quad -\alpha_l \left(\beta_l(e)h(\underline{u}_l(e)) + (1-\beta_l(e))\bar{u}_l(e)\right) \\
&= \pi^{initial}
\end{aligned}$$

where the first inequality follows from the strict convexity of h and the second inequality follows from

[15] Note that we did not use condition S1 for this part of the proof. Hence, the statement "($\mathrm{IG_h}$) cannot bind if $\bar{u}_h(e) > \underline{u}_h(e)$" holds in general.

$e \geq 0$.

Last we turn to the binding constraints. Until now we proved that only (IG_l) and (EH) can be binding. Suppose (IG_l) was not binding. Then marginally decreasing R_h increases profits and relaxes the only binding constraint (EH).[16] Similarly, suppose (EH) was slack. Then increasing both p_h and p_l by $\varepsilon > 0$ will increase profits without affecting the only binding constraint (IG_l) (this follows from the CARA preferences). It follows that both (IG_l) and (EH) must be binding. □

Before proceeding to optimal contracts, we still need to formally show that $e^h \geq e$, which is intuitively obvious but no longer as straightforward as in the paper as the optimal effort level depends not only on the Δ_is but also on utility levels.

Lemma S5. *Assume condition S1 holds. Then $e^h \geq e$.*

Proof. The proof is by contradiction. Suppose $e^h < e$. By (IR_l),

$$\beta_l(e)(-\mathbf{e}^{-\eta(w-p_l-D+R_l-e)} + \mathbf{e}^{-\eta(w-D-e)}) + (1-\beta_l(e))(-\mathbf{e}^{-\eta(w-p_l-e)} + \mathbf{e}^{-\eta(w-e)}) \geq 0.$$

Multiplying through with $\mathbf{e}^{-\eta(e-e^h)}$ and using $\beta_l(e^h) > \beta_l(e)$ gives

$$\beta_l(e^h)(-\mathbf{e}^{-\eta(w-p_l-D+R_l-e^h)} + \mathbf{e}^{-\eta(w-D-e^h)}) + (1-\beta_l(e^h))(-\mathbf{e}^{-\eta(w-p_l-e^h)} + \mathbf{e}^{-\eta(w-e^h)}) > 0,$$

which is equivalent to

$$\beta_l(e^h)\underline{u}_l(e^h) + (1-\beta_l(e^h))\bar{u}_l(e^h) > \beta_l(e^h)\underline{u}_0(e^h) + (1-\beta_l(e^h))\bar{u}_0(e^h). \tag{22}$$

By Lemma S4, (EH) is binding. Therefore,

$$\begin{aligned}&\alpha_h\left(\beta_h(e)\underline{u}_h(e) + (1-\beta_h(e))\bar{u}_h(e)\right) + \alpha_l\left(\beta_l(e)\underline{u}_l(e) + (1-\beta_l(e))\bar{u}_l(e)\right)\\ &\quad = \alpha_h\left(\beta_h(e^h)\underline{u}_h(e^h) + (1-\beta_h(e^h))\bar{u}_h(e^h)\right) + \alpha_l\left(\beta_l(e^h)\underline{u}_0(e^h) + (1-\beta_l(e^h))\bar{u}_0(e^h)\right).\end{aligned}$$

If we define g as

$$g(e) \equiv \alpha_h\left(\beta_h(e)\underline{u}_h(e) + (1-\beta_h(e))\bar{u}_h(e)\right) + \alpha_l\left(\beta_l(e)\underline{u}_l(e) + (1-\beta_l(e))\bar{u}_l(e)\right),$$

then the last equation yields $g(e) < g(e^h)$ because of (22). This contradicts the definition of e as the effort level maximizing the left-hand side of this inequality. □

10.4.2. Optimal contract

The main result we want to show here is that both contracts have less than full coverage.

Proposition S4. *Assume condition S1. The optimal contract menu includes only partial coverage contracts, i.e. $\bar{u}_h(e) - \underline{u}_h(e) > 0$.*

Proof. We first prove the proposition for a separating equilibrium. Suppose $R_h \geq D$, i.e. (more than) full coverage. We will show that the principal has a profitable deviation in this case. Using the implicit function theorem on (21), we get

$$\begin{aligned}\frac{de}{dp_h} &= \frac{-\eta^2\alpha_h\mathbf{e}^{\eta p_h} + \alpha_h\eta(\beta_h{}'(e) + \eta\beta_h(e))(\mathbf{e}^{\eta p_h} - \mathbf{e}^{\eta(p_h+D-R_h)})}{-SOC},\\ \frac{de}{dR_h} &= \frac{\eta\alpha_h\mathbf{e}^{\eta(p_h+D-R_h)}\left(\eta\beta_h(e) + \beta_h{}'(e)\right)}{-SOC} > 0,\end{aligned}$$

[16] This assumes $e^h > e$ as one should expect. As we did not prove this: If $e^h \leq e$, increasing R_h and p_h by $\varepsilon > 0$ increases profits and relaxes (EH).

where SOC is the derivative of (21) with respect to e, which must be negative by the second-order condition. Now consider changing contract h marginally while keeping contract l fixed. The principal's profits are $\alpha_h p_h + \alpha_l p_l - \alpha_h \beta_h(e) R_h - \alpha_l \beta_l(e) R_l$. The slope of the principal's isoprofit curve in the (R_h, p_h) plane is

$$\left.\frac{dp_h}{dR_h}\right|_{\pi=const} = \frac{\beta_h(e) + \frac{de}{dR_h}\beta_h{}'(e)\,(R_h - R_l)}{1 - \frac{de}{dp_h}\beta_h{}'(e)\,(R_h - R_l)} > \beta_h(e),$$

where the last inequality follows from the following steps: $1-\beta_h(e)-(\beta_h'(e))/\eta < 0$ because rearranging (21) gives[17]

$$\begin{aligned}
\beta_h(e) + \frac{\beta_h'(e)}{\eta} &= \frac{\alpha_h\left(\mathbf{e}^{-\eta(-p_h)} - \mathbf{e}^{-\eta(-p_l)}\right) + \beta\mathbf{e}^{-\eta(-p_l-D+R_l)} + (1-\beta)\mathbf{e}^{-\eta(-p_l)}}{-\alpha_h\mathbf{e}^{-\eta(-p_l)} + \alpha_h\mathbf{e}^{-\eta(-p_l-D+R_l)} + \alpha_h\mathbf{e}^{-\eta(-p_h)} - \alpha_h\mathbf{e}^{-\eta(-p_l-D+R_l)}} \\
&\geq \frac{\alpha_h\left(\mathbf{e}^{-\eta(-p_h)} - \mathbf{e}^{-\eta(-p_l)}\right) + \mathbf{e}^{-\eta(-p_l-D+R_l)}}{\alpha_h\left(\mathbf{e}^{-\eta(-p_h)} - \mathbf{e}^{-\eta(-p_l)}\right) + \alpha_h\mathbf{e}^{-\eta(-p_l-D+R_l)} - \alpha_h\mathbf{e}^{-\eta(-p_h-D+R_h)}} \\
&> \frac{\alpha_h\left(\mathbf{e}^{-\eta(-p_h)} - \mathbf{e}^{-\eta(-p_l)}\right) + \mathbf{e}^{-\eta(-p_l-D+R_l)}}{\alpha_h\left(\mathbf{e}^{-\eta(-p_h)} - \mathbf{e}^{-\eta(-p_l)}\right) + \mathbf{e}^{-\eta(-p_l-D+R_l)}} = 1.
\end{aligned}$$

But then using the expressions above, we have

$$\begin{aligned}
0 &> \mathbf{e}^{\eta p_h}(R_h - R_l)\,(1 - \beta_h(e) - (\beta_h'(e))/\eta) \\
\Leftrightarrow \frac{de}{dR_h}\beta_h{}'(e)\,(R_h - R_l) &> -\frac{de}{dp_h}\beta_h{}'(e)\,(R_h - R_l)\,.
\end{aligned}$$

This gives $\left.\frac{dp_h}{dR_h}\right|_{\pi=const} > \beta_h(e)$.

Similarly, we derive the slope of the ex ante indifference curve of the agent. Using an envelope argument and canceling terms lead to

$$\left.\frac{dp_h}{dR_h}\right|_{U=const} = \frac{\beta_h(e)}{\beta_h(e) + (1-\beta_h(e))\mathbf{e}^{\eta(R_h-D)}} \leq \beta_h(e),$$

where the inequality follows from $R_h \geq D$.

This shows that reducing R_h by $\varepsilon > 0$ and p_h by $\varepsilon \left.\frac{dp_h}{dR_h}\right|_{U=const}$ will increase the principal's payoff while leaving the agent's payoff unaffected (for $\varepsilon > 0$ small enough). The binding constraint (IG_l) is not affected by this change while the constraint (EH) is even relaxed: The slope of the indifference curve (in (R_h, p_h)-space) of an agent exerting effort e^h is $\frac{\beta_h(e^h)}{\beta_h(e^h)+(1-\beta_h(e^h))\mathbf{e}^{\eta(R_h-D)}}$ which is steeper than the indifference curve of an agent exerting effort e as $\beta_h(e^h) \geq \beta_h(e)$. Hence, the deviation is feasible and profitable.

The proof for a pooling equilibrium is similar. In a pooling equilibrium, $e = 0$ and $\frac{de}{dp} = \frac{de}{dR} = 0$. For $R \geq D$, we consequently get

$$\left.\frac{dp}{dR}\right|_{\pi=const} = \beta \geq \frac{\beta}{\beta + (1-\beta)\mathbf{e}^{\eta(R-D)}} = \left.\frac{dp}{dR}\right|_{U=const}.$$

Furthermore, the slope of the indifference curve of an agent exerting effort $e^h > 0$ (and buying insurance only when receiving a high signal) is $\frac{\beta_h(e^h)}{\beta_h(e^h)+(1-\beta_h(e^h))\mathbf{e}^{\eta(R-D)}}$, which is strictly higher than $\frac{\beta}{\beta+(1-\beta)\mathbf{e}^{\eta(R-D)}}$ as $\beta_h(e^h) > \beta$. Consequently, reducing R_h by $\varepsilon > 0$ and p_h by $\frac{\varepsilon\beta}{\beta+(1-\beta)\mathbf{e}^{\eta(R-D)}}$ will (i) keep the utility of the agent constant, (ii) (weakly) increase the principal's profit and (iii) strictly relax

[17] The step from the first to the second line assumes that $D \geq R_l$, i.e., that contract l provides at most full coverage. It is easy to show that this must be true: If both contracts had more than full coverage, the insurer could make higher profits by offering a full coverage pooling contract that gives the same expected utility to the agent as in the original situation. As coverage is lower in this deviation contract, (EH) is relaxed. As the agent's expected utility is the same, (IG_l) is unaffected. As the contract is more efficient (full coverage, no effort), it increases the insurer's profits.

the only binding constraint (EH). As the only binding constraint is strictly relaxed, decreasing R_h by slightly more than ε is still feasible and strictly increases the principal's payoff. $\square$

Given condition S1, we know that (IG_l) and (EH) are binding. This implies immediately proposition 2 in the paper.[18] Also the two binding constraints are again linear in utilities. The first-order conditions for e and e^h (as in (19)) are linear in utilities for CARA preferences. Therefore, it is straightforward to solve these four conditions for the utility levels $\bar{u}_h(e)$, $\underline{u}_h(e)$, $\bar{u}_l(e)$ and $\underline{u}_l(e)$. Using these expressions we can write the principal's maximization as a maximization problem over e and e^h under the constraint $e^h \geq e \geq 0$ which implies an equivalent of proposition A1 in the appendix of the paper. If we use γe instead of e in the monetary effort cost setup and let γ approach 0, the Stiglitz menu results (see section 4 in this supplementary material and our discussion in section 2.1 in the paper). This is true as the proof in the paper only relies on continuity of the utility/cost function which is unproblematic in the monetary effort cost framework.

[18] In the monetary effort cost setup, we would use $-\gamma e$ instead of $-e$ in the argument of the utility function to parameterize costs.

References

Boone, J. and C. Schottmüller, "Health Insurance without Single Crossing: Why Healthy People Have High Coverage," *Economic Journal* (forthcoming).

Rothschild, M. and J. Stiglitz, "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information," *Quarterly Journal of Economics* 90 (1976), 629–649.

Stiglitz, J. E., "Monopoly, Non-Linear Pricing and Imperfect Information: The Insurance Market," *Review of Economic Studies* 44 (1977), 407–430.