Supplementary Material to "Facilitating Consumer Learning in Insurance Markets—What Are the Welfare Effects?"*

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In this supplementary material, which is not meant to be published, we provide some proofs that were omitted from our paper "Facilitating Consumer Learning in Insurance Markets—What Are the Welfare Effects?" (forthcoming in the *Scandinavian Journal of Economics*). We also investigate the robustness of the results in our paper with respect to the assumption that the information acquisition cost enters the agent's payoff additively.

Section 1 proves Proposition 1 and the claim in the end of Section III of our paper that $\pi^{NI} > \pi^{SZ}_{max}$. Section 2 derives equation (12) in the paper. In Section 3 we study an alternative setup in which the information acquisition cost enters the argument of the agent's utility function. Assuming CARA preferences, we show that the results of that model are similar to the results in the paper.

1. Proof of Propositions 1 and the claim that $\pi^{NI} > \pi^{SZ}_{max}$

We first restate and prove Proposition 1 from the paper (note that this version of the proposition contains a couple of additional results).

Proposition 1 (The Stiglitz model). *Consider the Stiglitz model and suppose that P optimally interacts with both agent types. Then, at the optimum, the high-risk type is fully insured ($\overline{u}^{SZ}_A = \overline{u}^{SZ}_N \equiv \overline{u}^{SZ}$) and the low-risk type is underinsured ($\underline{u}^{SZ}_A < \underline{u}^{SZ}_N$). The ex post utility levels at the optimum ($\underline{u}^{SZ}_A$, $\underline{u}^{SZ}_N$, and $\overline{u}^{SZ}$) are implicitly defined by the two binding constraints (IR-low and IC-high) and by the equality*

$$\upsilon\underline{\theta}\left(1-\underline{\theta}\right)\left[h'\left(\underline{u}^{SZ}_N\right)-h'\left(\underline{u}^{SZ}_A\right)\right]=\left(1-\upsilon\right)\left(\overline{\theta}-\underline{\theta}\right)h'\left(\overline{u}^{SZ}\right). \quad (1)$$

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Moreover, we have the following relationships between the utility levels:

$$\underline{u}_A^{SZ} < \overline{u}^{SZ} < \underline{U}^* < \underline{u}_N^{SZ}, \tag{2}$$

$$(1-\underline{\theta})\,\underline{u}_N^{SZ} + \underline{\theta}\underline{u}_A^{SZ} = \underline{U}^* \quad \text{and} \quad \overline{U}^* < \overline{u}^{SZ}. \tag{3}$$

Proof. By Lemma A1 in the paper, we know that IR-high is implied by other constraints. We will also guess that IC-low is not binding at the optimum (and verify that later). The Lagrangian to P's problem can be written as

$$\begin{aligned}\mathcal{L} \;=\; & \widehat{w} - \upsilon\left[(1-\underline{\theta})\,h(\underline{u}_N) + \underline{\theta}h(\underline{u}_A)\right] - (1-\upsilon)\left[\left(1-\overline{\theta}\right)h(\overline{u}_N) + \overline{\theta}h(\overline{u}_A)\right] \\ & +\lambda\left[(1-\underline{\theta})\,\underline{u}_N + \underline{\theta}\underline{u}_A - \underline{U}^*\right] + \mu\left[\left(1-\overline{\theta}\right)\overline{u}_N + \overline{\theta}\overline{u}_A - \left(1-\overline{\theta}\right)\underline{u}_N - \overline{\theta}\underline{u}_A\right],\end{aligned}$$

where λ is the shadow price associated with IR-low and μ is the shadow price associated with IC-high.

The first-order condition with respect to $\overline{u}_N$ is

$$\frac{\partial\mathcal{L}}{\partial\overline{u}_N} = 0 \Leftrightarrow (1-\upsilon)\,h'(\overline{u}_N) = \mu. \tag{4}$$

This implies that $\mu > 0$; that is, IC-high binds at the optimum. The first-order condition with respect to $\underline{u}_N$ is

$$\frac{\partial\mathcal{L}}{\partial\underline{u}_N} = 0 \Leftrightarrow \upsilon(1-\underline{\theta})\,h'(\underline{u}_N) = \lambda(1-\underline{\theta}) - \mu\left(1-\overline{\theta}\right). \tag{5}$$

This implies that $\lambda > 0$; that is, also IR-low binds at the optimum.

The first-order conditions with respect to $\overline{u}_A$ and $\underline{u}_A$ are

$$\frac{\partial\mathcal{L}}{\partial\overline{u}_A} = 0 \Leftrightarrow (1-\upsilon)\,h'(\overline{u}_A) = \mu, \tag{6}$$

$$\frac{\partial\mathcal{L}}{\partial\underline{u}_A} = 0 \Leftrightarrow \upsilon\underline{\theta}h'(\underline{u}_A) = \lambda\underline{\theta} - \mu\overline{\theta}. \tag{7}$$

Combining (4) and (6), using the fact that $h'' > 0$, immediately yields $\overline{u}_N = \overline{u}_A \equiv \overline{u}$. That is, the high-risk type is fully insured. Next, multiply (5) by $\underline{\theta}$ and multiply (7) by $(1-\underline{\theta})$:

$$\upsilon\underline{\theta}(1-\underline{\theta})\,h'(\underline{u}_N) = \lambda\underline{\theta}(1-\underline{\theta}) - \mu\underline{\theta}\left(1-\overline{\theta}\right),$$

$$\upsilon\underline{\theta}(1-\underline{\theta})\,h'(\underline{u}_A) = \lambda\underline{\theta}(1-\underline{\theta}) - \mu\overline{\theta}(1-\underline{\theta}).$$

Subtracting the latter from the former and then simplifying, we obtain

$$\upsilon\underline{\theta}(1-\underline{\theta})\left[h'(\underline{u}_N) - h'(\underline{u}_A)\right] = \mu\left(\overline{\theta} - \underline{\theta}\right). \tag{8}$$

Since $\upsilon\underline{\theta}(1-\underline{\theta}) > 0$, $\overline{\theta} > \underline{\theta}$, $\mu > 0$, and $h'' > 0$, the above inequality implies that $\underline{u}_N > \underline{u}_A$. Hence the low type is underinsured at the optimum. Equation (1) in Proposition 1 is obtained by combining (8) and (4).

Given the analysis above, there are three things that we need to do in order to complete the proof of Proposition 1: (i) verify that IC-low is satisfied at the optimum, (ii) show that the inequalities in (2) hold, and (iii) show the relationships in (3). To show (i) note that, given that IR-low binds and that $\overline{u}_A^{SZ} = \overline{u}_N^{SZ} \equiv \overline{u}^{SZ}$, IC-low simplifies to $\underline{U}^* \geq \overline{u}^{SZ}$. But this inequality holds if (ii) holds, which we show next. We have already demonstrated that $\underline{u}_A^{SZ} < \underline{u}_N^{SZ}$. The binding IR-low constraint, $(1-\underline{\theta})\,\underline{u}_N^{SZ} + \underline{\theta}\underline{u}_A^{SZ} = \underline{U}^*$, then implies that $\underline{u}_A^{SZ} < \underline{U}^* < \underline{u}_N^{SZ}$. Next, the binding IC-high constraint, $\overline{u}^{SZ} = \left(1-\overline{\theta}\right)\underline{u}_N^{SZ} + \overline{\theta}\underline{u}_A^{SZ}$, implies that $\underline{u}_A^{SZ} < \overline{u}^{SZ} < \underline{u}_N^{SZ}$. It now only remains to show that $\overline{u}^{SZ} < \underline{U}^*$. Using the two binding constraints we get

$$\begin{aligned}
\underline{U}^* - \overline{u}^{SZ} &= \left[(1-\underline{\theta})\,\underline{u}_N^{SZ} + \underline{\theta}\underline{u}_A^{SZ}\right] - \left[\left(1-\overline{\theta}\right)\underline{u}_N^{SZ} + \overline{\theta}\underline{u}_A^{SZ}\right] \\
&= \left(\overline{\theta} - \underline{\theta}\right)\left(\underline{u}_N^{SZ} - \underline{u}_A^{SZ}\right),
\end{aligned}$$

which we know is strictly positive. Finally consider (iii). The first relationship in (3) follows from the binding IR-low. From IR-high, which we know is satisfied with a strict inequality, we also obtain the second relationship in (3), $\overline{u}^{SZ} > \overline{U}^*$. □

Next we will restate and prove the claim that $\pi^{NI} > \pi_{max}^{SZ}$. For easy reference, we first restate some equations from the paper:

$$\pi_{SD}^{SZ} = (1-\upsilon)\left[w - \overline{\theta}d - h\left(\overline{U}^*\right)\right], \tag{9}$$

$$\pi^{SZ} = \widehat{w} - \upsilon\left[(1-\underline{\theta})\,h\left(\underline{u}_N^{SZ}\right) + \underline{\theta}h\left(\underline{u}_A^{SZ}\right)\right] - (1-\upsilon)\,h\left(\overline{u}^{SZ}\right), \tag{10}$$

$$EU^{NI} = \upsilon\underline{U}^* + (1-\upsilon)\,\overline{U}^*, \tag{11}$$

$$\pi^{NI} = \widehat{w} - h\left(EU^{NI}\right). \tag{12}$$

We are now ready to prove that $\pi^{NI} > \pi_{max}^{SZ}$.

Proof that $\pi^{NI} > \pi_{max}^{SZ}$. We can write

$$\begin{aligned}
\pi^{NI} &= \widehat{w} - h\left(EU^{NI}\right) \\
&= \widehat{w} - h\left[(1-\upsilon)\,\overline{U}^* + \upsilon\underline{U}^*\right] \\
&> \widehat{w} - (1-\upsilon)\,h\left(\overline{U}^*\right) - \upsilon h\left(\underline{U}^*\right) \equiv \pi^{FI} \\
&\geq \widehat{w} - (1-\upsilon)\,h\left(\overline{u}^{SZ}\right) - \upsilon h\left[(1-\underline{\theta})\,\underline{u}_N^{SZ} + \underline{\theta}\underline{u}_A^{SZ}\right] \\
&> \widehat{w} - (1-\upsilon)\,h\left(\overline{u}^{SZ}\right) - \upsilon(1-\underline{\theta})\,h\left(\underline{u}_N^{SZ}\right) - \upsilon\underline{\theta}h\left(\underline{u}_A^{SZ}\right) \\
&= \pi^{SZ},
\end{aligned}$$

where the first equality uses (12), the second uses (11), the two strict inequalities use the strict convexity of $h(\cdot)$, the weak inequality uses the two IR constraints, and the last equality uses (10). We have thus shown that $\pi^{NI} > \pi^{FI} > \pi^{SZ}$. To complete the proof it thus suffices to show that, in addition, $\pi^{FI} > \pi_{SD}^{SZ}$ (which implies $\pi^{NI} > \pi_{SD}^{SZ}$). By using (9), the definition of π^{FI} on the third line above, and the equality $\widehat{w} = w - \left[(1-\upsilon)\,\overline{\theta} + \upsilon\underline{\theta}\right]d$, we can write

$$\pi^{FI} > \pi_{SD}^{SZ} \Leftrightarrow$$

$$w-\left[(1-\upsilon)\overline{\theta}+\upsilon\underline{\theta}\right]d-(1-\upsilon)h\left(\overline{U}^{*}\right)-\upsilon h\left(\underline{U}^{*}\right)>(1-\upsilon)\left[w-\overline{\theta}d-h\left(\overline{U}^{*}\right)\right]\Leftrightarrow$$

$$\upsilon w-\upsilon\underline{\theta}d-\upsilon h\left(\underline{U}^{*}\right)>0\Leftrightarrow w-\underline{\theta}d>h\left(\underline{U}^{*}\right)\Leftrightarrow$$

$$u\left(w-\underline{\theta}d\right)>\underline{U}^{*}=\left(1-\underline{\theta}\right)u\left(w\right)+\underline{\theta}u\left(w-d\right),$$

which always holds due to the strict concavity of u. □

2. Deriving equation (12)

We here derive equation (12), which gave us the coordinates of the crossing point C:

$$\left(u_{A},u_{N}\right)=\left(u\left(w-d\right)+(1-\theta^{e})k,u\left(w\right)-\theta^{e}k\right). \tag{13}$$

Proof. Calculate the crossing point between φ_{high} and φ_{ante}:

$$\frac{\upsilon\underline{U}^{*}-c}{\upsilon\left(1-\underline{\theta}\right)}-\frac{\underline{\theta}}{1-\underline{\theta}}u_{A}=\frac{\upsilon\underline{U}^{*}+(1-\upsilon)\overline{U}^{*}}{\upsilon\left(1-\underline{\theta}\right)+(1-\upsilon)\left(1-\overline{\theta}\right)}-\frac{\upsilon\underline{\theta}+(1-\upsilon)\overline{\theta}}{\upsilon\left(1-\underline{\theta}\right)+(1-\upsilon)\left(1-\overline{\theta}\right)}u_{A}\Leftrightarrow$$

$$\frac{\upsilon\underline{\theta}+(1-\upsilon)\overline{\theta}}{\upsilon\left(1-\underline{\theta}\right)+(1-\upsilon)\left(1-\overline{\theta}\right)}u_{A}-\frac{\underline{\theta}}{1-\underline{\theta}}u_{A}=\frac{\upsilon\underline{U}^{*}+(1-\upsilon)\overline{U}^{*}}{\upsilon\left(1-\underline{\theta}\right)+(1-\upsilon)\left(1-\overline{\theta}\right)}-\frac{\upsilon\underline{U}^{*}-c}{\upsilon\left(1-\underline{\theta}\right)}\Leftrightarrow$$

$$\begin{aligned}&\frac{\left(1-\underline{\theta}\right)\left[\upsilon\underline{\theta}+(1-\upsilon)\overline{\theta}\right]-\underline{\theta}\left[\upsilon\left(1-\underline{\theta}\right)+(1-\upsilon)\left(1-\overline{\theta}\right)\right]}{\left[\upsilon\left(1-\underline{\theta}\right)+(1-\upsilon)\left(1-\overline{\theta}\right)\right]\left(1-\underline{\theta}\right)}u_{A}\\=&\quad\frac{\left(1-\underline{\theta}\right)\left[\upsilon\underline{U}^{*}+(1-\upsilon)\overline{U}^{*}\right]-\left[\upsilon\left(1-\underline{\theta}\right)+(1-\upsilon)\left(1-\overline{\theta}\right)\right]\underline{U}^{*}}{\left[\upsilon\left(1-\underline{\theta}\right)+(1-\upsilon)\left(1-\overline{\theta}\right)\right]\left(1-\underline{\theta}\right)}+\frac{c}{\upsilon\left(1-\underline{\theta}\right)}\Leftrightarrow\end{aligned}$$

$$\frac{(1-\upsilon)\left[\left(1-\underline{\theta}\right)\overline{\theta}-\underline{\theta}\left(1-\overline{\theta}\right)\right]}{\left[\upsilon\left(1-\underline{\theta}\right)+(1-\upsilon)\left(1-\overline{\theta}\right)\right]}u_{A}=\frac{(1-\upsilon)\left[\left(1-\underline{\theta}\right)\overline{U}^{*}-\left(1-\overline{\theta}\right)\underline{U}^{*}\right]}{\left[\upsilon\left(1-\underline{\theta}\right)+(1-\upsilon)\left(1-\overline{\theta}\right)\right]}+\frac{c}{\upsilon}\Leftrightarrow$$

$$u_{A}=\frac{\left(1-\underline{\theta}\right)\overline{U}^{*}-\left(1-\overline{\theta}\right)\underline{U}^{*}}{\overline{\theta}-\underline{\theta}}+\frac{\left[\upsilon\left(1-\underline{\theta}\right)+(1-\upsilon)\left(1-\overline{\theta}\right)\right]c}{\upsilon\left(1-\upsilon\right)\left(\overline{\theta}-\underline{\theta}\right)}\Leftrightarrow$$

$$u_{A}=u\left(w-d\right)+\frac{\left[\upsilon\left(1-\underline{\theta}\right)+(1-\upsilon)\left(1-\overline{\theta}\right)\right]c}{\upsilon\left(1-\upsilon\right)\left(\overline{\theta}-\underline{\theta}\right)}. \tag{14}$$

In order to obtain the value of u_N at the crossing point, we plug (2) into the equation for

φ_{high} and simplify:

$$
\begin{aligned}
u_N &= \frac{\upsilon \underline{U}^* - c}{\upsilon\left(1-\underline{\theta}\right)} - \frac{\underline{\theta}}{1-\underline{\theta}} u_A \\
&= \frac{\upsilon \underline{U}^* - c}{\upsilon\left(1-\underline{\theta}\right)} - \frac{\underline{\theta}}{1-\underline{\theta}} \left[\frac{\left(1-\underline{\theta}\right)\overline{U}^* - \left(1-\overline{\theta}\right)\underline{U}^*}{\overline{\theta}-\underline{\theta}} + \frac{\left[\upsilon\left(1-\underline{\theta}\right) + \left(1-\upsilon\right)\left(1-\overline{\theta}\right)\right]c}{\upsilon\left(1-\upsilon\right)\left(\overline{\theta}-\underline{\theta}\right)} \right] \\
&= \frac{\underline{U}^*}{\left(1-\underline{\theta}\right)} - \frac{\underline{\theta}\left[\left(1-\underline{\theta}\right)\overline{U}^* - \left(1-\overline{\theta}\right)\underline{U}^*\right]}{\left(\overline{\theta}-\underline{\theta}\right)\left(1-\underline{\theta}\right)} - \frac{\underline{\theta}\left[\upsilon\left(1-\underline{\theta}\right) + \left(1-\upsilon\right)\left(1-\overline{\theta}\right)\right]c}{\upsilon\left(1-\upsilon\right)\left(1-\underline{\theta}\right)\left(\overline{\theta}-\underline{\theta}\right)} - \frac{c}{\upsilon\left(1-\underline{\theta}\right)} \\
&= \frac{\left[\overline{\theta}-\underline{\theta}+\underline{\theta}\left(1-\overline{\theta}\right)\right]\underline{U}^* - \underline{\theta}\left(1-\underline{\theta}\right)\overline{U}^*}{\left(\overline{\theta}-\underline{\theta}\right)\left(1-\underline{\theta}\right)} - \frac{\left[\upsilon\underline{\theta}\left(1-\underline{\theta}\right) + \left(1-\upsilon\right)\underline{\theta}\left(1-\overline{\theta}\right) + \left(1-\upsilon\right)\left(\overline{\theta}-\underline{\theta}\right)\right]c}{\upsilon\left(1-\upsilon\right)\left(1-\underline{\theta}\right)\left(\overline{\theta}-\underline{\theta}\right)} \\
&= \frac{\overline{\theta}\left(1-\underline{\theta}\right)\underline{U}^* - \underline{\theta}\left(1-\underline{\theta}\right)\overline{U}^*}{\left(\overline{\theta}-\underline{\theta}\right)\left(1-\underline{\theta}\right)} - \frac{\left[\upsilon\underline{\theta}\left(1-\underline{\theta}\right) + \left(1-\upsilon\right)\underline{\theta}\left(1-\overline{\theta}\right) + \left(1-\upsilon\right)\left(\overline{\theta}-\underline{\theta}\right)\right]c}{\upsilon\left(1-\upsilon\right)\left(1-\underline{\theta}\right)\left(\overline{\theta}-\underline{\theta}\right)} \\
&= \frac{\overline{\theta}\underline{U}^* - \underline{\theta}\overline{U}^*}{\overline{\theta}-\underline{\theta}} - \frac{\upsilon\underline{\theta}\left(1-\underline{\theta}\right) + (1-\upsilon)\overline{\theta}(1-\underline{\theta})}{\upsilon\left(1-\upsilon\right)\left(1-\underline{\theta}\right)\left(\overline{\theta}-\underline{\theta}\right)} c \Leftrightarrow
\end{aligned}
$$

$$
u_N = u\left(w\right) - \frac{\upsilon\underline{\theta} + (1-\upsilon)\overline{\theta}}{\upsilon\left(1-\upsilon\right)\left(\overline{\theta}-\underline{\theta}\right)} c.
$$

□

3. Monetary costs of information gathering

In this section, we look at an alternative setup where the cost of effort does not enter the utility function additively. Instead the effort cost c is monetary, and it is therefore subtracted from the agent's wealth (in the argument of the utility function). We will focus on the case of CARA preferences; that is, given a consumption level y, the agent's utility is

$$u(y) = -e^{-\eta y},$$

where $\eta > 0$ is a parameter. Hence, the expected utility of an agent gathering information, finding out his type is $\bar{\theta}$ and buying the insurance contract $(\bar{a}, \bar{p})$, is[1]

$$-\bar{\theta} e^{-\eta(w-\bar{p}-d+\bar{a}-c)} - (1-\bar{\theta}) e^{-\eta(w-\bar{p}-c)}.$$

We still use the utility notation of the paper but amend it in the following way: For example, $\bar{u}_A(1)$ is the utility when acquiring information, buying the "upperbar contract" and having an accident; i.e. $\bar{u}_A(1) = -e^{-\eta(w-\bar{p}-d+\bar{a}-c)}$. In contrast, $\bar{u}_A(0)$ is the utility when buying the "upperbar contract" and having an accident without acquiring information; i.e. $\bar{u}_A(0) = -e^{-\eta(w-\bar{p}-d+\bar{a})}$. The other utility levels are defined similarly.

Similarly to the structure of the paper, we first analyze the case where the principal wants to induce information gathering and then the case where he does not. We will show that in this setup the same constraints as in the setup of the paper are binding. This will imply that, qualitatively, also the same distortions as in the paper occur.

[1] In contrast to the paper, for convenience we here let a denote the gross (not the net) indemnity.

3.1. Inducing information gathering

Lemma S1 *IG-low implies IC-high.*

Proof. IG-low is written as

$$v[(1-\underline{\theta})\underline{u}_N(1)+\underline{\theta}\underline{u}_A(1)]+(1-v)[(1-\bar{\theta})\bar{u}_N(1)+\bar{\theta}\bar{u}_A(1)]$$
$$\geq v[(1-\underline{\theta})\underline{u}_N(0)+\underline{\theta}\underline{u}_A(0)]+(1-v)[(1-\bar{\theta})\underline{u}_N(0)+\bar{\theta}\underline{u}_A(0)].$$

This implies

$$v[(1-\underline{\theta})\underline{u}_N(1)+\underline{\theta}\underline{u}_A(1)]+(1-v)[(1-\bar{\theta})\bar{u}_N(1)+\bar{\theta}\bar{u}_A(1)]$$
$$> v[(1-\underline{\theta})\underline{u}_N(0)+\underline{\theta}\underline{u}_A(0)]+(1-v)[(1-\bar{\theta})\underline{u}_N(1)+\bar{\theta}\underline{u}_A(1)],$$

because $\underline{u}_N(1)<\underline{u}_N(0)$ and $\underline{u}_A(1)<\underline{u}_A(0)$. This in turn implies

$$(1-\bar{\theta})(\bar{u}_N(1)-\underline{u}_N(1))+\bar{\theta}(\bar{u}_A(1)-\underline{u}_A(1))$$
$$> \frac{v}{1-v}\left((1-\underline{\theta})(\underline{u}_N(0)-\underline{u}_N(1))+\underline{\theta}(\underline{u}_A(0)-\underline{u}_A(1))\right)>0,$$

which verifies IC-high. □

Lemma S2 *IR-ante is implied by IR-low and IG-low.*

Proof. IR-low is given by

$$\underline{\theta}\left(u(w-\underline{p}-d+\underline{a}-c)-u(w-d-c)\right)+(1-\underline{\theta})\left(u(w-\underline{p}-c)-u(w-c)\right)\geq 0.$$

Given that $\underline{p}\geq 0$ and $\underline{a}\geq\underline{p}$, this implies

$$\bar{\theta}\left(u(w-\underline{p}-d+\underline{a}-c)-u(w-d-c)\right)+(1-\bar{\theta})\left(u(w-\underline{p}-c)-u(w-c)\right)\geq 0,$$

which using our ususal notation can be written as

$$\bar{\theta}\underline{u}_A(1)+(1-\bar{\theta})\underline{u}_N(1)\geq\overline{U}^*(1).$$

With CARA preferences, this is equivalent to

$$\bar{\theta}\underline{u}_A(0)+(1-\bar{\theta})\underline{u}_N(0)\geq\overline{U}^*(0). \quad (15)$$

Also IR-low can, due to the CARA assumption, be restated as

$$(1-\underline{\theta})\underline{u}_N(0)+\underline{\theta}\underline{u}_A(0)\geq\underline{U}^*(0). \quad (16)$$

Now use (15) and (16) in IG-low:

$$v[(1-\underline{\theta})\underline{u}_N(1)+\underline{\theta}\underline{u}_A(1)]+(1-v)[(1-\bar{\theta})\bar{u}_N(1)+\bar{\theta}\bar{u}_A(1)]$$
$$\geq v[(1-\underline{\theta})\underline{u}_N(0)+\underline{\theta}\underline{u}_A(0)]+(1-v)[(1-\bar{\theta})\underline{u}_N(0)+\bar{\theta}\underline{u}_A(0)]$$
$$\geq v\underline{U}^*(0)+(1-v)\overline{U}^*(0),$$

which is IR-ante. □

Lemma S3 *If information acquisition is optimal, IG-high is lax and IR-low and IG-low are binding.*

Proof. Suppose, per contra, that IG-high is binding and that information acquisition is optimal. We will show that then there exists a contract that yields higher profits and does not induce information acquisition, which contradicts the optimality of information acquisition. The proof goes through two subcases.

First, assume $\bar{a} \leq d$, which means that the high coverage contract has at most full coverage. We claim that it is more profitable in this case to offer *only* the high coverage contract $(\bar{p}, \bar{a})$. As IG-high is binding by assumption, the consumer can achieve the same ex ante utility as before by buying the contract without gathering information. Since the fact that the low-coverage contract is not offered restricts the consumer's choice set, her ex ante utility cannot be higher than in the situation in which she could choose either contract. This implies that the consumer's optimal strategy when offered only $(\bar{p}, \bar{a})$ is to buy this contract without information acquisition. It remains to show that the insurer's profits are higher than in the situation where he offers two contracts. This is intuitively obvious as (i) the consumer surplus is the same in both situations but (ii) the information acquisition cost is saved and (iii) (some) consumers have more coverage. Welfare (in the sense "total amount of resources") must increase because of (ii) and (iii) but the consumer surplus remains the same. Hence, profits must increase.

Second, assume $\bar{a} > d$, which is equivalent to $\bar{u}_A > \bar{u}_N$. Then the insurer can offer a full coverage, pooling contract that gives the agent the same ex ante utility as before (but now without incurring the cost of information acquisition). This yields, we claim, a higher profit for the insurer. Call the utility level of this contract $u_p(0)$. Then the consumer does not want to deviate by exerting effort and buying insurance only in case she is a high-risk type: As IG-high is binding, $u_p(0) = \left(v\underline{\theta} + (1-v)\bar{\theta}\right)\bar{u}_A(0) + \left(v(1-\underline{\theta}) + (1-v)(1-\bar{\theta})\right)\bar{u}_N(0)$. This implies $u_p(0) < \bar{\theta}\bar{u}_A(0) + (1-\bar{\theta})\bar{u}_N(0)$, because of $\bar{u}_A(0) > \bar{u}_N(0)$ and $\bar{\theta} > \underline{\theta}$. By CARA, we then have $u_p(1) < \bar{\theta}\bar{u}_A(1) + (1-\bar{\theta})\bar{u}_N(1)$. This implies that the deviation "acquiring information and buying (high) coverage if type $\bar{\theta}$ while remaining uninsured if type $\underline{\theta}$" gives a lower payoff under the pooling contract than under the original menu. As the expected utility when not deviating is by definition of $u_p(0)$ the same, the deviation is not profitable in the pooling situation.

It remains to show that profits are higher under the pooling contract:

$$\begin{aligned}
\pi^{pooling} &= \hat{w} - h(u_p(0)) \\
&= \hat{w} - h\left(v\underline{\theta}\underline{u}_A(1) + v(1-\underline{\theta})\underline{u}_N(1) + (1-v)\bar{\theta}\bar{u}_A(1) + (1-v)(1-\bar{\theta})\bar{u}_N(1)\right) \\
&> \hat{w} - c - v\underline{\theta}h(\underline{u}_A(1)) - v(1-\underline{\theta})h(\underline{u}_N(1)) - (1-v)\bar{\theta}h(\bar{u}_A(1)) - (1-v)(1-\bar{\theta})h(\bar{u}_N(1)) \\
&= \pi^{original\ separating},
\end{aligned}$$

where the strict inequality follows from the strict convexity of h.

This proves that IG-high is lax. As in a standard problem also IR-high and IC-low are lax (we omit the formal proof of this as the result should be unsurprising). Hence, only IR-low and IG-low can be binding. If IR-low was lax, increasing $\underline{p}$ would clearly increase profits and relax the only potentially binding contraint IG-low. Hence, IR-low must bind. If IG-low was lax, increasing $\bar{p}$ would increase profits while not affecting the only binding constraint IR-low. Hence, IG-low must bind. □

The following proposition states that the low-risk type is underinsured and the high-risk type is fully insured if the principal finds it optimal to induce information acquisition.

Proposition S1 *If it is optimal for the principal to induce information acquisition, then* $\bar{u}_A(1) = \bar{u}_N(1)$ *and* $\underline{u}_A(1) < \underline{u}_N(1)$.

Proof. Note that due to the CARA assumption, $\bar{u}_i(0)e^{\eta c} = \bar{u}_i(1)$ for $i = N, A$ and similarly for $\underline{u}_i$. Given the constraints IR-low and IG-low, the Lagrangian of the principal's problem is

$$\begin{aligned}\mathcal{L} \;=\; & \widehat{w} - c - \upsilon\left[(1-\underline{\theta})\,h\left(\underline{u}_N(1)\right) + \underline{\theta}h\left(\underline{u}_A(1)\right)\right] - (1-\upsilon)\left[\left(1-\overline{\theta}\right)h\left(\overline{u}_N(1)\right) + \overline{\theta}h\left(\overline{u}_A(1)\right)\right] \\ & + \lambda\left[(1-\underline{\theta})\,\underline{u}_N(1) + \underline{\theta}\underline{u}_A(1) - \underline{U}^*(1)\right] \\ & + \underline{\mu}\left\{(1-\upsilon)\left[\left(1-\overline{\theta}\right)\left(\overline{u}_N(1) - \underline{u}_N(1)e^{-\eta c}\right) + \overline{\theta}\left(\overline{u}_A(1) - \underline{u}_A(1)e^{-\eta c}\right)\right]\right. \\ & \qquad \left. + \upsilon\left[(1-\underline{\theta})\underline{u}_N(1)(1-e^{-\eta c}) + \underline{\theta}\underline{u}_A(1)(1-e^{-\eta c})\right]\right\}. \end{aligned} \tag{17}$$

The first-order conditions with respect to $\underline{u}_A(1)$ and $\underline{u}_N(1)$ are

$$\frac{\partial \mathcal{L}}{\partial \underline{u}_A(1)} = 0 \;\Leftrightarrow\; \upsilon\underline{\theta}h'(\underline{u}_A(1)) = \lambda\underline{\theta} + \underline{\mu}\left[\upsilon\underline{\theta}(1-e^{-\eta c}) - (1-\upsilon)\bar{\theta}e^{-\eta c}\right] \tag{18}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial \underline{u}_N(1)} = 0 \;\Leftrightarrow\; & \upsilon(1-\underline{\theta})h'(\underline{u}_N(1)) \\ & = \lambda(1-\underline{\theta}) + \underline{\mu}\left[\upsilon(1-\underline{\theta})(1-e^{-\eta c}) - (1-\upsilon)(1-\bar{\theta})e^{-\eta c}\right]. \end{aligned} \tag{19}$$

Multiplying (18) by $(1-\underline{\theta})$ and (19) by $\underline{\theta}$ and subtracting the latter resulting equation from the former yields

$$\upsilon\underline{\theta}(1-\underline{\theta})\left(h'(\underline{u}_A(1)) - h'(\underline{u}_N(1))\right) = -\underline{\mu}(1-\upsilon)e^{-\eta c}(\bar{\theta} - \underline{\theta}),$$

which implies $\underline{u}_A < \underline{u}_N$ by the strict convexity of h and $\underline{\mu} > 0$ by the previous lemma.

The first-order condition with respect to $\bar{u}_N(1)$ and $\bar{u}_A(1)$ are

$$\frac{\partial \mathcal{L}}{\partial \bar{u}_N(1)} = 0 \;\Leftrightarrow\; (1-\upsilon)(1-\bar{\theta})h'(\bar{u}_N(1)) = \underline{\mu}(1-\upsilon)(1-\bar{\theta}), \tag{20}$$

$$\frac{\partial \mathcal{L}}{\partial \bar{u}_A(1)} = 0 \;\Leftrightarrow\; (1-\upsilon)\bar{\theta}h'(\bar{u}_A(1)) = \underline{\mu}\,(1-\upsilon)\,\bar{\theta}. \tag{21}$$

Combining equations (20) and (21) yields $h'(\bar{u}_N(1)) = \underline{\mu} = h'(\bar{u}_A(1))$, which implies $\bar{u}_A(1) = \bar{u}_N(1)$ by the strict convexity of h. □

Lemma S4 *If it is optimal for the principal to induce information acquisition, then* $\frac{\partial \pi^*_{x=1}}{\partial c} < 0$. *Furthermore,* $\lim_{c\to 0}\pi^*_{x=1} = \pi^{SB}_{max}$.

Proof. The first claim follows from the envelope theorem used on the Lagrangian (17). Note that the objective and IR-low are not affected by changes in c. However, IG-low is harder to satisfy for higher values of c.

To verify the second claim, note that for $c \to 0$ IG-low continuously becomes identical to IC-high. IR-low and the objective of the principal are not affected by changes in c. Hence, the maximization problem becomes identical to the Stiglitz problem as $c \to 0$. $\square$

3.2. Not inducing information gathering

Assuming CARA preferences and letting our u notation here denote the utility levels given $x = 0$, the principal's problem is to maximize

$$\hat{w} - \left[\upsilon(1-\underline{\theta}) + (1-\upsilon)(1-\bar{\theta})\right] h(u_N) - \left[\upsilon\underline{\theta} + (1-\upsilon)\bar{\theta}\right] h(u_A)$$

subject to

$$\upsilon(1-\underline{\theta})u_N + (1-\upsilon)(1-\bar{\theta})u_N(1-e^{\eta c}) + \upsilon\underline{\theta}u_A + (1-\upsilon)\bar{\theta}u_A(1-e^{\eta c}) - \upsilon e^{\eta c}\underline{U}^* \geq 0, \tag{IG-high}$$

$$\upsilon(1-\underline{\theta})u_N + (1-\upsilon)(1-\bar{\theta})u_N + \upsilon\underline{\theta}u_A + (1-\upsilon)\bar{\theta}u_A - \upsilon\underline{U}^* - (1-\upsilon)\overline{U}^* \geq 0. \tag{IR-ante}$$

The objective function is strictly concave and the domain is convex and compact as the constraints are weak inequalities that are linear in utilities. Consequently, the maximization problem has a unique solution. Furthermore, both constraints and objective are continuous in c and continuous in the utilities. Hence, the optimal contract will be continuous in c as well.

Note that utilities are negative with CARA preferences. Hence, it is obvious that IG-high is lax for $c \to \infty$. It is also clear that at least one of the constraints must bind (otherwise increasing the premium results in higher profits). For c high enough, IR-ante will, therefore, bind and IG-high will be lax.

Denoting the Lagrange multiplier of the IG-high constraint by μ and the Lagrange multiplier of the IR-ante constraint by λ, the first-order conditions of the maximization problem are

$$(1-A)h'(u_N) = \mu\left(1 - A - (1-\upsilon)(1-\bar{\theta})e^{\eta c}\right) + \lambda(1-A), \tag{22}$$

$$Ah'(u_A) = \mu\left(A - (1-\upsilon)\bar{\theta}e^{\eta c}\right) + \lambda A, \tag{23}$$

where A is defined as $A \equiv \upsilon\underline{\theta} + (1-\upsilon)\bar{\theta}$.[2] Note that $0 < \underline{\theta} < A < \bar{\theta} < 1$. Adding (22) and (23) yields

$$(1-A)h'(u_N) + Ah'(u_A) = \mu\left(1 - (1-\upsilon)e^{\eta c}\right) + \lambda.$$

[2]Note that in our paper we use the notation θ^e for A. We avoid that notation here since in this model the notation e represents the exponential function.

Solving this equation for λ and plugging it back into (23) gives

$$[h'(u_N) - h'(u_A)]A(1-A) = \mu(1-\upsilon)(\bar{\theta} - A)e^{\eta c}. \tag{24}$$

This leads to the following result:

Proposition S2 *Suppose that information gathering is not induced. Then the optimal contract provides full coverage if IG-high is slack and partial coverage if IG-high binds.*

For $c \to 0$, IG-high must bind. To see this, we can rewrite the constraints as $u_N \geq \varphi_{IGh}(u_A)$ and $u_N \geq \varphi_{ante}(u_A)$, where both φ functions are linear.[3] For $c = 0$, it is easy to see that φ_{ante} has a higher slope and intercept than φ_{IGh}. Furthermore, the two φ functions intersect at a point where $u_N > u_A$, i.e. above the 45-degree line. If IG-high did not bind, only IR-ante was binding and therefore $u_A = u_N$. Given that the intersection of the φ functions is above the 45-degree line, this would imply that IG-high is violated. This is impossible. As IG-high is continuous in c and IR-ante is not affected by c, the same must hold for $c > 0$ small enough.

Furthermore, it is clear from the derivation above that the continuity of the optimal contract in c implies that the Lagrange parameter λ and μ are also continuous in c. This is an important observation because it implies that there must be a range of c values for which both constraints are binding: If this was not the case, there would have to be a cutoff $\tilde{c}$ such that only IR-ante binds for $c > \tilde{c}$ and only IG-high binds for $c < \tilde{c}$. The continuity of λ and μ would then imply that $\lambda(\tilde{c}) = \mu(\tilde{c}) = 0$, which means that no constraint binds if $c = \tilde{c}$. Clearly, this is impossible.

3.2.1. Both constraints binding

Now we want to turn to the case where indeed both constraints are binding. The two constraints can then be solved for the utility levels u_N and u_A. Subtracting IG-high from IR-ante and rearranging gives

$$u_A = \frac{1}{\bar{\theta}(1-\upsilon)} \left((1-\upsilon)\overline{U}^* e^{-\eta c} + \upsilon \underline{U}^* \left(e^{-\eta c} - 1 \right) \right) - \frac{1-\bar{\theta}}{\bar{\theta}} u_N.$$

Plugging this back into IR-ante yields after rearranging

$$\upsilon \frac{\bar{\theta} - \underline{\theta}}{\bar{\theta}} u_N = \upsilon \underline{U}^* + (1-\upsilon)\overline{U}^* - \frac{\upsilon\underline{\theta} + (1-\upsilon)\bar{\theta}}{\bar{\theta}(1-\upsilon)} \left((1-\upsilon)\overline{U}^* e^{-\eta c} + \upsilon \underline{U}^* \left(e^{-\eta c} - 1 \right) \right).$$

The last two equations imply the following result (recall that all utilities are negative because of the CARA preferences):

[3] Alternatively, instead of going through the arguments in the next few sentences, we could note that for $c = 0$ the two constraints coincide with those in our paper (see in particular Figure 1 there and the discussion in connection to that figure). Therefore, for that case, the result that IG-high binds follows from the analysis in the paper. However, we will later in this supplementary material make use of the notation $\varphi_{IGh}(u_A)$ and $\varphi_{ante}(u_A)$, which is why we restate the arguments here.

Lemma S5 *If IG-high and IR-ante bind, then* $\frac{\partial u_N}{\partial c} < 0$ *and* $\frac{\partial u_A}{\partial c} > 0$.

Lemma S5 implies together with (24) that $\frac{\partial \mu}{\partial c} < 0$ whenever both constraints are binding. This and equation (23) yield then $\frac{\partial \lambda}{\partial c} > 0$ whenever both constraints are binding.

3.2.2. Only IG-high binds

In this case, $\lambda = 0$. Eliminating μ from (22) and (23) gives

$$\frac{A}{A-(1-\upsilon)\bar{\theta}e^{\eta c}}h'(u_A) = \frac{1-A}{1-A-(1-\upsilon)(1-\bar{\theta})e^{\eta c}}h'(u_N).$$

With CARA preferences $h'(u) = -1/(\eta u)$ and therefore the previous equation can be rewritten as

$$u_N = \frac{(1-A)(A-(1-\upsilon)\bar{\theta}e^{\eta c})}{A(1-A-(1-\upsilon)(1-\bar{\theta})e^{\eta c})}u_A. \tag{25}$$

Note that from IG-high we know that

$$u_A = -\frac{1-A-(1-\upsilon)(1-\bar{\theta})e^{\eta c}}{A-(1-\upsilon)\bar{\theta}e^{\eta c}}u_N + \frac{\upsilon e^{\eta c}}{A-(1-\upsilon)\bar{\theta}e^{\eta c}}\underline{U}^*. \tag{26}$$

Plugging (25) into (26) yields

$$u_A = \frac{A\upsilon e^{\eta c}}{A-(1-\upsilon)\bar{\theta}e^{\eta c}}\underline{U}^*,$$

which implies $\frac{\partial u_A}{\partial c} < 0$ (recall again that $\underline{U}^* < 0$). Solving (25) and (26) for u_N yields

$$u_N = \frac{(1-A)\upsilon e^{\eta c}}{1-A-(1-\upsilon)(1-\bar{\theta})e^{\eta c}}\underline{U}^*,$$

which implies $\frac{\partial u_N}{\partial c} < 0$. This leads to the following result:

Lemma S6 *The expected utility of the agent is*

- *strictly decreasing in* c *if only IG-high binds and*
- *constant in* c *if IR-ante binds.*

Proof. The first statement follows from the comparative statics results $\frac{\partial u_N}{\partial c} < 0$ and $\frac{\partial u_A}{\partial c} < 0$, which were just derived for the case where only IG-high binds. The second statement follows from the fact that ex ante consumer surplus is $\upsilon\underline{U}^* + (1-\upsilon)\overline{U}^*$ if IR-ante binds and this term is constant in c. □

3.2.3. Comparative statics with respect to c

Whether IR-ante is binding for small values of c depends on the shape of the utility function: The slope of an isoprofit curve is

$$\left.\frac{du_N}{du_A}\right|_{\pi_{x=0}=const} = \frac{-Ah'(u_A)}{(1-A)h'(u_N)}.$$

The slope of φ_{IGh} (for $c = 0$) is $-\underline{\theta}/(1-\underline{\theta})$. Let (u'_N, u'_A) be the point on φ_{IGh} such that $\frac{-Ah'(u'_A)}{(1-A)h'(u'_N)} = -\frac{\underline{\theta}}{(1-\underline{\theta})}$; that is, (u'_N, u'_A) is the tangency point of an isoprofit line with φ_{IGh}. If $u'_N > \varphi_{ante}(u'_A)$, then IR-ante is slack for $c > 0$ small enough. If $u'_N \leq \varphi_{ante}(u'_A)$, then IR-ante will bind for all values of c. The reason behind the last two statements is very simple: If IR-ante does not bind, then (u'_N, u'_A) is the optimal contract for $c = 0$. IR-ante does not bind at (u'_N, u'_A) only if $u'_N > \varphi_{ante}(u'_A)$. Note that IG-high changes continuously in c and therefore the same conclusion holds for $c > 0$ small enough.

Lemma S7 *If $u'_N > \varphi_{ante}(u'_A)$, then there exist a c' and a $c'' > c'$ such that*

- *only IG-high is binding if $c < c'$,*
- *IG-high and IR-ante are binding if $c \in (c', c'')$ and*
- *only IR-ante is binding if $c > c''$.*

If $u'_N \leq \varphi_{ante}(u'_A)$, then there exists a $c'' > 0$ such that

- *IG-high and IR-ante are binding if $c \in (0, c'')$ and*
- *only IR-ante is binding if $c > c''$.*

Proof. We already established that for c sufficiently small only IG-high is binding if $u'_N > \varphi_{ante}(u'_A)$. Since λ and μ are continuous in c and cannot both be zero for any c, it follows that there is a c' such that (i) $\lambda(c) = 0$ for all $c < c'$, (ii) $\lambda(c) > 0$ for $c \in (c', c' + \varepsilon)$ for some $\varepsilon > 0$ and (iii) $\mu(c') > 0$. For c slightly above c', both constraints bind and therefore λ is increasing in c and μ is decreasing in c. Let c'' be the lowest value of c where $\mu(c) = 0$. Note that μ is zero for all $c > c''$: Suppose otherwise and define $\hat{c} = \inf\{c : \mu(c) > 0 \text{ and } c > c''\}$. By the continuity of μ, $\mu(\hat{c}) = 0$ which implies that $\lambda(\hat{c}) > 0$ as at least one constraint must bind. By the continuity of λ, $\lambda(c) > 0$ for $c \in (\hat{c}, \hat{c} + \varepsilon)$ for some $\varepsilon > 0$. But this implies that μ cannot increase on $(\hat{c}, \hat{c} + \varepsilon)$ and therefore $\mu(c) = 0$ on $(\hat{c}, \hat{c} + \varepsilon)$. This contradicts the definition of $\hat{c}$.

The second statement is proven in the same way. □

The effect of a change of c on the profits π can be obtained from the envelope theorem:

$$\frac{\partial \pi^*_{x=0}}{\partial c} = -\mu\eta(1-\upsilon)((1-\bar{\theta})u_N + \bar{\theta}u_A)e^{\eta c} \geq 0.$$

Hence, profits (conditional on not inducing information gathering) are strictly increasing in c if IG-high binds and constant in c otherwise. This together with Lemma S6 gives the following result.

Proposition S3 *Assume no information gathering is induced. If $u'_N > \varphi_{ante}(u'_A)$, then*

- *for $c \in (0, c')$, expected utility of the agent is strictly decreasing in c and expected profits are strictly increasing in c;*

- *for $c \in (c', c'')$, expected utility is constant in c and expected profits are strictly increasing in c;*
- *for $c > c''$, expected utility and profits are constant in c.*

If $u'_N \leq \varphi_{ante}(u'_A)$, then

- *for $c \in (0, c'')$, expected utility is constant in c and expected profits are strictly increasing in c;*
- *for $c > c''$, expected utility and profits are constant in c.*

3.3. Combining the cases

We have shown that profits from inducing information acquisition are the profits of the Stiglitz model for $c = 0$. In the Stiglitz model, separation is always superior to pooling which implies that profits from information acquisition are higher than profits from not inducing information acquisition for $c = 0$. Profits are decreasing in c if the agent is induced to gather information while profits are increasing in c if the agent is induced not to gather information. For $c \to \infty$, obviously inducing no information acquisition leads to higher profits than inducing information acquisition. This implies that there must be a cost level $c^* > 0$ such that inducing information gathering is optimal for $c < c^*$ and not inducing information gathering is optimal for $c > c^*$. Note that $c^* < c''$ because profits are maximal for $c \geq c''$ (full insurance, no information gathering and the agent's individual rationality constraint is binding); that is, profits from not inducing information gathering are higher than the Stiglitz profits which are the highest profits achievable when inducing information gathering.

For c slightly below c'', profits are increasing in c and expected utility is constant in c (Proposition S3).[4] This implies that, for such values of c, a marginal reduction in c lowers welfare and leads to a Pareto inferior outcome.

[4] To be more precise: This holds for $c \in (\max\{c^*, c'\}, c'')$ if $u'_N > \varphi_{ante}(u'_A)$ and for $c \in (c^*, c'')$ if $u'_N \leq \varphi_{ante}(u'_A)$.