Supplementary Material to
"Strategic Gains from Discrimination"

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Contents

1 Introduction

In this Supplementary Material, which is not meant to be published, I provide proofs that were omitted from Lagerlöf (2019). In the next section, I prove Lemma 1 and Lemma 2 of Lagerlöf (2019). In Section 3, I state and prove six new lemmas (S1-S6) that will be used to prove Propositions 1 and and 5. Finally, in Section 4, I prove Propositions 1-5 of Lagerlöf (2019).

2 Proofs of Lemma 1 and Lemma 2

First, for convenience, let us copy in the following equations from Lagerlöf (2019):

$$R_i(w_{-i}) = \begin{cases} \frac{w_{-i}+p-t}{2} & \text{if } w_{-i} > p-3t \\ w_{-i}+t & \text{if } w_{-i} \leq p-3t, \end{cases} \tag{1}$$

$$\pi_2 = \begin{cases} (p-w_2)\left[\gamma_A\left(1-\overline{x}\right)+\gamma_B \frac{w_2}{t}\right] & \text{if } w_2 < t \\ (p-w_2)\left[\gamma_A\left(1-\overline{x}\right)+\gamma_B\right] & \text{if } w_2 \geq t, \end{cases} \tag{2}$$

$$w_{A|B} = w_{B|A} = \begin{cases} t & \text{if } \frac{t}{p} < \frac{1}{2} \\ \frac{p}{2} & \text{if } \frac{t}{p} \geq \frac{1}{2}. \end{cases} \tag{3}$$

In addition, the two large tables (Table 1 and Table 2) in Lagerlöf (2019) are copied in here.

2.1 Proof of Lemma 1 in Lagerlöf (2019)

In order to prove the lemma, it suffices to show the claims about the subgame $(y_1, y_2) = (A, C)$. The results for $(y_1, y_2) = (C, A)$ then follow by symmetry of the game.

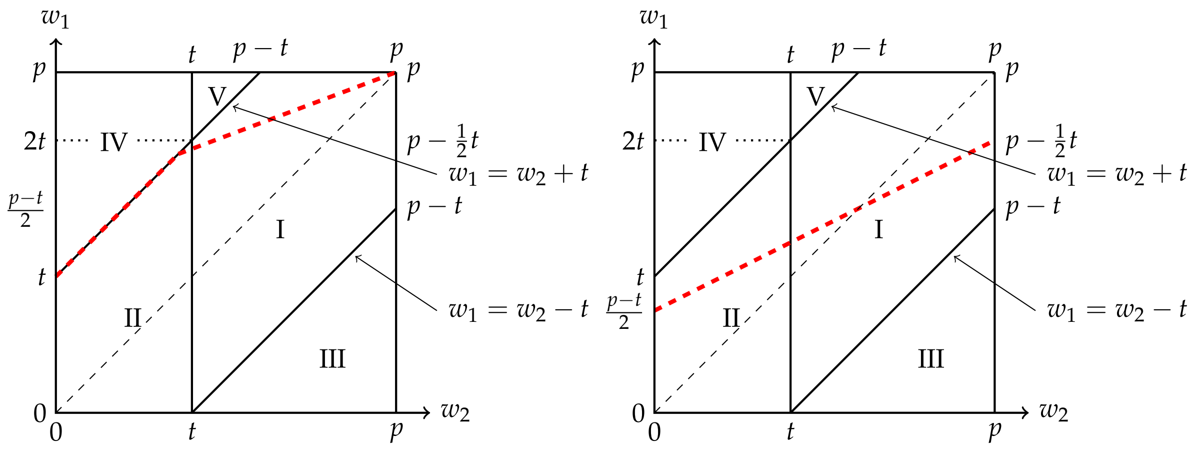
Thus consider the case $(y_1, y_2) = (A, C)$. Panels (a) and (b) of Fig. 1 illustrate the possible stage 2 outcomes in the (w_2, w_1)-space. These figures make use of some of the information stated in subsection 3.1 in Lagerlöf (2019)—for example, the fact that the threshold value $\overline{x}$ lies strictly inside the unit interval if and only if $w_1 \in (w_2 - t, w_2 + t)$. In region I of the figure, the A market is covered and shared by the two firms; moreover, the B market is covered. In region II, the A market is covered and shared by the two firms, but the B market is *not* covered. And so on for the other indicated regions. Firm 1's reaction function, as stated in equation (1), is graphed in Fig. 1 as a thick dashed (red) line; panel (a) shows the case where $t/p < 1/3$, meaning that for low enough values of w_2 firm 1 employs all workers in market A, while panel (b) shows the case where $t/p \geq 1/3$.[1]

It is clear that firm 1's reaction function passes through regions I and II. It may also be located on the line $w_1 = w_2 + t$. We can therefore conclude that an equilibrium must lie: (i) in the interior of region I; (ii) in the interior of region II; (iii) on the border between regions I and II, where $w_2 = t$; or (iv) on the line where $w_1 = w_2 + t$. Below I will investigate under what circumstances, if any, there is a pure strategy equilibrium in each one of these regions.

Finding an eq. in region I (where the B market is covered)

In (the interior of) region I there cannot be equilibrium where firm 1's wage choice is "in a corner" (i.e., given by the second line of (1)). Thus firm 1's best reply is interior (i.e., given by the first line of (1)). Given that we are in region I, firm 2's profit is given by the second line of (2) and, hence, the

[1]Both panels assume that $t/p < 1/2$. If $t/p \geq 1/2$, then region V in the figures disappears, but there are no qualitative changes that affect the reasoning below.



(a) The case $\frac{t}{p} < \frac{1}{3}$.

(b) The case $\frac{t}{p} \geq \frac{1}{3}$.

Figure 1: Finding an eq. of the subgame $(y_1, y_2) = (A, C)$.

associated first-order condition is:

$$\frac{\partial \pi_2}{\partial w_2} = -\left[1 - \frac{\gamma_A}{2t}(w_1 - w_2 + t)\right] + \frac{\gamma_A}{2t}(p - w_2) = 0,$$

which simplifies to

$$2t - \gamma_A (w_1 - w_2 + t) = \gamma_A (p - w_2). \tag{4}$$

Equation (4) and the first line of (1) define a linear equation system in w_1 and w_2. Solving this yields

$$w_1 = p - \frac{(2+\gamma_A)t}{3\gamma_A} = p - \frac{(3-\gamma_B)t}{3(1-\gamma_B)}, \qquad w_2 = p - \frac{(4-\gamma_A)t}{3\gamma_A} = p - \frac{(3+\gamma_B)t}{3(1-\gamma_B)}. \tag{5}$$

Also, using (5), we can compute firm 1's profit and firm 2's profit at the possible equilibrium:

$$\pi_1^* = \frac{\gamma_A}{2t}(p - w_1)^2 = \frac{\gamma_A}{2t}\left(\frac{2+\gamma_A}{3\gamma_A}t\right)^2 = \frac{t\,(2+\gamma_A)^2}{18\gamma_A} = \frac{t\,(3-\gamma_B)^2}{18(1-\gamma_B)}, \tag{6}$$

$$\pi_2^* = \frac{\gamma_A}{2t}(p - w_2)^2 = \frac{\gamma_A}{2t}\left(\frac{4-\gamma_A}{3\gamma_A}t\right)^2 = \frac{t\,(4-\gamma_A)^2}{18\gamma_A} = \frac{t\,(3+\gamma_B)^2}{18(1-\gamma_B)}. \tag{7}$$

We can now check the conditions that are required for being in (the interior of) region I. First,

$$w_1 > w_2 - t \Leftrightarrow p - \frac{(2+\gamma_A)t}{3\gamma_A} > p - \frac{(4-\gamma_A)t}{3\gamma_A} - t \Leftrightarrow 2 + \gamma_A > 0,$$

which always holds. Second,

$$w_1 < w_2 + t \Leftrightarrow p - \frac{(2+\gamma_A)t}{3\gamma_A} < p - \frac{(4-\gamma_A)t}{3\gamma_A} + t \Leftrightarrow \gamma_A > \frac{2}{5},$$

which also always holds. Third, the B market must indeed be covered:[2]

$$w_2 - t > 0 \Leftrightarrow p - \frac{(4-\gamma_A)t}{3\gamma_A} > t \Leftrightarrow \frac{t}{p} < \frac{3\gamma_A}{2(2+\gamma_A)}. \tag{8}$$

Finally, firm 1's best response must indeed be given by the first line of (1):

$$w_2 > p - 3t \Leftrightarrow p - \frac{(4-\gamma_A)t}{3\gamma_A} > p - 3t \Leftrightarrow \gamma_A > \frac{2}{5},$$

[2] This implies that also the A market is covered, since the worker who has the most distant location must travel farther in a monopsony market.

which again always holds.

There are two kinds of deviations that potentially could be profitable: Firm 2 could give up its ambition to hire anyone in the A market and instead choose the wage that maximizes its profits when hiring only in the B market; or firm 2 could stay in the A market but choose some wage $w_2 < t$, yielding a profit given by the first line of (2). The second kind of deviation is never profitable. If it were, the derivative of firm 2's profit function, as stated in the first line of (2) and evaluated at firm 1's wage and at $w_2 = t$, would be negative:

$$\frac{\partial \pi_2^{dev}}{\partial w_2} \Big|_{(w_1,w_2)=\left(p-\frac{(2+\gamma_A)t}{3\gamma_A},t\right)} < 0 \Leftrightarrow \frac{t}{p} > \frac{3}{7-\gamma_A}.$$

But the above inequality is inconsistent with (8).

Thus consider the first kind of deviation, where firm 2 gives up on the A market. Here firm 2 could choose $w_2 = t$ or it could choose some $w_2 \in (0, t)$. If making the latter deviation, the best deviation maximizes $\pi_2 = \frac{\gamma_B}{t}(p - w_2) w_2$, i.e., it is given by $w_2 = \frac{p}{2}$. For this wage to indeed be interior, we must have

$$\frac{p}{2} < t \Leftrightarrow \frac{t}{p} > \frac{1}{2}, \tag{9}$$

which is inconsistent with (8). This means that the best possible deviation is $w_2 = t$. Making this deviation, given that w_1 is given by (5), would yield the profit

$$\pi_2^{dev} = \gamma_B (p - w_2) = \gamma_B (p - t). \tag{10}$$

Thus, there is no incentive to deviate if, and only if,

$$\pi_2^* \geq \pi_2^{dev} \Leftrightarrow \frac{t(4-\gamma_A)^2}{18\gamma_A} \geq \gamma_B (p-t) \Leftrightarrow \frac{t}{p} \geq \frac{18\gamma_A\gamma_B}{(4-\gamma_A)^2 + 18\gamma_A\gamma_B} = \varphi(\gamma_B). \tag{11}$$

We can conclude that if (8) and (11) hold, then there is an equilibrium where the prices are given by (5), and the associated profit levels are given by (6) and (7). This yields the bottom line in Table 1.

Finding an eq. in region II (where the B market is not covered)

Again, in (the interior of) region II there cannot be an equilibrium where firm 1's wage choice is "in a corner" (i.e., given by the second line of (1)). Thus firm 1's best reply is interior (i.e., given by the first line of (1)). Given that we are in region II, firm 2's profit is given by the first line of (2) and, hence, the associated first-order condition is:

$$\frac{\partial \pi_2}{\partial w_2} = -\frac{1}{2t}\left[\gamma_A (w_2 - w_1 + t) + 2\gamma_B w_2\right] + \frac{\gamma_A + 2\gamma_B}{2t}(p - w_2) = 0,$$

which simplifies to

$$\gamma_A (w_2 - w_1 + t) + 2\gamma_B w_2 = (\gamma_A + 2\gamma_B)(p - w_2). \tag{12}$$

Equation (12) and the first line of (1) define a linear equation system in w_1 and w_2. Solving this yields

$$w_1 = \frac{3(1+\gamma_B)p - (3+\gamma_B)t}{3+5\gamma_B}, \qquad w_2 = \frac{(3+\gamma_B)p - 3(1-\gamma_B)t}{3+5\gamma_B}. \tag{13}$$

We can now check the conditions that are required for being in (the interior of) region II. First,

$$w_2 < t \Leftrightarrow \frac{(3+\gamma_B)p - 3(1-\gamma_B)t}{3+5\gamma_B} < t \Leftrightarrow \frac{t}{p} > \frac{1}{2}. \tag{14}$$

Table 1: Equilibrium wages and profits in the subgame where $(y_1, y_2) \in \{(A,C),(C,A)\}$

Eq. behavior	Condition	$w_{A\|C}$	$w_{C\|A}$	$\pi_{A\|C}$	$\pi_{C\|A}$
Low-wage eq.	$\frac{t}{p} \in \left(\frac{1}{2}, \frac{2}{3}\right]$	$\frac{3(1+\gamma_B)p-(3+\gamma_B)t}{3+5\gamma_B}$	$\frac{(3+\gamma_B)p-3(1-\gamma_B)t}{3+5\gamma_B}$	$\frac{\gamma_A}{2t}\left[\frac{2\gamma_B p+(3+\gamma_B)t}{3+5\gamma_B}\right]^2$	$\frac{1+\gamma_B}{2t}\left[\frac{4\gamma_B p+3(1-\gamma_B)t}{3+5\gamma_B}\right]^2$
Middle-wage eq.	$\frac{t}{p} \in \left[\frac{3(1-\gamma_B)}{2(3-\gamma_B)}, \frac{1}{2}\right]$	$\frac{p}{2}$	t	$\frac{(1-\gamma_B)p^2}{8t}$	$\frac{(p-t)[4t-(1-\gamma_B)p]}{4t}$
High-wage eq.	$\frac{t}{p} \in \left[\varphi(\gamma_B), \frac{3(1-\gamma_B)}{2(3-\gamma_B)}\right)$	$p - \frac{(3-\gamma_B)t}{3(1-\gamma_B)}$	$p - \frac{(3+\gamma_B)t}{3(1-\gamma_B)}$	$\frac{t(3-\gamma_B)^2}{18(1-\gamma_B)}$	$\frac{t(3+\gamma_B)^2}{18(1-\gamma_B)}$

Table 2: Equilibrium wages and profits in the subgame where $(y_1, y_2) \in \{(B,C),(C,B)\}$

Eq. behavior	Condition	$w_{B\|C}$	$w_{C\|B}$	$\pi_{B\|C}$	$\pi_{C\|B}$
Low-wage eq.	$\frac{t}{p} \in \left(\frac{1}{2}, \frac{2}{3}\right]$	$\frac{3(2-\gamma_B)p-(4-\gamma_B)t}{8-5\gamma_B}$	$\frac{(4-\gamma_B)p-3\gamma_B t}{8-5\gamma_B}$	$\frac{\gamma_B}{2t}\left[\frac{2(1-\gamma_B)p+(4-\gamma_B)t}{8-5\gamma_B}\right]^2$	$\frac{(2-\gamma_B)}{2t}\left[\frac{4(1-\gamma_B)p+3\gamma_B t}{8-5\gamma_B}\right]^2$
Middle-wage eq.	$\frac{t}{p} \in \left[\max\left\{\frac{3\gamma_B}{2(2+\gamma_B)}, \frac{1}{4}, \varphi(\gamma_B)\right\}, \frac{1}{2}\right]$	$\frac{p}{2}$	t	$\frac{\gamma_B p^2}{8t}$	$\frac{(p-t)(4t-\gamma_B p)}{4t}$
Full segmentation	$\frac{t}{p} \in \left[\varphi(\gamma_B), \frac{1}{4}\right]$	$2t$	t	$\gamma_B(p-2t)$	$\gamma_A(p-t)$
High-wage eq.	$\frac{t}{p} \in \left[\varphi(\gamma_B), \frac{3\gamma_B}{2(2+\gamma_B)}\right)$	$p - \frac{(2+\gamma_B)t}{3\gamma_B}$	$p - \frac{(4-\gamma_B)t}{3\gamma_B}$	$\frac{t(2+\gamma_B)^2}{18\gamma_B}$	$\frac{t(4-\gamma_B)^2}{18\gamma_B}$

Second,

$$w_1 < w_2 + t \Leftrightarrow \frac{3(1+\gamma_B)p - (3+\gamma_B)t}{3+5\gamma_B} < \frac{(3+\gamma_B)p - 3(1-\gamma_B)t}{3+5\gamma_B} + t \Leftrightarrow \frac{t}{p} > \frac{2\gamma_B}{3(1+3\gamma_B)},$$

which is implied by the condition above that $\frac{t}{p} > \frac{1}{2}$. Third, firm 1's best response must indeed be given by the first line of (1):

$$w_2 > p - 3t \Leftrightarrow \frac{(3+\gamma_B)p - 3(1-\gamma_B)t}{3+5\gamma_B} > p - 3t \Leftrightarrow \frac{t}{p} > \frac{2\gamma_B}{3(1+3\gamma_B)},$$

which is identical to the condition immediately above. Fourth, the A market must indeed be covered:

$$w_1 - t\overline{x} \geq 0 \Leftrightarrow w_1 \geq \frac{t}{2t}(w_1 - w_2 + t) \Leftrightarrow w_1 + w_2 \geq t \Leftrightarrow \frac{t}{p} \leq \frac{2(3+2\gamma_B)}{3(3+\gamma_B)},$$

which is implied by the assumption $\frac{t}{p} \leq \frac{2}{3}$.

Calculate firm 1's and firm 2's profit at the possible equilibrium:

$$\pi_1^* = \frac{\gamma_A}{2t}(p - w_1)^2 = \frac{\gamma_A}{2t}\left[p - \frac{3(1+\gamma_B)p - (3+\gamma_B)t}{3+5\gamma_B}\right]^2 = \frac{\gamma_A}{2t}\left[\frac{2\gamma_B p + (3+\gamma_B)t}{3+5\gamma_B}\right]^2, \quad (15)$$

$$\pi_2^* = \frac{(\gamma_A + 2\gamma_B)(p - w_2)^2}{2t} = \frac{(1+\gamma_B)}{2t}\left[p - \frac{(3+\gamma_B)p - 3(1-\gamma_B)t}{3+5\gamma_B}\right]^2 = \frac{(1+\gamma_B)}{2t}\left[\frac{4\gamma_B p + 3(1-\gamma_B)t}{3+5\gamma_B}\right]^2. \quad (16)$$

There is one kind of deviation that we must check: Firm 2 could give up its ambition to hire in the A market and instead choose the wage that maximizes its profit when hiring only in the B market. If making this deviation, the best deviation maximizes $\pi_2 = \frac{\gamma_B}{t}(p - w_2)w_2$, i.e., it is given by $w_2 = \frac{p}{2}$. (This wage is indeed interior, for $\frac{p}{2} < t \Leftrightarrow \frac{t}{p} > \frac{1}{2}$, which is identical to (14).) Making this deviation would yield the profit

$$\pi_2^{dev} = \gamma_B\left(p - \frac{p}{2}\right)\frac{p}{2t} = \frac{\gamma_B p^2}{4t}. \quad (17)$$

Thus, there is no incentive to deviate if, and only if,

$$\pi_2^* \geq \pi_2^{dev} \Leftrightarrow \frac{(1+\gamma_B)}{2t}\left[\frac{4\gamma_B p + 3(1-\gamma_B)t}{3+5\gamma_B}\right]^2 \geq \frac{\gamma_B p^2}{4t} \Leftrightarrow (1+\gamma_B)\left[\frac{8\gamma_B + 6(1-\gamma_B)\frac{t}{p}}{3+5\gamma_B}\right]^2 \geq 2\gamma_B. \quad (18)$$

It is easy to verify that the left-hand side of the last inequality is increasing in $\frac{t}{p}$ and, evaluated at $\frac{t}{p} = \frac{1}{2}$, equals $1 + \gamma_B$; hence the inequality holds for all $\frac{t}{p} > \frac{1}{2}$. This means that there is no profitable deviation.

We can conclude that if $\frac{t}{p} \in \left(\frac{1}{2}, \frac{2}{3}\right]$, then there is an equilibrium where the wages are given by (13), and the associated profit levels are given by (15) and (16). This yields the first line in Table 1.

Finding an eq. on the border between regions I and II (B market exactly covered)

In an equilibrium on the border between regions I and II, firm 2 chooses $w_2 = t$. Firm 1's reaction function is, as before, given by the first line of (1).[3] This means that in an equilibrium of this kind, firm

[3] The case under consideration (i.e., $w_2 = t$) is also consistent with firm 1's reaction function being given by the *second* line of (1). But, if so, we have $w_1 = w_2 + t$, which is the case dealt with below.

1's wage is given by $w_1 = \frac{w_2+p-t}{2} = \frac{p}{2}$. For $w_2 = t$ to be optimal for firm 2, given $w_1 = \frac{p}{2}$, the following two conditions must hold:

$$\frac{\partial \pi_2}{\partial w_2}\Big|_{(w_1,w_2)=(\frac{p}{2},t)} \geq 0 \Leftrightarrow \frac{1}{2t}\left[\gamma_A\left(t - \frac{p}{2} + t\right) + 2\gamma_B t\right] \leq \frac{\gamma_A + 2\gamma_B}{2t}(p-t) \Leftrightarrow \frac{t}{p} \leq \frac{1}{2},$$

$$\frac{\partial \pi_2}{\partial w_2}\Big|_{(w_1,w_2)=(\frac{p}{2},t)} \leq 0 \Leftrightarrow 1 - \frac{\gamma_A}{2t}\left(\frac{p}{2} - t + t\right) \geq \frac{\gamma_A}{2t}(p-t) \Leftrightarrow \frac{t}{p} \geq \frac{3\gamma_A}{2(2+\gamma_A)} = \frac{3(1-\gamma_B)}{2(3-\gamma_B)}. \quad (19)$$

The profit expression that is differentiated in the first condition is given by the first line of (2), while the profit expression that is differentiated in (19) is given by the second line of (2).

We can now check the remaining conditions that are required for $(w_1, w_2) = \left(\frac{p}{2}, t\right)$ to be an equilibrium. First, firm 1's best response must indeed be given by the first line of (1):

$$w_2 > p - 3t \Leftrightarrow t > p - 3t \Leftrightarrow \frac{t}{p} > \frac{1}{4},$$

which is implied by (19) above. Second, the A market must indeed be covered:

$$w_1 - t\overline{x} \geq 0 \Leftrightarrow w_1 \geq \frac{t}{2t}(w_1 - w_2 + t) \Leftrightarrow w_1 + w_2 \geq t \Leftrightarrow \frac{p}{2} + t \geq t,$$

which always holds.

Now calculate firm 1's profit at the equilibrium:

$$\pi_1^* = (p - w_1)\gamma_A \overline{x} = (p - \frac{p}{2})\gamma_A \frac{p}{4t} = \frac{\gamma_A p^2}{8t} = \frac{(1-\gamma_B)p^2}{8t}. \quad (20)$$

And calculate firm 2's profit at the equilibrium:

$$\pi_2^* = (p - w_2)(1 - \gamma_A \overline{x}) = (p-t)\left(1 - \gamma_A \frac{p}{4t}\right) = \frac{(p-t)\left[4t - (1-\gamma_B)p\right]}{4t}. \quad (21)$$

We can conclude that if $\frac{t}{p} \in \left[\frac{3(1-\gamma_B)}{2(3-\gamma_B)}, \frac{1}{2}\right]$, then there is an equilibrium where the wages are given by $(w_1, w_2) = \left(\frac{p}{2}, t\right)$, and the associated profit levels are given by (20) and (21). This yields the middle line in Table 1.

Finding an equilibrium where $w_1 = w_2 + t$ holds

Consider finally the possibility of an equilibrium where the equality $w_1 = w_2 + t$ holds (and, as before, the A market is covered). In such an equilibrium, firm 1 is the only one hiring in the A market (cf. panel (a) of Fig. 1).

A first condition that must be satisfied for this kind of equilibrium to exist is that firm 1's reaction function is given by $w_1 = w_2 + t$, i.e., by the second line of (1). This requires that $w_2 < p - 3t$. Note that for this inequality to hold for some $w_2 \geq 0$, we must have $\frac{t}{p} < \frac{1}{3}$. We also know that firm 2 is active only in the B market, and it is a monopsonist in that market. Therefore firm 2's optimally chosen wage must equal $w_2 = t$ (this follows from (3) and the fact that $\frac{t}{p} < \frac{1}{3}$ implies $\frac{t}{p} < \frac{1}{2}$). This in turn means, since $w_1 = w_2 + t$, that $w_1 = 2t$. Firm 2's profits if $(w_1, w_2) = (2t, t)$ are given by

$$\pi_2 = \gamma_B(p - t).$$

When is indeed $(w_1, w_2) = (2t, t)$ an equilibrium? A first requirement is that, evaluated at $w_2 = t$, we have $w_2 \leq p - 3t$; this is equivalent to $\frac{t}{p} \leq \frac{1}{4}$. Second, firm 2 must not have an incentive to make a global deviation by entering the A market. An entry into the A market must involve an increase of w_2 from $w_2 = t$ to some higher wage, which in particular means that firm 2 will still employ all workers in the B market. The optimal deviation thus maximizes the profit expression in the second line of (2), and the associated first-order condition is given by (4). Plugging $w_1 = 2t$ into this first-order condition and then solving for w_2, we have

$$w_2^{\text{dev}} = \frac{\gamma_A p - (2 - 3\gamma_A)t}{2\gamma_A}. \tag{22}$$

One can verify that $\frac{t}{p} < \frac{1}{3}$ and $\gamma_A > \frac{1}{2}$ guarantee that $w_2^{\text{dev}} > t$ holds. Firm 2's profit if deviating to w_2^{dev} is

$$\pi_2^{\text{dev}} = \frac{\gamma_A}{2t}(p - w_2^{\text{dev}})^2 = \frac{\gamma_A}{2t}\left[\frac{\gamma_A p + (2 - 3\gamma_A)t}{2\gamma_A}\right]^2.$$

Therefore firm 2 has no incentive to deviate if, and only if,

$$\pi_2 \geq \pi_2^{\text{dev}} \Leftrightarrow \gamma_B(p - t) \geq \frac{\gamma_A}{2t}\left[\frac{\gamma_A p + (2 - 3\gamma_A)t}{2\gamma_A}\right]^2$$
$$\Leftrightarrow 8\gamma_A\gamma_B(p - t)t \geq [\gamma_A(p - t) + 2(1 - \gamma_A)t]^2 \Leftrightarrow [\gamma_A(p - t) - 2(1 - \gamma_A)t]^2 \leq 0.$$

The last inequality is always violated (it holds with equality if $\frac{t}{p} = \frac{\gamma_A}{2 - \gamma_A}$, but this is inconsistent with $\frac{t}{p} \leq \frac{1}{4}$ and $\gamma_A > \frac{1}{2}$). We can conclude that there does not exist an equilibrium with $w_1 = w_2 + t$. □

2.2 Proof of Lemma 2 in Lagerlöf (2019)

In order to prove the lemma, it suffices to show the claims about the subgame $(y_1, y_2) = (B, C)$. The results for the subgame $(y_1, y_2) = (C, B)$ then follow by symmetry of the game.

Thus suppose that $(y_1, y_2) = (B, C)$; that is, firm 1 discriminates in hiring against the majority group, group A, while firm 2 does not discriminate at all. The analysis of this case is very similar to the analysis in the proof of Lemma 1. Basically, we have to replace γ_A with γ_B (and vice versa) everywhere in our previous analysis. We also must re-examine the conditions for the various kinds of equilibria to exist, since these may now look different (for we have $\gamma_A > \frac{1}{2}$, while $\gamma_B < \frac{1}{2}$).

First consider an equilibrium in (the interior of) region I. By using (5), and by replacing γ_A with γ_B, we have

$$w_1 = p - \frac{(2 + \gamma_B)t}{3\gamma_B} \quad \text{and} \quad w_2 = p - \frac{(4 - \gamma_B)t}{3\gamma_B}. \tag{23}$$

Similarly, using (6) and (7), we obtain the following profit expressions:

$$\pi_1^* = \frac{t\,(2 + \gamma_B)^2}{18\gamma_B} \quad \text{and} \quad \pi_2^* = \frac{t\,(4 - \gamma_B)^2}{18\gamma_B}. \tag{24}$$

We now check all the conditions. The requirement that $w_1 > w_2 - t$ still always holds. The requirement that $w_1 < w_2 + t$ is equivalent to $\gamma_B > \frac{2}{5}$. The condition in (8) now becomes

$$w_2 - t > 0 \Leftrightarrow \frac{t}{p} < \frac{3\gamma_B}{2(2 + \gamma_B)}. \tag{25}$$

One can check that the next few arguments in the proof of Lemma 1 do not add any new condition to the analysis here. For example, the condition in (11) becomes

$$\frac{t}{p} \geq \frac{18(1-\gamma_B)\gamma_B}{(4-\gamma_B)^2 + 18(1-\gamma_B)\gamma_B}, \tag{26}$$

which is implied by the assumption $\frac{t}{p} \geq \varphi(\gamma_B)$. Moreover, one can verify that the two inequalities (25) and (26) jointly imply $\gamma_B > \frac{2}{5}$. We can thus conclude that if $\frac{t}{p} \in \left[\varphi(\gamma_B), \frac{3\gamma_B}{2(2+\gamma_B)}\right)$, then there is an equilibrium where the wages are given by (23), and the associated profit levels are given by (24). This yields the bottom line in Table 2.

Next consider an equilibrium in (the interior of) region II. By using (13) and by replacing γ_A with γ_B, we have

$$w_1 = \frac{3(1+\gamma_A)p - (3+\gamma_A)t}{3+5\gamma_A} = \frac{3(2-\gamma_B)p - (4-\gamma_B)t}{8-5\gamma_B}, \tag{27}$$

$$w_2 = \frac{(3+\gamma_A)p - 3(1-\gamma_A)t}{3+5\gamma_A} = \frac{(4-\gamma_B)p - 3\gamma_B t}{8-5\gamma_B}. \tag{28}$$

Similarly, using (15) and (16), we obtain the following profit expressions:

$$\pi_1^* = \frac{\gamma_B}{2t}\left[\frac{2\gamma_A p + (3+\gamma_A)t}{3+5\gamma_A}\right]^2 = \frac{\gamma_B}{2t}\left[\frac{2(1-\gamma_B)p + (4-\gamma_B)t}{8-5\gamma_B}\right]^2, \tag{29}$$

$$\pi_2^* = \frac{(1+\gamma_A)}{2t}\left[\frac{4\gamma_A p + 3(1-\gamma_A)t}{3+5\gamma_A}\right]^2 = \frac{2-\gamma_B}{2t}\left[\frac{4(1-\gamma_B)p + 3\gamma_B t}{8-5\gamma_B}\right]^2. \tag{30}$$

We now check all the conditions. The requirement that $w_2 < t$ still holds if, and only, if $\frac{t}{p} > \frac{1}{2}$. The requirement that $w_1 < w_2 + t$ is equivalent to

$$\frac{t}{p} > \frac{2\gamma_A}{3(1+3\gamma_A)} = \frac{2(1-\gamma_B)}{3(4-3\gamma_B)},$$

which is implied by $\frac{t}{p} > \frac{1}{2}$. The condition that firm 1's best response is given by the first line of (1) is, as before, identical to the condition immediately above. The requirement that the B market (this is, for the subgame under consideration, the market in which both firms are active) is covered can be written as

$$w_1 - t\overline{x} \geq 0 \Leftrightarrow \frac{t}{p} \leq \frac{2(3+2\gamma_A)}{3(3+\gamma_A)} = \frac{2(5-2\gamma_B)}{3(4-\gamma_B)},$$

which is implied by the assumption $\frac{t}{p} \leq \frac{2}{3}$. Finally consider the condition required for firm 2 not to have an incentive to deviate globally (by giving up its ambition to hire in the B market). It is clear that the arguments in the proof of Lemma 1 apply also here: There is no profitable such deviation (to see this, note that if we replace γ_A with γ_B in (18), the resulting inequality always holds, given $\frac{t}{p} > \frac{1}{2}$ and $\gamma_A < 1$). We can thus conclude that if $\frac{t}{p} \in \left(\frac{1}{2}, \frac{2}{3}\right]$, then there is an equilibrium where the wages are given by (27) and (28), and the associated profit levels are given by (29) and (30). This yields the first line in Table 2.

Next consider an equilibrium on the border between regions I and II. Here, as in the proof of Lemma 1, the wages are given by $(w_1, w_2) = \left(\frac{p}{2}, t\right)$. The profits are obtained by swapping γ_A and γ_B in (20) and (21):

$$\pi_1 = \pi_{B|C} = \frac{\gamma_B p^2}{8t}, \qquad \pi_2 = \pi_{C|B} = (p-t)\left(1 - \gamma_B \frac{p}{4t}\right). \tag{31}$$

Among the conditions required for $(w_1, w_2) = \left(\frac{p}{2}, t\right)$ to be an equilibrium, only one is affected when we replace γ_A with γ_B. This is condition (19), which now becomes:

$$\frac{\partial \pi_2}{\partial w_2} \Big|_{(w_1,w_2)=\left(\frac{p}{2},t\right)} \leq 0 \Leftrightarrow \frac{t}{p} \geq \frac{3\gamma_B}{2(2+\gamma_B)}. \tag{32}$$

The conditions that are the same as in the proof of Lemma 1 are $\frac{t}{p} \leq \frac{1}{2}$ and $\frac{t}{p} > \frac{1}{4}$. In addition, we have assumed that $\frac{t}{p} \geq \varphi(\gamma_B)$. Of the two latter conditions and of the condition in (32), either one can (depending the value of γ_B) be the most stringent one. We can thus conclude that if

$$\frac{t}{p} \in \left[\max\left\{\frac{3\gamma_B}{2(2+\gamma_B)}, \frac{1}{4}, \varphi(\gamma_B)\right\}, \frac{1}{2}\right],$$

then there is an equilibrium where $(w_1, w_2) = \left(\frac{p}{2}, t\right)$, and the associated profit levels are given by (31). This yields the second line in Table 2.

Finally we must investigate the possibility of an equilibrium where $w_1 = w_2 + t$ holds. It follows from the arguments in the proof of Lemma 1 that in this kind of equilibrium, $(w_1, w_2) = (2t, t)$. Moreover, it follows that we must have $\frac{t}{p} \leq \frac{1}{4}$. Similarly to the Lemma 1 proof, firm 2 must not have an incentive to make a global deviation by entering the B market. An entry into the B market must involve an increase of w_2 from $w_2 = t$ to some higher wage (so firm 2 would still employ all A workers). The optimal deviation must be given by (22), but with γ_A replaced by γ_B:

$$w_2^{\text{dev}} = \frac{\gamma_B p - (2 - 3\gamma_B)t}{2\gamma_B}.$$

This expression does not exceed t if, and only if,

$$\frac{t}{p} \geq \frac{\gamma_B}{2-\gamma_B}. \tag{33}$$

One can show that, given $\frac{t}{p} \leq \frac{1}{4}$, (33) is implied by the assumption that $\frac{t}{p} \geq \varphi(\gamma_B)$. Hence, $\frac{t}{p} \in \left[\varphi(\gamma_B), \frac{1}{4}\right]$ guarantees that firm 2 does not have a profitable deviation and therefore that $(w_1, w_2) = (2t, t)$ is an equilibrium. We can thus conclude that if $\frac{t}{p} \in \left[\varphi(\gamma_B), \frac{1}{4}\right]$, then there is an equilibrium where $(w_1, w_2) = (2t, t)$. The associated profit levels can be computed as $\pi_1 = \pi_{B|C} = \gamma_B(p - 2t)$ and $\pi_2 = \pi_{C|B} = \gamma_A(p - t)$. This yields the third line in Table 2. □

3 Proofs of Six New Lemmas to Be Used When Proving Propositions 1 and 5

In this section, I state and prove six lemmas. The lemmas relate various profit expressions to each other, for different parts of the parameter space. Knowledge about these relationships will then be used when proving Propositions 1 and 5.

Lemma S1. *Suppose $\frac{t}{p} \in \left(\varphi(\gamma_B), \frac{2}{3}\right)$. We then have the following relationships:*

a) $\pi_{D|A} = \pi_{A|A} + \pi_{B|A}$ *(and hence $\pi_{D|A} > \pi_{A|A}$ and $\pi_{D|A} > \pi_{B|A}$);*

b) $\pi_{D|B} = \pi_{A|B} + \pi_{B|B}$ *(and hence $\pi_{D|B} > \pi_{A|B}$ and $\pi_{D|B} > \pi_{B|B}$);*

c) $\pi_{C|C} = \pi_{C|D} = \pi_{D|C} = \pi_{D|D} > \pi_{A|D} > \pi_{B|D}$.

Proof of Lemma S1. Follows immediately from the results stated in Section 3 of Lagerlöf (2019). □

Lemma S2. *Suppose* $\frac{t}{p} \in \left[\frac{1}{2}, \frac{2}{3}\right)$. *We then have the following relationships:*

a) $\pi_{C|A} > \pi_{A|A}$, $\pi_{C|A} > \pi_{B|A}$;

b) $\pi_{C|C} > \pi_{A|C} > \pi_{B|C}$;

c) $\pi_{D|B} > \pi_{C|B}$;

d) $\pi_{C|B} > \pi_{A|B} > \pi_{B|B}$.

Proof of Lemma S2. Throughout, I make use of results stated in Section 3 of Lagerlöf (2019). First we have

$$\pi_{C|A} > \pi_{A|A} \Leftrightarrow \frac{1+\gamma_B}{2t}\left[\frac{4\gamma_B p + 3(1-\gamma_B)t}{3+5\gamma_B}\right]^2 > \frac{(1-\gamma_B)t}{2}$$
$$\Leftrightarrow (1+\gamma_B)\left[4\gamma_B \frac{p}{t} + 3(1-\gamma_B)\right]^2 - (1-\gamma_B)(3+5\gamma_B)^2 > 0.$$

But this inequality must hold for all $\frac{t}{p} \leq \frac{2}{3}$, because the left-hand side is increasing in $\frac{p}{t}$ and evaluated at $\frac{p}{t} = \frac{3}{2}$ it equals

$$9(1+\gamma_B)(1+\gamma_B)^2 - (1-\gamma_B)(3+5\gamma_B)^2,$$

which can be shown to be strictly positive for all $\gamma_B \in (0, \frac{1}{2})$. Second, we have

$$\pi_{C|A} > \pi_{B|A} \Leftrightarrow \frac{1+\gamma_B}{2t}\left[\frac{4\gamma_B p + 3(1+\gamma_B)t}{3+5\gamma_B}\right]^2 > \frac{\gamma_B p^2}{4t} \Leftrightarrow 2(1+\gamma_B)\left[4\gamma_B + 3(1+\gamma_B)\frac{t}{p}\right]^2 > \gamma_B(3+5\gamma_B)^2.$$

But this inequality must hold for all $\frac{t}{p} \geq \frac{1}{2}$, because the left-hand side is increasing in $\frac{t}{p}$ and the inequality holds (with some margin) at $\frac{t}{p} = \frac{1}{2}$. Third, we can write

$$\pi_{C|C} > \pi_{A|C} \Leftrightarrow \frac{t}{2} > \frac{\gamma_A}{2t}\left[\frac{2\gamma_B p + (3-\gamma_B)t}{3+5\gamma_B}\right]^2 \Leftrightarrow (3+5\gamma_B)^2 > \gamma_A[2\gamma_B \frac{p}{t} + (3-\gamma_B)]^2.$$

This inequality must hold for all $\frac{t}{p} \geq \frac{1}{2}$: The right-hand side is increasing in $\frac{p}{t}$, and evaluated at $\frac{p}{t} = 2$ the inequality is equivalent to $1 > \gamma_A$. Fourth, we can write

$$\pi_{A|C} > \pi_{B|C} \Leftrightarrow \frac{\gamma_A}{2t}\left[\frac{2\gamma_B p + (3+\gamma_B)t}{3+5\gamma_B}\right]^2 > \frac{\gamma_B}{2t}\left[\frac{2(1-\gamma_B)p + (4-\gamma_B)t}{8-5\gamma_B}\right]^2 \Leftrightarrow$$
$$\Psi\left(\gamma_B, \frac{t}{p}\right) \stackrel{\text{def}}{=} \ln\left[(1-\gamma_B)\left(\frac{2\gamma_B + (3+\gamma_B)\frac{t}{p}}{3+5\gamma_B}\right)^2\right] - \ln\left[\gamma_B\left(\frac{2(1-\gamma_B) + (4-\gamma_B)\frac{t}{p}}{8-5\gamma_B}\right)^2\right] > 0.$$

Note that

$$\frac{\partial \Psi}{\partial\left(\frac{t}{p}\right)} = \frac{2(3+\gamma_B)}{2\gamma_B + (3+\gamma_B)\frac{t}{p}} - \frac{2(4-\gamma_B)}{2(1-\gamma_B) + (4-\gamma_B)\frac{t}{p}} = \frac{4\left[(3+\gamma_B)(1-\gamma_B) - (4-\gamma_B)\gamma_B\right]}{\left[2\gamma_B + (3+\gamma_B)\frac{t}{p}\right]\left[2(1-\gamma_B) + (4-\gamma_B)\frac{t}{p}\right]},$$

which is strictly positive thanks to the assumption that $\gamma_B < \frac{1}{2}$. It thus suffices to show that, evaluated at $\frac{t}{p} = \frac{1}{2}$, $\Psi\left(\gamma_B, \frac{t}{p}\right)$ is strictly positive. But it is easy to see that $\Psi\left(\gamma_B, \frac{1}{2}\right) = \ln(1-\gamma_B) - \ln(\gamma_B)$, which is strictly positive for all $\gamma_B < \frac{1}{2}$.

Fifth, we can write

$$\pi_{D|B} > \pi_{C|B} \Leftrightarrow \frac{\gamma_A p^2}{2t} + \frac{\gamma_B t}{2} > \frac{2-\gamma_B}{2t}\left[\frac{4(1-\gamma_B)p + 3\gamma_B t}{8-5\gamma_B}\right]^2 \Leftrightarrow$$

$$\varkappa(\gamma_B, \frac{t}{p}) \stackrel{\text{def}}{=} 1-\gamma_B + \gamma_B\left(\frac{t}{p}\right)^2 - (2-\gamma_B)\left[\frac{4(1-\gamma_B) + 3\gamma_B\left(\frac{t}{p}\right)}{8-5\gamma_B}\right]^2 > 0.$$

The function $\varkappa(\gamma_B, \frac{t}{p})$ is strictly increasing in $\frac{t}{p}$ if, and only if,

$$\frac{t}{p} > \frac{12(1-\gamma_B)(2-\gamma_B)}{(8-5\gamma_B)^2 - 9\gamma_B(2-\gamma_B)^2},$$

which is implied by $\frac{t}{p} > \frac{1}{2}$. It therefore suffices to show that $\varkappa(\gamma_B, \frac{1}{2}) > 0$ or, equivalently,

$$1-\gamma_B + \frac{\gamma_B}{4} - (2-\gamma_B)\left[\frac{4(1-\gamma_B) + \frac{3}{2}\gamma_B}{8-5\gamma_B}\right]^2 > 0,$$

which simplifies to $\gamma_B < 1$ and thus always hold.

Sixth, we have

$$\pi_{C|B} > \pi_{A|B} \Leftrightarrow \frac{2-\gamma_B}{2t}\left[\frac{4(1-\gamma_B)p + 3\gamma_B t}{8-5\gamma_B}\right]^2 > \frac{\gamma_A p^2}{4t}$$

$$\Leftrightarrow 2(2-\gamma_B)\left[4(1-\gamma_B) + 3\gamma_B\frac{t}{p}\right]^2 - (1-\gamma_B)(8-5\gamma_B)^2 > 0.$$

But this inequality must hold for all $\frac{t}{p} \geq \frac{1}{2}$, because the left-hand side is increasing in $\frac{t}{p}$ and evaluated at $\frac{p}{t} = \frac{1}{2}$ it equals

$$\frac{2-\gamma_B}{2}(8-5\gamma_B)^2 - (1-\gamma_B)(8-5\gamma_B)^2,$$

which is strictly positive for all $\gamma_B \in (0, \frac{1}{2})$.

Finally, we can write

$$\pi_{A|B} > \pi_{B|B} \Leftrightarrow \frac{\gamma_A p^2}{4t} > \frac{\gamma_B t}{2} \Leftrightarrow 1-\gamma_B > 2\gamma_B\left(\frac{t}{p}\right)^2.$$

But this inequality must hold for all $\frac{t}{p} \leq \frac{2}{3}$, because the right-hand side is increasing in $\frac{t}{p}$ and evaluated at $\frac{p}{t} = \frac{2}{3}$ the inequality becomes

$$9(1-\gamma_B) > 8\gamma_B,$$

which can be shown to hold for all $\gamma_B \in (0, \frac{1}{2})$. □

Lemma S3. *Suppose $\frac{t}{p} \in \left[\frac{3(1-\gamma_B)}{2(3-\gamma_B)}, \frac{1}{2}\right)$. We then have the following relationships:*

a) $\pi_{C|A} \gtreqless \pi_{D|A}$ *as* $\frac{t}{p} \gtreqless \frac{1}{3}$;

b) $\pi_{D|C} \gtreqless \pi_{A|C}$ *as* $\frac{t}{p} \gtreqless \frac{\sqrt{1-\gamma_B}}{2}$;

c) $\pi_{C|B} \gtreqless \pi_{D|B}$ *as* $\frac{t}{p} \gtreqless \frac{1}{3}$;

d) $\pi_{A|C} > \pi_{B|C}$;

e) $\pi_{C|A} > \pi_{B|A}$ *and* $\pi_{A|B} > \pi_{B|B}$;

f) If $\gamma_B > \frac{3}{7}$*, then* $\pi_{B|A} > \pi_{A|A}$*;*

g) $\pi_{C|B} > \pi_{A|B}$*.*

Proof of Lemma S3. First, we can write

$$\pi_{C|A} \geq \pi_{D|A} \Leftrightarrow \frac{(p-t)\left[4t-(1-\gamma_B)p\right]}{4t} \geq \gamma_B(p-t) + \frac{\gamma_A t}{2}$$
$$\Leftrightarrow (p-t)\left[4t-(1-\gamma_B)p\right] \geq 4\gamma_B(p-t)t + 2(1-\gamma_B)t^2$$
$$\Leftrightarrow (1-\gamma_B)(p-t)p + 2(1-\gamma_B)t^2 - 4(1-\gamma_B)(p-t)t \leq 0$$
$$\Leftrightarrow 1 - \frac{t}{p} + 2\left(\frac{t}{p}\right)^2 - 4\left(1-\frac{t}{p}\right)\frac{t}{p} \leq 0 \Leftrightarrow \left(\frac{3t}{p}-1\right)\left(\frac{2t}{p}-1\right) \leq 0.$$

Since, by assumption, $\frac{t}{p} < \frac{1}{2}$, the last inequality is equivalent to $\frac{t}{p} \geq \frac{1}{3}$. Second, we can write

$$\pi_{D|C} > \pi_{A|C} \Leftrightarrow \frac{t}{2} > \frac{(1-\gamma_B)p^2}{8t} \Leftrightarrow \frac{t}{p} > \frac{\sqrt{1-\gamma_B}}{2},$$

from which the claim follows. Third, we can write

$$\pi_{C|B} \geq \pi_{D|B} \Leftrightarrow \frac{(p-t)\left(4t-\gamma_B p\right)}{4t} \geq \gamma_A(p-t) + \frac{\gamma_B t}{2}.$$

Note that this inequality is identical to the one above (see the calculations for $\pi_{C|A} \geq \pi_{D|A}$), except that γ_A and γ_B have swapped places. Therefore the result there, which did not depend on the particular value of $\gamma_B \in (0,1)$, applies here, too, and the claim follows. Fourth, the claim that $\pi_{A|C} > \pi_{B|C}$ follows immediately from Tables 1 and 2 in Lagerlöf (2019) and the assumption that $\gamma_B < \frac{1}{2}$. Fifth, we can write

$$\pi_{C|A} > \pi_{B|A} \Leftrightarrow \frac{(p-t)\left[4t-(1-\gamma_B)p\right]}{4t} > \gamma_B\left(p-t\right) \Leftrightarrow \frac{t}{p} > \frac{1}{4},$$

which always holds because $\frac{t}{p} \geq \frac{3(1-\gamma_B)}{2(3-\gamma_B)} > \frac{3}{10}$ for all $\gamma_B \in (0, \frac{1}{2})$. Sixth, we can write

$$\pi_{A|B} > \pi_{B|B} \Leftrightarrow \left(1-\gamma_B\right)\left(p-t\right) > \frac{\gamma_B t}{2} \Leftrightarrow \frac{t}{p} < \frac{2(1-\gamma_B)}{2-\gamma_B},$$

which holds for all $\gamma_B \in (0, \frac{1}{2})$. Seventh, we can write

$$\pi_{B|A} > \pi_{A|A} \Leftrightarrow \gamma_B\left(p-t\right) > \frac{\left(1-\gamma_B\right)t}{2} \Leftrightarrow \frac{t}{p} < \frac{2\gamma_B}{1+\gamma_B},$$

which holds for all $\gamma_B > \frac{3}{7}$ (indeed, it holds for all $\gamma_B > \frac{1}{3}$). Eighth, we can write

$$\pi_{C|B} > \pi_{A|B} \Leftrightarrow \frac{(p-t)\left(4t-\gamma_B p\right)}{4t} > (1-\gamma_B)(p-t) \Leftrightarrow \frac{t}{p} > \frac{1}{4},$$

which always holds under the assumptions of the lemma. □

Lemma S4. *Suppose* $\frac{t}{p} \in \left(\max\left\{\varphi(\gamma_B), \frac{3\gamma_B}{2(2+\gamma_B)}, \frac{1}{4}\right\}, \frac{3(1-\gamma_B)}{2(3-\gamma_B)}\right]$*. We then have the following relationships:*

a) $\pi_{C|A} \gtreqqless \pi_{D|A}$ *as* $\frac{t}{p} \gtreqqless \frac{9(1-\gamma_B)}{21-13\gamma_B}$*;*

b) $\pi_{A|C} > \pi_{C|C}$*;*

c) $\pi_{A|C} \gtreqqless \pi_{B|C}$ *as* $\frac{t}{p} \gtreqqless \frac{3\sqrt{\gamma_B(1-\gamma_B)}}{2(3-\gamma_B)}$*;*

d) $\pi_{C|B} \gtreqqless \pi_{D|B}$ *as* $\frac{t}{p} \gtreqqless \frac{1}{3}$;

e) $\pi_{C|A} > \pi_{A|A}$, $\pi_{C|A} > \pi_{B|A}$, $\pi_{C|B} > \pi_{A|B}$, *and* $\pi_{C|B} > \pi_{B|B}$.

Proof of Lemma S4. First, we can write

$$\pi_{C|A} \geq \pi_{D|A} \Leftrightarrow \frac{t\,(3+\gamma_B)^2}{18(1-\gamma_B)} \geq \gamma_B(p-t) + \gamma_A \frac{t}{2}$$
$$\Leftrightarrow \left[(3+\gamma_B)^2 + 18\gamma_B(1-\gamma_B) - 9(1-\gamma_B)^2\right] t \geq 18\gamma_B(1-\gamma_B)p$$
$$\Leftrightarrow 2(21-13\gamma_B)\gamma_B t \geq 18\gamma_B(1-\gamma_B)p \Leftrightarrow \frac{t}{p} \geq \frac{9(1-\gamma_B)}{21-13\gamma_B}.$$

Second, we can write

$$\pi_{A|C} > \pi_{C|C} \Leftrightarrow \frac{t\,(3-\gamma_B)^2}{18(1-\gamma_B)} > \frac{t}{2} \Leftrightarrow (3-\gamma_B)^2 > 9(1-\gamma_B),$$

which can be shown to hold for all $\gamma_B > 0$. Third, we can write

$$\pi_{A|C} \geq \pi_{B|C} \Leftrightarrow \frac{t\,(3-\gamma_B)^2}{18(1-\gamma_B)} \geq \frac{\gamma_B\, p^2}{8t} \Leftrightarrow 4\,(3-\gamma_B)^2\, t^2 \geq 9\gamma_B\,(1-\gamma_B)\, p^2$$

or

$$\frac{t}{p} \geq \frac{3\sqrt{\gamma_B(1-\gamma_B)}}{2(3-\gamma_B)}.$$

Fourth, we can write

$$\pi_{C|B} \geq \pi_{D|B} \Leftrightarrow \frac{(p-t)\,(4t-\gamma_B p)}{4t} \geq \gamma_A(p-t) + \gamma_B \frac{t}{2}$$
$$\Leftrightarrow [4t - \gamma_B p - 4(1-\gamma_B)t\,]\,(p-t) \geq 2\gamma_B t^2 \Leftrightarrow (4t-p)\,(p-t) \geq 2t^2 \Leftrightarrow (p-3t)\,(p-2t) \leq 0.$$

It follows from the assumptions in the lemma that $\frac{t}{p} < \frac{1}{2}$. Therefore, the above inequality is equivalent to $\frac{t}{p} \geq \frac{1}{3}$. Fifth, we can write

$$\pi_{C|A} > \pi_{A|A} \Leftrightarrow \frac{t\,(3+\gamma_B)^2}{18(1-\gamma_B)} > \frac{(1-\gamma_B)t}{2} \Leftrightarrow (3+\gamma_B)^2 - 9\,(1-\gamma_B)^2 > 0.$$

The left-hand side of the last inequality is strictly increasing in γ_B and it equals zero at $\gamma_B = 0$; hence it holds for all $\gamma_B \in (0, \frac{1}{2})$. Sixth, we can write

$$\pi_{C|A} > \pi_{B|A} \Leftrightarrow \frac{t\,(3+\gamma_B)^2}{18(1-\gamma_B)} > \gamma_B\,(p-t) \Leftrightarrow \frac{t}{p} > \varphi(\gamma_B),$$

which always holds. Seventh, we can write

$$\pi_{C|B} > \pi_{A|B} \Leftrightarrow \frac{(p-t)\,(4t-\gamma_B p)}{4t} > (1-\gamma_B)\,(p-t) \Leftrightarrow \frac{t}{p} > \frac{1}{4},$$

which is satisfied under the assumptions of the lemma. Eighth and finally, we can write

$$\pi_{C|B} > \pi_{B|B} \Leftrightarrow \frac{(p-t)\,(4t-\gamma_B p)}{4t} > \frac{\gamma_B t}{2} \Leftrightarrow -2(2+\gamma_B)\left(\frac{t}{p}\right)^2 + (4+\gamma_B)\frac{t}{p} - \gamma_B > 0.$$

The right-hand side of the last inequality is increasing in γ_B (since $\frac{t}{p} < \frac{1}{2}$). Moreover, the inequality clearly holds when evaluated at $\gamma_B = 0$ (since $\frac{t}{p} < 1$). Thus, the inequality holds for all $\gamma_B \in (0, \frac{1}{2})$. □

Lemma S5. *Suppose* $\frac{t}{p} \in \left(\varphi(\gamma_B), \frac{3\gamma_B}{2(2+\gamma_B)}\right]$*. We then have the following relationships:*

a) $\pi_{D|A} > \pi_{C|A} > \pi_{B|A}$ *and* $\pi_{C|A} > \pi_{A|A}$*;*

b) $\pi_{B|C} > \pi_{A|C} > \pi_{C|C}$*;*

c) $\pi_{D|B} > \pi_{C|B} > \pi_{B|B}$ *and* $\pi_{C|B} > \pi_{A|B}$*.*

Proof of Lemma S5. The relationships $\pi_{D|A} > \pi_{C|A}$ and $\pi_{A|C} > \pi_{C|C}$ are already shown in the proof of Lemma S4 (the arguments in question are valid also for this part of the parameter space). Consider the relationship $\pi_{C|A} > \pi_{B|A}$. We can write

$$\pi_{C|A} > \pi_{B|A} \Leftrightarrow \frac{t\,(3+\gamma_B)^2}{18(1-\gamma_B)} > \gamma_B\,(p-t) \Leftrightarrow \frac{t}{p} > \varphi(\gamma_B),$$

which holds for all $\gamma_B \in (0, \frac{1}{2})$. We can also write

$$\pi_{C|A} > \pi_{A|A} \Leftrightarrow \frac{t\,(3+\gamma_B)^2}{18(1-\gamma_B)} > \frac{(1-\gamma_B)\,t}{2} \Leftrightarrow (3+\gamma_B)^2 > 9(1-\gamma_B)^2,$$

which again holds for all $\gamma_B \in (0, \frac{1}{2})$. Next consider the relationship $\pi_{B|C} > \pi_{A|C}$. We can write

$$\pi_{B|C} > \pi_{A|C} \Leftrightarrow \frac{t\,(2+\gamma_B)^2}{18\gamma_B} > \frac{t\,(3-\gamma_B)^2}{18(1-\gamma_B)} \Leftrightarrow (1-2\gamma_B)(4-\gamma_B+\gamma_B^2) > 0,$$

which holds for all $\gamma_B \in (0, \frac{1}{2})$. Similarly, we can write

$$\begin{aligned}\pi_{C|B} < \pi_{D|B} &\Leftrightarrow \frac{t\,(4-\gamma_B)^2}{18\gamma_B} < \gamma_A(p-t) + \gamma_B\frac{t}{2}\\ &\Leftrightarrow \left[(4-\gamma_B)^2 + 18\gamma_B(1-\gamma_B) - 9\gamma_B^2\right] t < 18\gamma_B(1-\gamma_B)p\\ &\Leftrightarrow \left[16 + 10\gamma_B - 26\gamma_B^2\right] t = 2(8+13\gamma_B)(1-\gamma_B)t < 18\gamma_B(1-\gamma_B)p \Leftrightarrow \frac{t}{p} < \frac{9\gamma_B}{8+13\gamma_B}.\end{aligned}$$

But the last inequality is implied by $\frac{t}{p} \leq \frac{3\gamma_B}{2(2+\gamma_B)}$. Now consider the relationship $\pi_{C|B} > \pi_{B|B}$:

$$\pi_{C|B} > \pi_{B|B} \Leftrightarrow \frac{t\,(4-\gamma_B)^2}{18\gamma_B} > \gamma_B\frac{t}{2} \Leftrightarrow (4-\gamma_B)^2 > 9\gamma_B^2,$$

which holds for all $\gamma_B \in (0, \frac{1}{2})$. Finally write

$$\pi_{C|B} > \pi_{A|B} \Leftrightarrow \frac{t\,(4-\gamma_B)^2}{18\gamma_B} > (1-\gamma_B)(p-t) \Leftrightarrow \frac{t}{p} > \frac{18\gamma_B(1-\gamma_B)}{(4-\gamma_B)^2 + 18\gamma_B(1-\gamma_B)},$$

which is implied by $\frac{t}{p} > \varphi(\gamma_B)$. □

Lemma S6. *Suppose* $\frac{t}{p} \in \left(\varphi(\gamma_B), \frac{1}{4}\right]$*. We then have the following relationships:*

a) $\pi_{D|A} > \pi_{C|A}$*;*

b) $\pi_{A|C} > \pi_{C|C}$ *and* $\pi_{A|C} > \pi_{B|C}$*;*

c) $\pi_{D|B} > \pi_{C|B}$*;*

d) $\pi_{C|A} > \pi_{B|A}$ *and* $\pi_{C|A} > \pi_{A|A}$*;*

e) $\pi_{A|B} = \pi_{C|B} > \pi_{B|B}$.

Proof of Lemma S6. First note that $\frac{t}{p} \leq \frac{1}{4}$ implies $\frac{t}{p} < \frac{1}{3}$. The claims that $\pi_{D|A} > \pi_{C|A}$ and $\pi_{A|C} > \pi_{C|C}$ are already shown in the proof of Lemma S4 (the arguments in question are valid also for this part of the parameter space). Similarly, the relationships $\pi_{C|A} > \pi_{B|A}$ and $\pi_{C|A} > \pi_{A|A}$ are already shown in the proof of Lemma S5 (the arguments are valid also for this part of the parameter space).

Thus consider the relationship $\pi_{A|C} > \pi_{B|C}$. We can write

$$\pi_{A|C} > \pi_{B|C} \Leftrightarrow \frac{t\,(3-\gamma_B)^2}{18(1-\gamma_B)} > \gamma_B(p-2t) \Leftrightarrow \frac{t}{p} > \frac{18\gamma_B(1-\gamma_B)}{(3-\gamma_B)^2 + 36\gamma_B(1-\gamma_B)}.$$

However, the last inequality is, given $\frac{t}{p} < \frac{1}{3}$, implied by our assumption that $\frac{t}{p} > \varphi(\gamma_B)$. Similarly, we can write

$$\pi_{D|B} > \pi_{C|B} \Leftrightarrow \gamma_A(p-t) + \gamma_B\frac{t}{2} > \gamma_A(p-t),$$

which trivially always holds. Next, it is clear that we have $\pi_{A|B} = \pi_{C|B}$, since both profit levels equal $\gamma_A(p-t)$. Finally we can write

$$\pi_{C|B} > \pi_{B|B} \Leftrightarrow \gamma_A(p-t) > \gamma_B\frac{t}{2} \Leftrightarrow \frac{t}{p} < \frac{2(1-\gamma_B)}{2-\gamma_B},$$

which holds for all $\gamma_B \in (0, \frac{1}{2})$. □

4 Proofs of Propositions 1-5

4.1 Proof of Proposition 1 in Lagerlöf (2019)

Proof of Proposition 1, part (i). First suppose $\frac{t}{p} \in \left[\frac{1}{2}, \frac{2}{3}\right)$. Consider the game matrix in Figure 1 in Lagerlöf (2019). Let us study one column of the matrix at a time, while using the results in Lemmas S1 and S2.

- In column A, firm 1 prefers C to A and B (for we have $\pi_{C|A} > \pi_{A|A}$ and $\pi_{C|A} > \pi_{B|A}$), and it prefers D to A and B (for we have $\pi_{D|A} > \pi_{A|A}$ and $\pi_{D|A} > \pi_{B|A}$). However, neither $(y_1, y_2) = (C, A)$ nor $(y_1, y_2) = (D, A)$ can be an equilibrium, as firm 2 would in both cases have an incentive to deviate to C or to D (for we have $\pi_{C|C} = \pi_{D|C} > \pi_{A|C}$ and $\pi_{C|D} = \pi_{D|D} > \pi_{A|D}$).

- In column B, firm 1 prefers D to A, B and C (for we have $\pi_{D|B} > \pi_{A|B}$, $\pi_{D|B} > \pi_{B|B}$ and $\pi_{D|B} > \pi_{C|B}$). But $(y_1, y_2) = (D, B)$ cannot be an equilibrium, as firm 2 would have an incentive to deviate to C or D (for we have $\pi_{C|D} = \pi_{D|D} > \pi_{B|D}$).

- In column C, firm 1 is indifferent between C and D and prefers each one of these choices to A and B (for we have $\pi_{C|C} = \pi_{D|C} > \pi_{A|C} > \pi_{B|C}$). By symmetry (in particular, by the inequalities stated in the previous sentence), if firm 1 chooses C, then firm 2 has a (weak) incentive to also choose C. Hence $(y_1, y_2) = (C, C)$ is an equilibrium. If firm 1 chooses D, then firm 2 has a weak incentive to choose C (for we have $\pi_{C|D} = \pi_{D|D} > \pi_{A|D} > \pi_{B|D}$). Hence $(y_1, y_2) = (D, C)$ is an equilibrium.

- In column D, firm 1 is indifferent between C and D and prefers each one of these choices to A and B (for we have $\pi_{C|D} = \pi_{D|D} > \pi_{A|D} > \pi_{B|D}$). Indeed, since the game is symmetric and we have found that $(y_1, y_2) = (D, C)$ is an equilibrium, so is $(y_1, y_2) = (C, D)$. Moreover, again by symmetry, since firm 1 weakly prefers D in column D, firm 2 weakly prefers D in row D. Hence $(y_1, y_2) = (D, D)$ is an equilibrium.

Next suppose $\frac{t}{p} \in \left(\frac{\sqrt{1-\gamma_B}}{2}, \frac{1}{2}\right)$, and again study one column of the game matrix at a time, while using the results in Lemmas S1 and S3. Note that $\frac{t}{p} > \frac{\sqrt{1-\gamma_B}}{2}$ implies $\frac{t}{p} > \frac{1}{3}$.

- In column A, firm 1 prefers C to A, B and D (for we have $\pi_{C|A} > \pi_{D|A} = \pi_{A|A} + \pi_{B|A}$). However, $(y_1, y_2) = (C, A)$ cannot be an equilibrium, as firm 2 would have an incentive to deviate to D (for we have $\pi_{D|C} > \pi_{A|C}$).

- In column B, firm 1 again prefers C to A, B and D (for we have $\pi_{C|B} > \pi_{D|B} = \pi_{A|B} + \pi_{B|B}$). But $(y_1, y_2) = (C, B)$ cannot be an equilibrium, as firm 2 would have an an incentive to deviate to D (for we have $\pi_{D|C} > \pi_{B|C}$).

- In column C, firm 1 is indifferent between C and D and prefers each one of these choices to A and B (for we have $\pi_{C|C} = \pi_{D|C} > \pi_{A|C} > \pi_{B|C}$). By symmetry (in particular, by the inequalities stated in the previous sentence), if firm 1 chooses C, then firm 2 has a (weak) incentive to also choose C. Hence $(y_1, y_2) = (C, C)$ is an equilibrium. If firm 1 chooses D, then firm 2 has a weak incentive to choose C (for we have $\pi_{C|D} = \pi_{D|D} > \pi_{A|D} > \pi_{B|D}$). Hence $(y_1, y_2) = (D, C)$ is an equilibrium.

- In column D, firm 1 is indifferent between C and D and prefers each one of these choices to A and to B (for we have $\pi_{C|D} = \pi_{D|D} > \pi_{A|D} > \pi_{B|D}$). Indeed, since the game is symmetric and we have found that $(y_1, y_2) = (D, C)$ is an equilibrium, so is $(y_1, y_2) = (C, D)$. Moreover, again by symmetry, since firm 1 weakly prefers D in column D, firm 2 weakly prefers D in row D. Hence $(y_1, y_2) = (D, D)$ is an equilibrium.

Proof of Proposition 1, part (ii). First suppose $\frac{t}{p} \in \left(\max\left\{\frac{3(1-\gamma_B)}{2(3-\gamma_B)}, \frac{1}{3}\right\}, \frac{\sqrt{1-\gamma_B}}{2}\right)$. Let us study one column of the matrix in Figure 1 in Lagerlöf (2019) at a time, while using the results in Lemmas S1 and S3.

- In column A, firm 1 prefers C to A, B and D (for we have $\pi_{C|A} > \pi_{D|A} = \pi_{A|A} + \pi_{B|A}$). Given that firm 1 chooses C, firm 2 does not have an incentive to deviate (for we have $\pi_{A|C} > \pi_{D|C} = \pi_{C|C}$ and $\pi_{A|C} > \pi_{B|C}$). Hence $(y_1, y_2) = (C, A)$ is an equilibrium.

- In column B, firm 1 again prefers C to A, B and D (for we have $\pi_{C|B} > \pi_{D|B} = \pi_{A|B} + \pi_{B|B}$). But $(y_1, y_2) = (C, B)$ cannot be an equilibrium, as firm 2 would have an an incentive to deviate to A (for we have $\pi_{A|C} > \pi_{B|C}$).

- In column C, by symmetry (cf. the above arguments about columns A and B) the following must hold: $(y_1, y_2) = (A, C)$ is an equilibrium, but $(y_1, y_2) = (B, C)$ is not. Moreover, neither $(y_1, y_2) = (C, C)$ nor $(y_1, y_2) = (D, C)$ is an equilibrium, as firm 1 would have an incentive to deviate to A (for we have $\pi_{A|C} > \pi_{D|C} = \pi_{C|C}$).

- In column D, firm 1 is indifferent between C and D and prefers each one of these choices to A and to B (for we have $\pi_{C|D} = \pi_{D|D} > \pi_{A|D} > \pi_{B|D}$). Since the game is symmetric and we have found that $(y_1, y_2) = (D, C)$ is not an equilibrium, nor is $(y_1, y_2) = (C, D)$. Moreover, again by symmetry, since firm 1 weakly prefers D in column D, firm 2 weakly prefers D in row D. Hence $(y_1, y_2) = (D, D)$ is an equilibrium.

Next suppose $\frac{t}{p} \in \left(\frac{9(1-\gamma_B)}{21-13\gamma_B}, \frac{3(1-\gamma_B)}{2(3-\gamma_B)}\right]$, and again study one column of the game matrix at a time, using the results in Lemmas S1 and S4. Note that $\frac{t}{p} > \frac{9(1-\gamma_B)}{21-13\gamma_B}$ implies $\frac{t}{p} > \frac{3\sqrt{\gamma_B(1-\gamma_B)}}{2(3-\gamma_B)}$ and that $\frac{t}{p} \in \left(\frac{9(1-\gamma_B)}{21-13\gamma_B}, \frac{3(1-\gamma_B)}{2(3-\gamma_B)}\right)$ implies $\frac{t}{p} > \frac{1}{3}$.

- In column A, firm 1 prefers C to A, B and D (for we have $\pi_{C|A} > \pi_{D|A} = \pi_{A|A} + \pi_{B|A}$). Given that firm 1 chooses C, firm 2 does not have an incentive to deviate (for we have $\pi_{A|C} > \pi_{C|C} = \pi_{D|C}$ and $\pi_{A|C} > \pi_{B|C}$). Hence $(y_1, y_2) = (C, A)$ is an equilibrium.

- In column B, firm 1 again prefers C to A, B and D (for we have $\pi_{C|B} > \pi_{D|B} = \pi_{A|B} + \pi_{B|B}$). But $(y_1, y_2) = (C, B)$ cannot be an equilibrium, as firm 2 would have an incentive to deviate to A (for we have $\pi_{A|C} > \pi_{B|C}$).

- In column C, by symmetry (cf. the above arguments about columns A and B) the following must hold: $(y_1, y_2) = (A, C)$ is an equilibrium, but $(y_1, y_2) = (B, C)$ is not. Moreover, neither $(y_1, y_2) = (C, C)$ nor $(y_1, y_2) = (D, C)$ is an equilibrium, as firm 1 would have an incentive to deviate to A (for we have $\pi_{A|C} > \pi_{C|C} = \pi_{D|C}$).

- In column D, firm 1 is indifferent between C and D and prefers each one of these choices to A and to B (for we have $\pi_{C|D} = \pi_{D|D} > \pi_{A|D} > \pi_{B|D}$). Since the game is symmetric and we have found that $(y_1, y_2) = (D, C)$ is not an equilibrium, nor is $(y_1, y_2) = (C, D)$. Moreover, again by symmetry, since firm 1 weakly prefers D in column D, firm 2 weakly prefers D in row D. Hence $(y_1, y_2) = (D, D)$ is an equilibrium.

Proof of Proposition 1 , part (iii). Suppose $\frac{t}{p} \in \left(\max\left\{\varphi(\gamma_B), \frac{3\gamma_B}{2(2+\gamma_B)}, \frac{1}{4}\right\}, \min\left\{\frac{3(1-\gamma_B)}{2(3-\gamma_B)}, \frac{9(1-\gamma_B)}{21-13\gamma_B}\right\}\right)$. Let us study one column of the matrix in Figure 1 in Lagerlöf (2019) at a time, while using the results in Lemmas S1 and S4.

- In column A, firm 1 prefers D to A, B and C (for we have $\pi_{D|A} > \pi_{C|A}$ and $\pi_{D|A} = \pi_{A|A} + \pi_{B|A}$). However, $(y_1, y_2) = (D, A)$ cannot be an equilibrium, as firm 2 would have an incentive to deviate to D (for we have $\pi_{D|D} > \pi_{A|D}$).

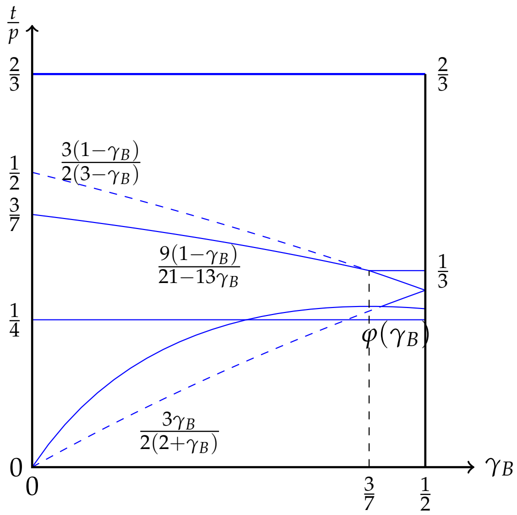


Figure 2: An illustration of different parts of the parameter space, which may be useful when studying the proofs of Propositions 1 and 3. See also panels (a) and (b) of Fig. 1.

- In column B, firm 1 prefers, depending on parameter values, either C or D to A and to B (for we have $\pi_{C|B} \gtreqless \pi_{D|B}$ as $\frac{t}{p} \gtreqless \frac{1}{3}$, and we always have $\pi_{D|B} = \pi_{A|B} + \pi_{B|B}$). But neither $(y_1, y_2) = (C, B)$ nor $(y_1, y_2) = (D, B)$ can be an equilibrium. In the former case firm 2 would have an incentive to deviate to A (for if $\pi_{C|B} \geq \pi_{D|B}$, then we must have $\pi_{A|C} > \pi_{B|C}$). In the latter case firm 2 would have an incentive to deviate to D (for we always have $\pi_{D|D} > \pi_{B|D}$).

- In column C, firm 1 prefers A to C and D (for we have $\pi_{A|C} > \pi_{C|C} = \pi_{D|C}$). Depending on parameter values, firm 1 may prefer A to B, or B to A. However, by symmetry of the game, neither $(y_1, y_2) = (A, C)$ nor $(y_1, y_2) = (B, C)$ can be an equilibrium since we showed above that $(y_1, y_2) = (C, A)$ and $(y_1, y_2) = (C, B)$ are not.

- In column D, firm 1 is indifferent between C and D and prefers each one of these choices to A and to B (for we have $\pi_{C|D} = \pi_{D|D} > \pi_{A|D} > \pi_{B|D}$). Since the game is symmetric and we have found that $(y_1, y_2) = (D, C)$ is not an equilibrium, nor is $(y_1, y_2) = (C, D)$. Moreover, again by symmetry, since firm 1 weakly prefers D in column D, firm 2 weakly prefers D in row D. Hence $(y_1, y_2) = (D, D)$ is an equilibrium.

Next suppose $\frac{t}{p} \in \left(\frac{3(1-\gamma_B)}{2(3-\gamma_B)}, \frac{1}{3}\right)$, and again study one column of the game matrix at a time, while using the results in Lemmas S1 and S3. Note that $\frac{t}{p} < \frac{1}{3}$ implies $\frac{t}{p} < \frac{\sqrt{1-\gamma_B}}{2}$.

- In column A, firm 1 prefers D to A, B and C (for we have $\pi_{D|A} > \pi_{C|A}$ and $\pi_{D|A} = \pi_{A|A} + \pi_{B|A}$). However, $(y_1, y_2) = (D, A)$ cannot be an equilibrium, as firm 2 would have an incentive to deviate to D (for we have $\pi_{D|D} > \pi_{A|D}$).

- In column B, firm 1 again prefers D to A, B and C (for we have $\pi_{D|B} > \pi_{C|B}$ and $\pi_{D|B} = \pi_{A|B} + \pi_{B|B}$). But $(y_1, y_2) = (D, B)$ cannot be an equilibrium, as firm 2 would have an incentive to deviate to D (for we have $\pi_{D|D} > \pi_{B|D}$).

- In column C, firm 1 prefers A to B, C and D (for we have $\pi_{A|C} > \pi_{D|C} = \pi_{C|C}$ and $\pi_{A|C} > \pi_{B|C}$). However, by symmetry of the game, $(y_1, y_2) = (A, C)$ cannot be an equilibrium since we showed above that $(y_1, y_2) = (C, A)$ is not.

- In column D, firm 1 is indifferent between C and D and prefers each one of these choices to A and to B (for we have $\pi_{C|D} = \pi_{D|D} > \pi_{A|D} > \pi_{B|D}$). By symmetry, if firm 1 plays D, then firm 2 does not have an incentive to deviate. Hence $(y_1, y_2) = (D, D)$ is an equilibrium. However, again by symmetry of the game, $(y_1, y_2) = (C, D)$ cannot be an equilibrium since we showed above that $(y_1, y_2) = (D, C)$ is not.

Now suppose $\frac{t}{p} \in \left(\varphi(\gamma_B), \frac{3\gamma_B}{2(2+\gamma_B)}\right)$, and again study one column of the game matrix at a time, while using the results in Lemmas S1 and S5.

- In column A, firm 1 prefers D to A, B and C (for we have $\pi_{D|A} > \pi_{C|A}$ and $\pi_{D|A} = \pi_{A|A} + \pi_{B|A}$). However, $(y_1, y_2) = (D, A)$ cannot be an equilibrium, as firm 2 would have an incentive to deviate to D (for we have $\pi_{D|D} > \pi_{A|D}$).

- In column B, firm 1 again prefers D to A, B and C (for we have $\pi_{D|B} > \pi_{C|B}$ and $\pi_{D|B} = \pi_{A|B} + \pi_{B|B}$). But $(y_1, y_2) = (D, B)$ cannot be an equilibrium, as firm 2 would have an incentive to deviate to D (for we have $\pi_{D|D} > \pi_{B|D}$).

- In column C, firm 1 prefers B to A, C and D (for we have $\pi_{B|C} > \pi_{A|C} > \pi_{C|C} = \pi_{D|C}$). However, by symmetry of the game, $(y_1, y_2) = (B, C)$ cannot be an equilibrium since we showed above that $(y_1, y_2) = (C, B)$ is not.

- In column D, firm 1 is indifferent between C and D and prefers each one of these choices to A and to B (for we have $\pi_{C|D} = \pi_{D|D} > \pi_{A|D} > \pi_{B|D}$). By symmetry, if firm 1 plays D, then firm 2 does not have an incentive to deviate. Hence $(y_1, y_2) = (D, D)$ is an equilibrium. However, again by symmetry of the game, $(y_1, y_2) = (C, D)$ cannot be an equilibrium since we showed above that $(y_1, y_2) = (D, C)$ is not.

Finally suppose $\frac{t}{p} \in \left(\varphi(\gamma_B), \frac{1}{4}\right)$, and again study one column of the game matrix at a time, while using the results in Lemmas S1 and S6.

- In column A, firm 1 prefers D to A, B and C (for we have $\pi_{D|A} > \pi_{C|A}$ and $\pi_{D|A} = \pi_{A|A} + \pi_{B|A}$). However, $(y_1, y_2) = (D, A)$ cannot be an equilibrium, as firm 2 would have an incentive to deviate to D (for we have $\pi_{D|D} > \pi_{A|D}$).

- In column B, firm 1 again prefers D to A, B and C (for we have $\pi_{D|B} > \pi_{C|B}$ and $\pi_{D|B} = \pi_{A|B} + \pi_{B|B}$). But $(y_1, y_2) = (D, B)$ cannot be an equilibrium, as firm 2 would have an incentive to deviate to D (for we have $\pi_{D|D} > \pi_{B|D}$).

- In column C, firm 1 prefers A to B, C and D (for we have $\pi_{A|C} > \pi_{B|C}$ and $\pi_{A|C} > \pi_{C|C} = \pi_{D|C}$). However, by symmetry of the game, $(y_1, y_2) = (A, C)$ cannot be an equilibrium since we showed above that $(y_1, y_2) = (C, A)$ is not.

- In column D, firm 1 is indifferent between C and D and prefers each one of these choices to A and to B (for we have $\pi_{C|D} = \pi_{D|D} > \pi_{A|D} > \pi_{B|D}$). By symmetry, if firm 1 plays D, then firm 2 does not have an incentive to deviate. Hence $(y_1, y_2) = (D, D)$ is an equilibrium. However, again by symmetry of the game, $(y_1, y_2) = (C, D)$ cannot be an equilibrium since we showed above that $(y_1, y_2) = (D, C)$ is not.

□

4.2 Proof of Proposition 2 in Lagerlöf (2019)

To prove the claim it suffices to show that, given $\frac{t}{p} \in \Omega_{II}$, we have $\pi_{C|A} \geq \pi_{A|C} > \pi_{D|D}$. From subsection 3.1.2 in Lagerlöf (2019), we know that $\pi_{D|D} = \frac{t}{2}$. The expressions for $\pi_{A|C}$ and $\pi_{C|A}$ depend on whether (i) $\frac{t}{p} \in \left(\frac{9(1-\gamma_B)}{21-13\gamma_B}, \frac{3(1-\gamma_B)}{2(3-\gamma_B)}\right]$ or (ii) $\frac{t}{p} \in \left(\max\left\{\frac{3(1-\gamma_B)}{2(3-\gamma_B)}, \frac{1}{3}\right\}, \frac{\sqrt{1-\gamma_B}}{2}\right)$. For case (i) we have a high-wage equilibrium and, by Table 1 in Lagerlöf (2019),

$$\pi_{A|C} > \pi_{D|D} \Leftrightarrow \frac{t\,(3-\gamma_B)^2}{18\,(1-\gamma_B)} > \frac{t}{2} \Leftrightarrow (3-\gamma_B)^2 > 9\,(1-\gamma_B) \Leftrightarrow 3\gamma_B + \gamma_B^2 > 0$$

and

$$\pi_{C|A} > \pi_{A|C} \Leftrightarrow \frac{t\,(3+\gamma_B)^2}{18\,(1-\gamma_B)} > \frac{t\,(3-\gamma_B)^2}{18\,(1-\gamma_B)}.$$

Clearly, both conditions hold for all $\gamma_B \in \left(0, \frac{1}{2}\right)$.

For case (ii) we have a middle-wage equilibrium and, by Table 1,

$$\pi_{A|C} > \pi_{D|D} \Leftrightarrow \frac{(1-\gamma_B)\,p^2}{8t} > \frac{t}{2} \Leftrightarrow \frac{t}{p} < \frac{\sqrt{1-\gamma_B}}{2}$$

and

$$\pi_{C|A} > \pi_{A|C} \Leftrightarrow \frac{(p-t)\,[4t-(1-\gamma_B)\,p]}{4t} > \frac{(1-\gamma_B)\,p^2}{8t} \Leftrightarrow 2\left(1-\frac{t}{p}\right)\left[4\frac{t}{p}-(1-\gamma_B)\right] > 1-\gamma_B.$$

The first condition clearly holds for all $\gamma_B \in \left(0, \frac{1}{2}\right)$. The left-hand side of the second condition is increasing in $\frac{t}{p}$ for all $\gamma_B \in \left(0, \frac{1}{2}\right)$; hence the condition holds if it is satisfied when evaluated at the lowest possible value of $\frac{t}{p}$, namely $\frac{t}{p} = \max\left\{\frac{3(1-\gamma_B)}{2(3-\gamma_B)}, \frac{1}{3}\right\}$. Indeed, it suffices to check that it holds for $\frac{t}{p} = \frac{3(1-\gamma_B)}{2(3-\gamma_B)}$:

$$2\left(1-\frac{3\,(1-\gamma_B)}{2\,(3-\gamma_B)}\right)\left[4\frac{3\,(1-\gamma_B)}{2\,(3-\gamma_B)}-(1-\gamma_B)\right] = \frac{(3+\gamma_B)^2\,(1-\gamma_B)}{(3-\gamma_B)^2} > 1-\gamma_B,$$

which is satisfied for all $\gamma_B \in \left(0, \frac{1}{2}\right)$. □

4.3 Proof of Proposition 3 in Lagerlöf (2019)

First suppose that $\frac{t}{p} \in \Omega_I \cup \Omega_{III}$. Then, by Proposition 1, $(y_1^*, y_2^*) \in \{(C,C), (C,D), (D,C), (D,D)\}$. Therefore the firms address the same segments of workers and, by the analysis in subsection 3.1.2 in the paper, $w_1^* = w_2^* = p - t$ and $\pi_1^* = \pi_2^* = \frac{t}{2}$. The claims in part (i) of Proposition 3 follow immediately from these expressions.

Next suppose that $\frac{t}{p} \in \Omega_{II}$. Then, given that a payoff-dominated equilibrium is not played, it follows from Propositions 1 and 2 that $(y_1^*, y_2^*) \in \{(A, C), (C, A)\}$. That is, one firm discriminates in hiring against the minority group ($y_j = A$) and the other firm does not discriminate at all ($y_{-j} = C$). If $\frac{t}{p} > \frac{3(1-\gamma_B)}{2(3-\gamma_B)}$, then it follows from subsection 3.1.4 and Lemma 1 in the paper that the equilibrium wages are $w_j^* = \frac{p}{2}$ and $w_{-j}^* = t$, and the equilibrium profits are

$$\pi_j^* = \frac{(1-\gamma_B)p^2}{8t} \quad \text{and} \quad \pi_{-j}^* = \frac{(p-t)\left[4t - (1-\gamma_B)p\right]}{4t}.$$

The claims in part (ii) of Proposition 3 follow immediately from these expressions (differentiating yields $\frac{\partial \pi_{-j}^*}{\partial t} > 0 \Leftrightarrow \frac{t}{p} < \frac{\sqrt{1-\gamma_B}}{2}$, which is implied by $\frac{t}{p} \in \Omega_{II}$). If $\frac{t}{p} < \frac{3(1-\gamma_B)}{2(3-\gamma_B)}$, then it follows from subsection 3.1.4 and Lemma 1 in the paper that the equilibrium wages are $w_j^* = p - \frac{(3-\gamma_B)t}{3(1-\gamma_B)}$ and $w_{-j}^* = p - \frac{(3+\gamma_B)t}{3(1-\gamma_B)}$, and the equilibrium profits are

$$\pi_j^* = \frac{t\,(3-\gamma_B)^2}{18(1-\gamma_B)} \quad \text{and} \quad \pi_{-j}^* = \frac{t\,(3+\gamma_B)^2}{18(1-\gamma_B)}.$$

The claims in part (iii) of Proposition 3 follow immediately from these expressions. □

4.4 Proof of Proposition 4 in Lagerlöf (2019)

To prove claim (i) it suffices to show that $\pi_{C|A} \geq \pi_{A|C} > \pi_{C|C}$. But, since $\pi_{C|C} = \pi_{D|D}$, this follows from the proof of Proposition 2.

To prove claim (ii), note from eq. (1) in the paper that the workers care about their wage and their mismatch cost. Also note that, given $\frac{t}{p} \in \Omega_{II}$, all workers are employed, both with and without the anti-discrimination policy described in the proposition; hence, in both scenarios, all workers earn a wage and incur a mismatch cost. From subsection 3.1.2 and Table 1 in Lagerlöf (2019), it follows that, for all $\frac{t}{p} \in \Omega_{II}$, $w_{C|C} > w_{A|C} > w_{C|A}$. That is, the wage utility that accrues to any given worker is higher with the policy. Moreover, with the policy the two firms' wages are the same, which means that the threshold value $\overline{x}$ defined in eq. (5) in Lagerlöf (2019) is given by one-half: All workers left (right, respectively) of the midpoint of the unit interval chooses firm 1 (2, respectively). On the other hand, without the policy some workers will choose an employer that is farther away, while others choose the same employer as with the policy. That is, the mismatch cost that any given worker incurs is either the same or strictly lower with the policy. Those things imply that each one of the workers is strictly better off with the policy than without.

To prove claim (iii), note again that, given $\frac{t}{p} \in \Omega_{II}$, all workers are employed both with and without the policy. Moreover, the wage does not matter for total surplus, since it is only a transfer from a firm to a worker. Those things imply that only the aggregate mismatch costs matter for total surplus. As argued in the paragraph immediately above, however, these mismatch costs are strictly higher without the policy. □

4.5 Proof of Proposition 5 in Lagerlöf (2019)

The structure of the proof is very similar to the one in the proof of Proposition 1, except that here there is no D action.

Proof of Proposition 5, part (i). First suppose $\frac{t}{p} \in \left[\frac{1}{2}, \frac{2}{3}\right)$. Consider the game matrix in Figure 1 in Lagerlöf (2019). Let us study one column of the matrix at a time, while using the results in Lemmas S1 and S2. Since we now have $S \in \{A, B, C\}$, we ignore the D column and the D row.

- In column A, firm 1 prefers C to A and B (for we have $\pi_{C|A} > \pi_{A|A}$ and $\pi_{C|A} > \pi_{B|A}$). However, $(y_1, y_2) = (C, A)$ cannot be an equilibrium, as firm 2 would have an incentive to deviate to C (for we have $\pi_{C|C} > \pi_{A|C}$).
- In column B, firm 1 prefers C to A and B (for we have $\pi_{C|B} > \pi_{A|B} > \pi_{B|B}$). But $(y_1, y_2) = (C, B)$ cannot be an equilibrium, as firm 2 would have an incentive to deviate to C (for we have $\pi_{C|C} > \pi_{B|C}$).
- In column C, firm 1 prefers C to A and B (for we have $\pi_{C|C} > \pi_{A|C} > \pi_{B|C}$). By symmetry (in particular, by the inequalities stated in the previous sentence), if firm 1 chooses C, then firm 2 has an incentive to also choose C. Hence $(y_1, y_2) = (C, C)$ is an equilibrium.

Next suppose $\frac{t}{p} \in \left(\frac{\sqrt{1-\gamma_B}}{2}, \frac{1}{2}\right)$, and again study one column of the game matrix at a time, while using the results in Lemmas S1 and S3. Note that $\frac{t}{p} > \frac{\sqrt{1-\gamma_B}}{2}$ implies $\frac{t}{p} > \frac{1}{3}$.

- In column A, firm 1 prefers C to A and B (for we have $\pi_{C|A} > \pi_{D|A} = \pi_{A|A} + \pi_{B|A}$). However, $(y_1, y_2) = (C, A)$ cannot be an equilibrium, as firm 2 would have an incentive to deviate to C (for we have $\pi_{C|C} > \pi_{A|C}$).
- In column B, firm 1 again prefers C to A and B (for we have $\pi_{C|B} > \pi_{D|B} = \pi_{A|B} + \pi_{B|B}$). But $(y_1, y_2) = (C, B)$ cannot be an equilibrium, as firm 2 would have an an incentive to deviate to C (for we have $\pi_{C|C} > \pi_{A|C} > \pi_{B|C}$).
- In column C, firm 1 prefers C to A and B (for we have $\pi_{C|C} > \pi_{A|C} > \pi_{B|C}$). By symmetry (in particular, by the inequalities stated in the previous sentence), if firm 1 chooses C, then firm 2 has an incentive to also choose C. Hence $(y_1, y_2) = (C, C)$ is an equilibrium.

Proof of Proposition 5, part (ii). First suppose $\frac{t}{p} \in \left(\max\left\{\frac{3(1-\gamma_B)}{2(3-\gamma_B)}, \frac{1}{3}\right\}, \frac{\sqrt{1-\gamma_B}}{2}\right)$. Let us study one column of the matrix in Figure 1 in Lagerlöf (2019) at a time, while using the results in Lemmas S1 and S3.

- In column A, firm 1 prefers C to A and B (for we have $\pi_{C|A} > \pi_{D|A} = \pi_{A|A} + \pi_{B|A}$). Given that firm 1 chooses C, firm 2 does not have an incentive to deviate to C or B (for we have $\pi_{A|C} > \pi_{D|C} = \pi_{C|C}$ and $\pi_{A|C} > \pi_{B|C}$). Hence $(y_1, y_2) = (C, A)$ is an equilibrium.

- In column B, firm 1 again prefers C to A and B (for we have $\pi_{C|B} > \pi_{D|B} = \pi_{A|B} + \pi_{B|B}$). But $(y_1, y_2) = (C, B)$ cannot be an equilibrium, as firm 2 would have an an incentive to deviate to A (for we have $\pi_{A|C} > \pi_{B|C}$).

- In column C, by symmetry (cf. the above arguments about columns A and B) the following must hold: $(y_1, y_2) = (A, C)$ is an equilibrium, but $(y_1, y_2) = (B, C)$ is not. Moreover, $(y_1, y_2) = (C, C)$ is not an equilibrium, as firm 1 would have an incentive to deviate to A (for we have $\pi_{A|C} > \pi_{D|C} = \pi_{C|C}$).

Next suppose $\frac{t}{p} \in \left(\frac{9(1-\gamma_B)}{21-13\gamma_B}, \frac{3(1-\gamma_B)}{2(3-\gamma_B)}\right)$, and again study one column of the game matrix at a time, using the results in Lemmas S1 and S4. Note that $\frac{t}{p} > \frac{9(1-\gamma_B)}{21-13\gamma_B}$ implies $\frac{t}{p} > \frac{3\sqrt{\gamma_B(1-\gamma_B)}}{2(3-\gamma_B)}$ and that $\frac{t}{p} \in \left(\frac{9(1-\gamma_B)}{21-13\gamma_B}, \frac{3(1-\gamma_B)}{2(3-\gamma_B)}\right)$ implies $\frac{t}{p} > \frac{1}{3}$.

- In column A, firm 1 prefers C to A and B (for we have $\pi_{C|A} > \pi_{D|A} = \pi_{A|A} + \pi_{B|A}$). Given that firm 1 chooses C, firm 2 does not have an incentive to deviate to C or B (for we have $\pi_{A|C} > \pi_{C|C}$ and $\pi_{A|C} > \pi_{B|C}$). Hence $(y_1, y_2) = (C, A)$ is an equilibrium.

- In column B, firm 1 again prefers C to A and B (for we have $\pi_{C|B} > \pi_{D|B} = \pi_{A|B} + \pi_{B|B}$). But $(y_1, y_2) = (C, B)$ cannot be an equilibrium, as firm 2 would have an incentive to deviate to A (for we have $\pi_{A|C} > \pi_{B|C}$).

- In column C, by symmetry (cf. the above arguments about columns A and B) the following must hold: $(y_1, y_2) = (A, C)$ is an equilibrium, but $(y_1, y_2) = (B, C)$ is not. Moreover, $(y_1, y_2) = (C, C)$ is not an equilibrium, as firm 1 would have an incentive to deviate to A (for we have $\pi_{A|C} > \pi_{C|C}$).

Now suppose $\frac{t}{p} \in \left(\max\left\{\varphi(\gamma_B), \frac{3\gamma_B}{2(2+\gamma_B)}, \frac{1}{4}\right\}, \min\left\{\frac{3(1-\gamma_B)}{2(3-\gamma_B)}, \frac{9(1-\gamma_B)}{21-13\gamma_B}\right\}\right)$. Let us study one column of the matrix in Figure 1 in Lagerlöf (2019) at a time, while using the results in Lemmas S1 and S4.

- In column A, firm 1 prefers C to A and B (for we have $\pi_{C|A} > \pi_{A|A}$ and $\pi_{C|A} > \pi_{B|A}$). Given that firm 1 chooses C, firm 2 never has an incentive to deviate to C (for we have $\pi_{A|C} > \pi_{C|C}$) and firm 2 has no incentive to deviate to B if and only if $\frac{t}{p} \geq \frac{3\sqrt{\gamma_B(1-\gamma_B)}}{2(3-\gamma_B)}$ (by part c) of Lemma S4). Hence $(y_1, y_2) = (C, A)$ is an equilibrium if and only if $\frac{t}{p} \geq \frac{3\sqrt{\gamma_B(1-\gamma_B)}}{2(3-\gamma_B)}$.

- In column B, firm 1 prefers C to A and B (for we have $\pi_{C|B} > \pi_{A|B}$ and $\pi_{C|B} > \pi_{B|B}$). Given that firm 1 chooses C, firm 2's best deviation cannot be C (for we have $\pi_{A|C} > \pi_{C|C}$) and firm 2 has no incentive to deviate to A if and only if $\frac{t}{p} \leq \frac{3\sqrt{\gamma_B(1-\gamma_B)}}{2(3-\gamma_B)}$ (by part c) of Lemma S4). Hence $(y_1, y_2) = (C, B)$ is an equilibrium if and only if $\frac{t}{p} \leq \frac{3\sqrt{\gamma_B(1-\gamma_B)}}{2(3-\gamma_B)}$.

- In column C, by symmetry (cf. the above arguments about columns A and B) the following must hold: $(y_1, y_2) = (A, C)$ is an equilibrium if and only if $\frac{t}{p} \geq \frac{3\sqrt{\gamma_B(1-\gamma_B)}}{2(3-\gamma_B)}$; and $(y_1, y_2) = (B, C)$ is an equilibrium if and only if $\frac{t}{p} \leq \frac{3\sqrt{\gamma_B(1-\gamma_B)}}{2(3-\gamma_B)}$. Moreover, $(y_1, y_2) = (C, C)$ is not an equilibrium, as firm 1 would have an incentive to deviate to A (for we have $\pi_{A|C} > \pi_{C|C}$).

Next suppose $\frac{t}{p} \in \left(\frac{3(1-\gamma_B)}{2(3-\gamma_B)}, \frac{1}{3}\right)$, and again study one column of the game matrix at a time, while using the results in Lemmas S1 and S3. Note that $\frac{t}{p} < \frac{1}{3}$ implies $\frac{t}{p} < \frac{\sqrt{1-\gamma_B}}{2}$. Also, $\frac{t}{p} \in \left(\frac{3(1-\gamma_B)}{2(3-\gamma_B)}, \frac{1}{3}\right)$ implies that $\gamma_B > \frac{3}{7}$.

- In column A, firm 1 prefers C to A and B (for we have $\pi_{C|A} > \pi_{B|A} > \pi_{A|A}$). Given that firm 1 chooses C, firm 2 does not have an incentive to deviate to B or C (for we have $\pi_{A|C} > \pi_{B|C}$ and $\pi_{A|C} > \pi_{D|C} = \pi_{C|C}$). Hence $(y_1, y_2) = (C, A)$ is an equilibrium.
- In column B, firm 1 prefers C to A and B (for we have $\pi_{C|B} > \pi_{A|B} > \pi_{B|B}$). But $(y_1, y_2) = (C, B)$ cannot be an equilibrium, as firm 2 would have an incentive to deviate to A (for we have $\pi_{A|C} > \pi_{B|C}$).
- In column C, firm 1 prefers A to B and C (for we have $\pi_{A|C} > \pi_{B|C}$ and $\pi_{A|C} > \pi_{D|C} = \pi_{C|C}$). Given that firm 1 chooses A, firm 2 does not have an incentive to deviate to A or B (for we have $\pi_{C|A} > \pi_{B|A} > \pi_{A|A}$). Hence $(y_1, y_2) = (A, C)$ is an equilibrium.

Now suppose $\frac{t}{p} \in \left(\varphi(\gamma_B), \frac{3\gamma_B}{2(2+\gamma_B)}\right]$, and again study one column of the game matrix at a time, while using the results in Lemmas S1 and S5.

- In column A, firm 1 prefers C to A and B (for we have $\pi_{C|A} > \pi_{B|A}$ and $\pi_{C|A} > \pi_{A|A}$). However, $(y_1, y_2) = (C, A)$ cannot be an equilibrium, as firm 2 would have an incentive to deviate to B (for we have $\pi_{B|C} > \pi_{A|C}$).
- In column B, firm 1 again prefers C to A and B (for we have $\pi_{C|B} > \pi_{B|B}$ and $\pi_{C|B} > \pi_{A|B}$). Given that firm 1 chooses C, firm 2 does not have an incentive to deviate to A or C (for we have $\pi_{B|C} > \pi_{A|C} > \pi_{C|C}$). Hence $(y_1, y_2) = (C, B)$ is an equilibrium.
- In column C, by symmetry (cf. the above arguments about columns A and B) the following must hold: $(y_1, y_2) = (B, C)$ is an equilibrium, but $(y_1, y_2) = (A, C)$ is not. Moreover, $(y_1, y_2) = (C, C)$ is not an equilibrium, as firm 1 would have an incentive to deviate to B (for we have $\pi_{B|C} > \pi_{C|C}$).

Finally suppose $\frac{t}{p} \in \left(\varphi(\gamma_B), \frac{1}{4}\right]$, and again study one column of the game matrix at a time, while using the results in Lemmas S1 and S6.

- In column A, firm 1 prefers C to A and B (for we have $\pi_{C|A} > \pi_{B|A}$ and $\pi_{C|A} > \pi_{A|A}$). Given that firm 1 chooses C, firm 2 does not have an incentive to deviate to B or C (for we have $\pi_{A|C} > \pi_{B|C}$ and $\pi_{A|C} > \pi_{C|C}$). Hence $(y_1, y_2) = (C, A)$ is an equilibrium.
- In column B, firm 1 prefers A and C to B (for we have $\pi_{A|B} = \pi_{C|B} > \pi_{B|B}$). But $(y_1, y_2) = (C, B)$ cannot be an equilibrium, as firm 2 would have an incentive to deviate to A (for we have $\pi_{A|C} > \pi_{B|C}$). Nor can $(y_1, y_2) = (A, B)$ be an equilibrium, as firm 2 would have an incentive to deviate to C (for we have $\pi_{C|A} > \pi_{B|A}$).
- In column C, by symmetry (cf. the above arguments about columns A and B) the following must hold: $(y_1, y_2) = (A, C)$ is an equilibrium, but $(y_1, y_2) = (B, C)$ is not. Moreover, $(y_1, y_2) = (C, C)$

is not an equilibrium, as firm 1 would have an incentive to deviate to A (for we have $\pi_{A|C} > \pi_{C|C}$).

□

References

Lagerlöf, Johan N. M. (2019) "Strategic Gains from Discrimination," September, Mimeo, University of Copenhagen.