Supplementary Material to "Strategic Gains from Discrimination"

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Contents

1. Introduction

In this Supplementary Material, which is not meant to be published, I provide some proofs that were omitted from Lagerlöf (2018). In the next section, I first state and prove some useful lemmas. After that I prove Propositions 1, 3, and 5 of Lagerlöf (2018).

2. Proofs of results not proven in the paper

2.1. Lemmas to be used when proving Propositions 1 and 5

In this first subsection, I state and prove six lemmas. The lemmas relate various profit expressions to each other, for different parts of the parameter space. Knowledge about these relationships will then be used when proving Propositions 1 and 5.

Lemma S1. *Suppose* $\frac{t}{p} \in \left(\varphi(\gamma_B), \frac{2}{3}\right)$. *We then have the following relationships:*

a) $\pi_{D|A} = \pi_{A|A} + \pi_{B|A}$ *(and hence* $\pi_{D|A} > \pi_{A|A}$ *and* $\pi_{D|A} > \pi_{B|A}$*);*

b) $\pi_{D|B} = \pi_{A|B} + \pi_{B|B}$ *(and hence* $\pi_{D|B} > \pi_{A|B}$ *and* $\pi_{D|B} > \pi_{B|B}$*);*

c) $\pi_{C|C} = \pi_{C|D} = \pi_{D|C} = \pi_{D|D} > \pi_{A|D} > \pi_{B|D}$.

Proof of Lemma S1. Follows immediately from the results stated in Section 3 of Lagerlöf (2018). □

Lemma S2. *Suppose* $\frac{t}{p} \in \left[\frac{1}{2}, \frac{2}{3}\right)$*. We then have the following relationships:*

a) $\pi_{C|A} > \pi_{A|A}$, $\pi_{C|A} > \pi_{B|A}$;

b) $\pi_{C|C} > \pi_{A|C} > \pi_{B|C}$;

c) $\pi_{D|B} > \pi_{C|B}$;

d) $\pi_{C|B} > \pi_{A|B} > \pi_{B|B}$.

Proof of Lemma S2. Throughout, I make use of results stated in Section 3 of Lagerlöf (2018). First we have

$$\pi_{C|A} > \pi_{A|A} \Leftrightarrow \frac{1+\gamma_B}{2t}\left[\frac{4\gamma_B p + 3(1-\gamma_B)t}{3+5\gamma_B}\right]^2 > \frac{(1-\gamma_B)t}{2}$$
$$\Leftrightarrow (1+\gamma_B)\left[4\gamma_B \frac{p}{t} + 3(1-\gamma_B)\right]^2 - (1-\gamma_B)(3+5\gamma_B)^2 > 0.$$

But this inequality must hold for all $\frac{t}{p} \leq \frac{2}{3}$, because the left-hand side is increasing in $\frac{p}{t}$ and evaluated at $\frac{p}{t} = \frac{3}{2}$ it equals

$$9(1+\gamma_B)(1+\gamma_B)^2 - (1-\gamma_B)(3+5\gamma_B)^2,$$

which can be shown to be strictly positive for all $\gamma_B \in (0, \frac{1}{2})$. Second, we have

$$\pi_{C|A} > \pi_{B|A} \Leftrightarrow \frac{1+\gamma_B}{2t}\left[\frac{4\gamma_B p + 3(1+\gamma_B)t}{3+5\gamma_B}\right]^2 > \frac{\gamma_B p^2}{4t} \Leftrightarrow 2(1+\gamma_B)\left[4\gamma_B + 3(1+\gamma_B)\frac{t}{p}\right]^2 > \gamma_B(3+5\gamma_B)^2.$$

But this inequality must hold for all $\frac{t}{p} \geq \frac{1}{2}$, because the left-hand side is increasing in $\frac{t}{p}$ and the inequality holds (with some margin) at $\frac{t}{p} = \frac{1}{2}$. Third, we can write

$$\pi_{C|C} > \pi_{A|C} \Leftrightarrow \frac{t}{2} > \frac{\gamma_A}{2t}\left[\frac{2\gamma_B p + (3-\gamma_B)t}{3+5\gamma_B}\right]^2 \Leftrightarrow (3+5\gamma_B)^2 > \gamma_A[2\gamma_B \frac{p}{t} + (3-\gamma_B)]^2.$$

But this inequality must hold for all $\frac{t}{p} \geq \frac{1}{2}$: The right-hand side is increasing in $\frac{p}{t}$, and evaluated at $\frac{p}{t} = 2$ the inequality is equivalent to $1 > \gamma_A$. Fourth, we can write

$$\pi_{A|C} > \pi_{B|C} \Leftrightarrow \frac{\gamma_A}{2t}\left[\frac{2\gamma_B p + (3+\gamma_B)t}{3+5\gamma_B}\right]^2 > \frac{\gamma_B}{2t}\left[\frac{2(1-\gamma_B)p + (4-\gamma_B)t}{8-5\gamma_B}\right]^2 \Leftrightarrow$$
$$\Psi\left(\gamma_B, \frac{t}{p}\right) \stackrel{\text{def}}{=} \ln\left[(1-\gamma_B)\left(\frac{2\gamma_B + (3+\gamma_B)\frac{t}{p}}{3+5\gamma_B}\right)^2\right] - \ln\left[\gamma_B\left(\frac{2(1-\gamma_B) + (4-\gamma_B)\frac{t}{p}}{8-5\gamma_B}\right)^2\right] > 0.$$

Note that

$$\frac{\partial\Psi}{\partial\left(\frac{t}{p}\right)} = \frac{2(3+\gamma_B)}{2\gamma_B + (3+\gamma_B)\frac{t}{p}} - \frac{2(4-\gamma_B)}{2(1-\gamma_B) + (4-\gamma_B)\frac{t}{p}} = \frac{4\left[(3+\gamma_B)(1-\gamma_B) - (4-\gamma_B)\gamma_B\right]}{\left[2\gamma_B + (3+\gamma_B)\frac{t}{p}\right]\left[2(1-\gamma_B) + (4-\gamma_B)\frac{t}{p}\right]},$$

which is strictly positive thanks to the assumption that $\gamma_B < \frac{1}{2}$. It thus suffices to show that, evaluated at $\frac{t}{p} = \frac{1}{2}$, $\Psi\left(\gamma_B, \frac{t}{p}\right)$ is strictly positive. But it is easy to see that $\Psi\left(\gamma_B, \frac{1}{2}\right) = \ln(1-\gamma_B) - \ln(\gamma_B)$, which is strictly positive for all $\gamma_B < \frac{1}{2}$.

Fifth, we can write

$$\pi_{D|B} > \pi_{C|B} \Leftrightarrow \frac{\gamma_A p^2}{2t} + \frac{\gamma_B t}{2} > \frac{2-\gamma_B}{2t}\left[\frac{4(1-\gamma_B)p + 3\gamma_B t}{8-5\gamma_B}\right]^2 \Leftrightarrow$$
$$\varkappa(\gamma_B, \frac{t}{p}) \stackrel{\text{def}}{=} 1 - \gamma_B + \gamma_B\left(\frac{t}{p}\right)^2 - (2-\gamma_B)\left[\frac{4(1-\gamma_B) + 3\gamma_B\left(\frac{t}{p}\right)}{8-5\gamma_B}\right]^2 > 0.$$

The function $\varkappa(\gamma_B, \frac{t}{p})$ is strictly increasing in $\frac{t}{p}$ if, and only if,

$$\frac{t}{p} > \frac{12(1-\gamma_B)(2-\gamma_B)}{(8-5\gamma_B)^2 - 9\gamma_B(2-\gamma_B)^2},$$

which is implied by $\frac{t}{p} > \frac{1}{2}$. It therefore suffices to show that $\varkappa(\gamma_B, \frac{1}{2}) > 0$ or, equivalently,

$$1 - \gamma_B + \frac{\gamma_B}{4} - (2-\gamma_B)\left[\frac{4(1-\gamma_B) + \frac{3}{2}\gamma_B}{8-5\gamma_B}\right]^2 > 0,$$

which simplifies to $\gamma_B < 1$ and thus always hold.

Sixth, we have

$$\pi_{C|B} > \pi_{A|B} \Leftrightarrow \frac{2-\gamma_B}{2t}\left[\frac{4(1-\gamma_B)p + 3\gamma_B t}{8-5\gamma_B}\right]^2 > \frac{\gamma_A p^2}{4t}$$
$$\Leftrightarrow 2(2-\gamma_B)\left[4(1-\gamma_B) + 3\gamma_B\frac{t}{p}\right]^2 - (1-\gamma_B)(8-5\gamma_B)^2 > 0.$$

But this inequality must hold for all $\frac{t}{p} \geq \frac{1}{2}$, because the left-hand side is increasing in $\frac{t}{p}$ and evaluated at $\frac{p}{t} = \frac{1}{2}$ it equals

$$\frac{2-\gamma_B}{2}(8-5\gamma_B)^2 - (1-\gamma_B)(8-5\gamma_B)^2,$$

which is strictly positive for all $\gamma_B \in (0, \frac{1}{2})$.

Finally, we can write

$$\pi_{A|B} > \pi_{B|B} \Leftrightarrow \frac{\gamma_A p^2}{4t} > \frac{\gamma_B t}{2} \Leftrightarrow 1 - \gamma_B > 2\gamma_B\left(\frac{t}{p}\right)^2.$$

But this inequality must hold for all $\frac{t}{p} \leq \frac{2}{3}$, because the right-hand side is increasing in $\frac{t}{p}$ and evaluated at $\frac{p}{t} = \frac{2}{3}$ the inequality becomes

$$9(1-\gamma_B) > 8\gamma_B,$$

which can be shown to hold for all $\gamma_B \in (0, \frac{1}{2})$. □

Lemma S3. *Suppose $\frac{t}{p} \in \left[\frac{3(1-\gamma_B)}{2(3-\gamma_B)}, \frac{1}{2}\right)$. We then have the following relationships:*

a) $\pi_{C|A} \gtreqqless \pi_{D|A}$ *as* $\frac{t}{p} \gtreqqless \frac{1}{3}$;

b) $\pi_{D|C} \gtreqqless \pi_{A|C}$ *as* $\frac{t}{p} \gtreqqless \frac{\sqrt{1-\gamma_B}}{2}$;

c) $\pi_{C|B} \gtreqqless \pi_{D|B}$ *as* $\frac{t}{p} \gtreqqless \frac{1}{3}$;

d) $\pi_{A|C} > \pi_{B|C}$;

e) $\pi_{C|A} > \pi_{B|A}$ *and* $\pi_{A|B} > \pi_{B|B}$.

f) If $\gamma_B > \frac{3}{7}$, *then* $\pi_{B|A} > \pi_{A|A}$;

g) $\pi_{C|B} > \pi_{A|B}$.

Proof of Lemma S3. First, we can write

$$\pi_{C|A} \geq \pi_{D|A} \Leftrightarrow \frac{(p-t)\,[4t-(1-\gamma_B)p]}{4t} \geq \gamma_B(p-t) + \frac{\gamma_A t}{2}$$
$$\Leftrightarrow (p-t)\,[4t-(1-\gamma_B)p] \geq 4\gamma_B(p-t)t + 2(1-\gamma_B)t^2$$
$$\Leftrightarrow (1-\gamma_B)(p-t)p + 2(1-\gamma_B)t^2 - 4(1-\gamma_B)(p-t)t \leq 0$$
$$\Leftrightarrow 1 - \frac{t}{p} + 2\left(\frac{t}{p}\right)^2 - 4\left(1-\frac{t}{p}\right)\frac{t}{p} \leq 0 \Leftrightarrow \left(\frac{3t}{p}-1\right)\left(\frac{2t}{p}-1\right) \leq 0.$$

Since, by assumption, $\frac{t}{p} < \frac{1}{2}$, the last inequality is equivalent to $\frac{t}{p} \geq \frac{1}{3}$. Second, we can write

$$\pi_{D|C} > \pi_{A|C} \Leftrightarrow \frac{t}{2} > \frac{(1-\gamma_B)p^2}{8t} \Leftrightarrow \frac{t}{p} > \frac{\sqrt{1-\gamma_B}}{2},$$

from which the claim follows. Third, we can write

$$\pi_{C|B} \geq \pi_{D|B} \Leftrightarrow \frac{(p-t)\,(4t-\gamma_B p)}{4t} \geq \gamma_A(p-t) + \frac{\gamma_B t}{2}.$$

Note that this inequality is identical to the one above (see the calculations for $\pi_{C|A} \geq \pi_{D|A}$), except that γ_A and γ_B have swapped places. Therefore the result there, which did not depend on the particular value of $\gamma_B \in (0,1)$, applies here, too, and the claim follows. Fourth, the claim that $\pi_{A|C} > \pi_{B|C}$ follows immediately from Tables 1 and 2 in Lagerlöf (2018) and the assumption that $\gamma_B < \frac{1}{2}$. Fifth, we can write

$$\pi_{C|A} > \pi_{B|A} \Leftrightarrow \frac{(p-t)\,[4t-(1-\gamma_B)p]}{4t} > \gamma_B\,(p-t) \Leftrightarrow \frac{t}{p} > \frac{1}{4},$$

which always holds because $\frac{t}{p} \geq \frac{3(1-\gamma_B)}{2(3-\gamma_B)} > \frac{3}{10}$ for all $\gamma_B \in (0, \frac{1}{2})$. Sixth, we can write

$$\pi_{A|B} > \pi_{B|B} \Leftrightarrow \; (1-\gamma_B)\,(p-t) > \frac{\gamma_B t}{2} \Leftrightarrow \frac{t}{p} < \frac{2(1-\gamma_B)}{2-\gamma_B},$$

which holds for all $\gamma_B \in (0, \frac{1}{2})$. Seventh, we can write

$$\pi_{B|A} > \pi_{A|A} \Leftrightarrow \gamma_B\,(p-t) > \frac{(1-\gamma_B)\,t}{2} \Leftrightarrow \frac{t}{p} < \frac{2\gamma_B}{1+\gamma_B},$$

which holds for all $\gamma_B > \frac{3}{7}$ (indeed, it holds for all $\gamma_B > \frac{1}{3}$). Eighth, we can write

$$\pi_{C|B} > \pi_{A|B} \Leftrightarrow \frac{(p-t)\,(4t-\gamma_B p)}{4t} > (1-\gamma_B)(p-t) \Leftrightarrow \frac{t}{p} > \frac{1}{4},$$

which always holds under the assumptions of the lemma. □

Lemma S4. *Suppose* $\frac{t}{p} \in \left(\max\left\{\varphi(\gamma_B), \frac{3\gamma_B}{2(2+\gamma_B)}, \frac{1}{4}\right\}, \frac{3(1-\gamma_B)}{2(3-\gamma_B)}\right]$. *We then have the following relationships:*

a) $\pi_{C|A} \gtreqless \pi_{D|A}$ *as* $\frac{t}{p} \gtreqless \frac{9(1-\gamma_B)}{21-13\gamma_B}$;

b) $\pi_{A|C} > \pi_{C|C}$;

c) $\pi_{A|C} \gtreqless \pi_{B|C}$ *as* $\frac{t}{p} \gtreqless \frac{3\sqrt{\gamma_B(1-\gamma_B)}}{2(3-\gamma_B)}$;

d) $\pi_{C|B} \gtreqless \pi_{D|B}$ *as* $\frac{t}{p} \gtreqless \frac{1}{3}$;

e) $\pi_{C|A} > \pi_{A|A}$, $\pi_{C|A} > \pi_{B|A}$, $\pi_{C|B} > \pi_{A|B}$, *and* $\pi_{C|B} > \pi_{B|B}$.

Proof of Lemma S4. First, we can write

$$\pi_{C|A} \geq \pi_{D|A} \Leftrightarrow \frac{t\,(3+\gamma_B)^2}{18(1-\gamma_B)} \geq \gamma_B(p-t) + \gamma_A \frac{t}{2}$$
$$\Leftrightarrow \left[(3+\gamma_B)^2 + 18\gamma_B(1-\gamma_B) - 9(1-\gamma_B)^2\right] t \geq 18\gamma_B(1-\gamma_B)p$$
$$\Leftrightarrow 2(21-13\gamma_B)\gamma_B t \geq 18\gamma_B(1-\gamma_B)p \Leftrightarrow \frac{t}{p} \geq \frac{9(1-\gamma_B)}{21-13\gamma_B}.$$

Second, we can write

$$\pi_{A|C} > \pi_{C|C} \Leftrightarrow \frac{t\,(3-\gamma_B)^2}{18(1-\gamma_B)} > \frac{t}{2} \Leftrightarrow (3-\gamma_B)^2 > 9(1-\gamma_B),$$

which can be shown to hold for all $\gamma_B > 0$. Third, we can write

$$\pi_{A|C} \geq \pi_{B|C} \Leftrightarrow \frac{t\,(3-\gamma_B)^2}{18(1-\gamma_B)} \geq \frac{\gamma_B\, p^2}{8t} \Leftrightarrow 4\,(3-\gamma_B)^2\, t^2 \geq 9\gamma_B\,(1-\gamma_B)\,p^2$$

or

$$\frac{t}{p} \geq \frac{3\sqrt{\gamma_B(1-\gamma_B)}}{2(3-\gamma_B)}.$$

Fourth, we can write

$$\pi_{C|B} \geq \pi_{D|B} \Leftrightarrow \frac{(p-t)\,(4t-\gamma_B p)}{4t} \geq \gamma_A(p-t) + \gamma_B \frac{t}{2}$$
$$\Leftrightarrow [4t - \gamma_B p - 4(1-\gamma_B)t\,]\,(p-t) \geq 2\gamma_B t^2 \Leftrightarrow (4t-p)\,(p-t) \geq 2t^2 \Leftrightarrow (p-3t)\,(p-2t) \leq 0.$$

It follows from the assumptions in the lemma that $\frac{t}{p} < \frac{1}{2}$. Therefore, the above inequality is equivalent to $\frac{t}{p} \geq \frac{1}{3}$. Fifth, we can write

$$\pi_{C|A} > \pi_{A|A} \Leftrightarrow \frac{t\,(3+\gamma_B)^2}{18(1-\gamma_B)} > \frac{(1-\gamma_B)t}{2} \Leftrightarrow (3+\gamma_B)^2 - 9\,(1-\gamma_B)^2 > 0.$$

The left-hand side of the last inequality is strictly increasing in γ_B and it equals zero at $\gamma_B = 0$; hence it holds for all $\gamma_B \in (0, \frac{1}{2})$. Sixth, we can write

$$\pi_{C|A} > \pi_{B|A} \Leftrightarrow \frac{t\,(3+\gamma_B)^2}{18(1-\gamma_B)} > \gamma_B\,(p-t) \Leftrightarrow \frac{t}{p} > \varphi(\gamma_B),$$

which always holds. Seventh, we can write

$$\pi_{C|B} > \pi_{A|B} \Leftrightarrow \frac{(p-t)\,(4t-\gamma_B p)}{4t} > (1-\gamma_B)\,(p-t) \Leftrightarrow \frac{t}{p} > \frac{1}{4},$$

which is satisfied under the assumptions of the lemma. Eighth and finally, we can write

$$\pi_{C|B} > \pi_{B|B} \Leftrightarrow \frac{(p-t)\,(4t-\gamma_B p)}{4t} > \frac{\gamma_B t}{2} \Leftrightarrow -2(2+\gamma_B)\left(\frac{t}{p}\right)^2 + (4+\gamma_B)\frac{t}{p} - \gamma_B > 0.$$

The right-hand side of the last inequality is increasing in γ_B (since $\frac{t}{p} < \frac{1}{2}$). Moreover, the inequality clearly holds when evaluated at $\gamma_B = 0$ (since $\frac{t}{p} < 1$). Thus, the inequality holds for all $\gamma_B \in (0, \frac{1}{2})$. $\square$

Lemma S5. *Suppose* $\frac{t}{p} \in \left(\varphi(\gamma_B), \frac{3\gamma_B}{2(2+\gamma_B)}\right]$. *We then have the following relationships:*

a) $\pi_{D|A} > \pi_{C|A} > \pi_{B|A}$ *and* $\pi_{C|A} > \pi_{A|A}$*;*

b) $\pi_{B|C} > \pi_{A|C} > \pi_{C|C}$;

c) $\pi_{D|B} > \pi_{C|B} > \pi_{B|B}$ *and* $\pi_{C|B} > \pi_{A|B}$.

Proof of Lemma S5. The relationships $\pi_{D|A} > \pi_{C|A}$ and $\pi_{A|C} > \pi_{C|C}$ are already shown in the proof of Lemma S4 (the arguments in question are valid also for this part of the parameter space). Consider the relationship $\pi_{C|A} > \pi_{B|A}$. We can write

$$\pi_{C|A} > \pi_{B|A} \Leftrightarrow \frac{t\,(3+\gamma_B)^2}{18(1-\gamma_B)} > \gamma_B\,(p-t) \Leftrightarrow \frac{t}{p} > \varphi(\gamma_B),$$

which holds for all $\gamma_B \in (0, \frac{1}{2})$. We can also write

$$\pi_{C|A} > \pi_{A|A} \Leftrightarrow \frac{t\,(3+\gamma_B)^2}{18(1-\gamma_B)} > \frac{(1-\gamma_B)\,t}{2} \Leftrightarrow (3+\gamma_B)^2 > 9(1-\gamma_B)^2,$$

which again holds for all $\gamma_B \in (0, \frac{1}{2})$. Next consider the relationship $\pi_{B|C} > \pi_{A|C}$. We can write

$$\pi_{B|C} > \pi_{A|C} \Leftrightarrow \frac{t\,(2+\gamma_B)^2}{18\gamma_B} > \frac{t\,(3-\gamma_B)^2}{18(1-\gamma_B)} \Leftrightarrow (1-2\gamma_B)(4-\gamma_B+\gamma_B^2) > 0,$$

which holds for all $\gamma_B \in (0, \frac{1}{2})$. Similarly, we can write

$$\begin{aligned}\pi_{C|B} < \pi_{D|B} &\Leftrightarrow \frac{t\,(4-\gamma_B)^2}{18\gamma_B} < \gamma_A(p-t) + \gamma_B\frac{t}{2}\\ &\Leftrightarrow \left[(4-\gamma_B)^2 + 18\gamma_B(1-\gamma_B) - 9\gamma_B^2\right] t < 18\gamma_B(1-\gamma_B)p\\ \Leftrightarrow \left[16+10\gamma_B-26\gamma_B^2\right] t &= 2(8+13\gamma_B)(1-\gamma_B)t < 18\gamma_B(1-\gamma_B)p \Leftrightarrow \frac{t}{p} < \frac{9\gamma_B}{8+13\gamma_B}.\end{aligned}$$

But the last inequality is implied by $\frac{t}{p} \le \frac{3\gamma_B}{2(2+\gamma_B)}$. Now consider the relationship $\pi_{C|B} > \pi_{B|B}$:

$$\pi_{C|B} > \pi_{B|B} \Leftrightarrow \frac{t\,(4-\gamma_B)^2}{18\gamma_B} > \gamma_B\frac{t}{2} \Leftrightarrow (4-\gamma_B)^2 > 9\gamma_B^2,$$

which holds for all $\gamma_B \in (0, \frac{1}{2})$. Finally write

$$\pi_{C|B} > \pi_{A|B} \Leftrightarrow \frac{t\,(4-\gamma_B)^2}{18\gamma_B} > (1-\gamma_B)(p-t) \Leftrightarrow \frac{t}{p} > \frac{18\gamma_B(1-\gamma_B)}{(4-\gamma_B)^2 + 18\gamma_B(1-\gamma_B)},$$

which is implied by $\frac{t}{p} > \varphi(\gamma_B)$. □

Lemma S6. *Suppose* $\frac{t}{p} \in \left(\varphi(\gamma_B), \frac{1}{4}\right]$. *We then have the following relationships:*

a) $\pi_{D|A} > \pi_{C|A}$;

b) $\pi_{A|C} > \pi_{C|C}$ *and* $\pi_{A|C} > \pi_{B|C}$;

c) $\pi_{D|B} > \pi_{C|B}$;

d) $\pi_{C|A} > \pi_{B|A}$ *and* $\pi_{C|A} > \pi_{A|A}$;

e) $\pi_{A|B} = \pi_{C|B} > \pi_{B|B}$.

Proof of Lemma S6. First note that $\frac{t}{p} \leq \frac{1}{4}$ implies $\frac{t}{p} < \frac{1}{3}$. The claims that $\pi_{D|A} > \pi_{C|A}$ and $\pi_{A|C} > \pi_{C|C}$ are already shown in the proof of Lemma S4 (the arguments in question are valid also for this part of the parameter space). Similarly, the relationships $\pi_{C|A} > \pi_{B|A}$ and $\pi_{C|A} > \pi_{A|A}$ are already shown in the proof of Lemma S5 (the arguments are valid also for this part of the parameter space).

Thus consider the relationship $\pi_{A|C} > \pi_{B|C}$. We can write

$$\pi_{A|C} > \pi_{B|C} \Leftrightarrow \frac{t\,(3-\gamma_B)^2}{18(1-\gamma_B)} > \gamma_B(p-2t) \Leftrightarrow \frac{t}{p} > \frac{18\gamma_B(1-\gamma_B)}{(3-\gamma_B)^2+36\gamma_B(1-\gamma_B)}.$$

However, the last inequality is, given $\frac{t}{p} < \frac{1}{3}$, implied by our assumption that $\frac{t}{p} > \varphi(\gamma_B)$. Similarly, we can write

$$\pi_{D|B} > \pi_{C|B} \Leftrightarrow \gamma_A(p-t) + \gamma_B\frac{t}{2} > \gamma_A(p-t),$$

which trivially always holds. Next, it is clear that we have $\pi_{A|B} = \pi_{C|B}$, since both profit levels equal $\gamma_A(p-t)$. Finally we can write

$$\pi_{C|B} > \pi_{B|B} \Leftrightarrow \gamma_A(p-t) > \gamma_B\frac{t}{2} \Leftrightarrow \frac{t}{p} < \frac{2(1-\gamma_B)}{2-\gamma_B},$$

which holds for all $\gamma_B \in (0, \frac{1}{2})$. □

2.2. Proof of Proposition 1

Proof of Proposition 1, part (i). First suppose $\frac{t}{p} \in \left[\frac{1}{2}, \frac{2}{3}\right)$. Consider the game matrix in Figure 1 in Lagerlöf (2018). Let us study one column of the matrix at a time, while using the results in Lemmas S1 and S2.

- In column A, firm 1 prefers C to A and B (for we have $\pi_{C|A} > \pi_{A|A}$ and $\pi_{C|A} > \pi_{B|A}$), and it prefers D to A and B (for we have $\pi_{D|A} > \pi_{A|A}$ and $\pi_{D|A} > \pi_{B|A}$). However, neither $(y_1, y_2) = (C, A)$ nor $(y_1, y_2) = (D, A)$ can be an equilibrium, as firm 2 would in both cases have an incentive to deviate to C or to D (for we have $\pi_{C|C} = \pi_{D|C} > \pi_{A|C}$ and $\pi_{C|D} = \pi_{D|D} > \pi_{A|D}$).

- In column B, firm 1 prefers D to A, B and C (for we have $\pi_{D|B} > \pi_{A|B}$, $\pi_{D|B} > \pi_{B|B}$ and $\pi_{D|B} > \pi_{C|B}$). But $(y_1, y_2) = (D, B)$ cannot be an equilibrium, as firm 2 would have an incentive to deviate to C or D (for we have $\pi_{C|D} = \pi_{D|D} > \pi_{B|D}$).

- In column C, firm 1 is indifferent between C and D and prefers each one of these choices to A and B (for we have $\pi_{C|C} = \pi_{D|C} > \pi_{A|C} > \pi_{B|C}$). By symmetry (in particular, by the inequalities stated in the previous sentence), if firm 1 chooses C, then firm 2 has a (weak) incentive to also choose C. Hence $(y_1, y_2) = (C, C)$ is an equilibrium. If firm 1 chooses D, then firm 2 has a weak incentive to choose C (for we have $\pi_{C|D} = \pi_{D|D} > \pi_{A|D} > \pi_{B|D}$). Hence $(y_1, y_2) = (D, C)$ is an equilibrium.

- In column D, firm 1 is indifferent between C and D and prefers each one of these choices to A and B (for we have $\pi_{C|D} = \pi_{D|D} > \pi_{A|D} > \pi_{B|D}$). Indeed, since the game is symmetric and we have found that $(y_1, y_2) = (D, C)$ is an equilibrium, so is $(y_1, y_2) = (C, D)$. Moreover, again by symmetry, since firm 1 weakly prefers D in column D, firm 2 weakly prefers D in row D. Hence $(y_1, y_2) = (D, D)$ is an equilibrium.

Next suppose $\frac{t}{p} \in \left(\frac{\sqrt{1-\gamma_B}}{2}, \frac{1}{2}\right)$, and again study one column of the game matrix at a time, while using the results in Lemmas S1 and S3. Note that $\frac{t}{p} > \frac{\sqrt{1-\gamma_B}}{2}$ implies $\frac{t}{p} > \frac{1}{3}$.

- In column A, firm 1 prefers C to A, B and D (for we have $\pi_{C|A} > \pi_{D|A} = \pi_{A|A} + \pi_{B|A}$). However, $(y_1, y_2) = (C, A)$ cannot be an equilibrium, as firm 2 would have an incentive to deviate to D (for we have $\pi_{D|C} > \pi_{A|C}$).

- In column B, firm 1 again prefers C to A, B and D (for we have $\pi_{C|B} > \pi_{D|B} = \pi_{A|B} + \pi_{B|B}$). But $(y_1, y_2) = (C, B)$ cannot be an equilibrium, as firm 2 would have an an incentive to deviate to D (for we have $\pi_{D|C} > \pi_{B|C}$).

- In column C, firm 1 is indifferent between C and D and prefers each one of these choices to A and B (for we have $\pi_{C|C} = \pi_{D|C} > \pi_{A|C} > \pi_{B|C}$). By symmetry (in particular, by the inequalities stated in the previous sentence), if firm 1 chooses C, then firm 2 has a (weak) incentive to also choose C. Hence $(y_1, y_2) = (C, C)$ is an equilibrium. If firm 1 chooses D, then firm 2 has a weak incentive to choose C (for we have $\pi_{C|D} = \pi_{D|D} > \pi_{A|D} > \pi_{B|D}$). Hence $(y_1, y_2) = (D, C)$ is an equilibrium.

- In column D, firm 1 is indifferent between C and D and prefers each one of these choices to A and to B (for we have $\pi_{C|D} = \pi_{D|D} > \pi_{A|D} > \pi_{B|D}$). Indeed, since the game is symmetric and we have found that $(y_1, y_2) = (D, C)$ is an equilibrium, so is $(y_1, y_2) = (C, D)$. Moreover, again by symmetry, since firm 1 weakly prefers D in column D, firm 2 weakly prefers D in row D. Hence $(y_1, y_2) = (D, D)$ is an equilibrium.

Proof of Proposition 1, part (ii). First suppose $\frac{t}{p} \in \left(\max\left\{\frac{3(1-\gamma_B)}{2(3-\gamma_B)}, \frac{1}{3}\right\}, \frac{\sqrt{1-\gamma_B}}{2}\right)$. Let us study one column of the matrix in Figure 1 in Lagerlöf (2018) at a time, while using the results in Lemmas S1 and S3.

- In column A, firm 1 prefers C to A, B and D (for we have $\pi_{C|A} > \pi_{D|A} = \pi_{A|A} + \pi_{B|A}$). Given that firm 1 chooses C, firm 2 does not have an incentive to deviate (for we have $\pi_{A|C} > \pi_{D|C} = \pi_{C|C}$ and $\pi_{A|C} > \pi_{B|C}$). Hence $(y_1, y_2) = (C, A)$ is an equilibrium.

- In column B, firm 1 again prefers C to A, B and D (for we have $\pi_{C|B} > \pi_{D|B} = \pi_{A|B} + \pi_{B|B}$). But $(y_1, y_2) = (C, B)$ cannot be an equilibrium, as firm 2 would have an an incentive to deviate to A (for we have $\pi_{A|C} > \pi_{B|C}$).

- In column C, by symmetry (cf. the above arguments about columns A and B) the following must hold: $(y_1, y_2) = (A, C)$ is an equilibrium, but $(y_1, y_2) = (B, C)$ is not. Moreover, neither $(y_1, y_2) = (C, C)$ nor $(y_1, y_2) = (D, C)$ is an equilibrium, as firm 1 would have an incentive to deviate to A (for we have $\pi_{A|C} > \pi_{D|C} = \pi_{C|C}$).

- In column D, firm 1 is indifferent between C and D and prefers each one of these choices to A and to B (for we have $\pi_{C|D} = \pi_{D|D} > \pi_{A|D} > \pi_{B|D}$). Since the game is symmetric and we have found that $(y_1, y_2) = (D, C)$ is not an equilibrium, nor is $(y_1, y_2) = (C, D)$. Moreover, again by symmetry, since firm 1 weakly prefers D in column D, firm 2 weakly prefers D in row D. Hence $(y_1, y_2) = (D, D)$ is an equilibrium.

Next suppose $\frac{t}{p} \in \left(\frac{9(1-\gamma_B)}{21-13\gamma_B}, \frac{3(1-\gamma_B)}{2(3-\gamma_B)}\right]$, and again study one column of the game matrix at a time, using the results in Lemmas S1 and S4. Note that $\frac{t}{p} > \frac{9(1-\gamma_B)}{21-13\gamma_B}$ implies $\frac{t}{p} > \frac{3\sqrt{\gamma_B(1-\gamma_B)}}{2(3-\gamma_B)}$ and that $\frac{t}{p} \in \left(\frac{9(1-\gamma_B)}{21-13\gamma_B}, \frac{3(1-\gamma_B)}{2(3-\gamma_B)}\right)$ implies $\frac{t}{p} > \frac{1}{3}$.

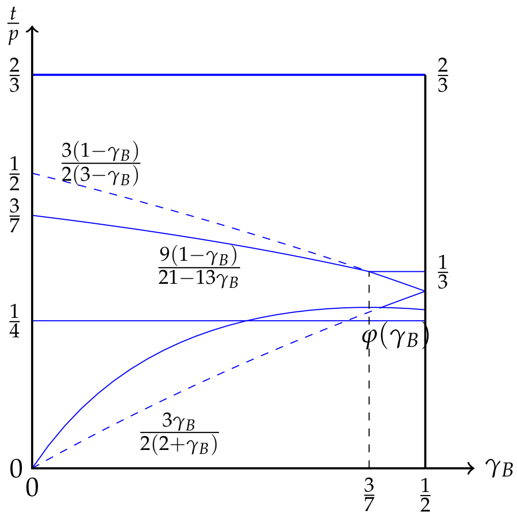

Figure 1: An illustration of different parts of the parameter space, which may be useful when studying the proofs of Propositions 1 and 3. See also panels (a) and (b) of Fig. 3 in Lagerlöf (2018) .

- In column A, firm 1 prefers C to A, B and D (for we have $\pi_{C|A} > \pi_{D|A} = \pi_{A|A} + \pi_{B|A}$). Given that firm 1 chooses C, firm 2 does not have an incentive to deviate (for we have $\pi_{A|C} > \pi_{C|C} = \pi_{D|C}$ and $\pi_{A|C} > \pi_{B|C}$). Hence $(y_1, y_2) = (C, A)$ is an equilibrium.

- In column B, firm 1 again prefers C to A, B and D (for we have $\pi_{C|B} > \pi_{D|B} = \pi_{A|B} + \pi_{B|B}$). But $(y_1, y_2) = (C, B)$ cannot be an equilibrium, as firm 2 would have an incentive to deviate to A (for we have $\pi_{A|C} > \pi_{B|C}$).

- In column C, by symmetry (cf. the above arguments about columns A and B) the following must hold: $(y_1, y_2) = (A, C)$ is an equilibrium, but $(y_1, y_2) = (B, C)$ is not. Moreover, neither $(y_1, y_2) = (C, C)$ nor $(y_1, y_2) = (D, C)$ is an equilibrium, as firm 1 would have an incentive to deviate to A (for we have $\pi_{A|C} > \pi_{C|C} = \pi_{D|C}$).

- In column D, firm 1 is indifferent between C and D and prefers each one of these choices to A and to B (for we have $\pi_{C|D} = \pi_{D|D} > \pi_{A|D} > \pi_{B|D}$). Since the game is symmetric and we have found that $(y_1, y_2) = (D, C)$ is not an equilibrium, nor is $(y_1, y_2) = (C, D)$. Moreover, again by symmetry, since firm 1 weakly prefers D in column D, firm 2 weakly prefers D in row D. Hence $(y_1, y_2) = (D, D)$ is an equilibrium.

Proof of Proposition 1 , part (iii). Suppose $\frac{t}{p} \in \left(\max\left\{\varphi(\gamma_B), \frac{3\gamma_B}{2(2+\gamma_B)}, \frac{1}{4}\right\}, \min\left\{\frac{3(1-\gamma_B)}{2(3-\gamma_B)}, \frac{9(1-\gamma_B)}{21-13\gamma_B}\right\}\right)$. Let us study one column of the matrix in Figure 1 in Lagerlöf (2018) at a time, while using the results in Lemmas S1 and S4.

- In column A, firm 1 prefers D to A, B and C (for we have $\pi_{D|A} > \pi_{C|A}$ and $\pi_{D|A} = \pi_{A|A} + \pi_{B|A}$). However, $(y_1, y_2) = (D, A)$ cannot be an equilibrium, as firm 2 would have an incentive to deviate to D (for we have $\pi_{D|D} > \pi_{A|D}$).

- In column B, firm 1 prefers, depending on parameter values, either C or D to A and to B (for we have $\pi_{C|B} \gtreqless \pi_{D|B}$ as $\frac{t}{p} \gtreqless \frac{1}{3}$, and we always have $\pi_{D|B} = \pi_{A|B} + \pi_{B|B}$). But neither $(y_1, y_2) = (C, B)$ nor $(y_1, y_2) = (D, B)$ can be an equilibrium. In the former case firm 2 would have an incentive to deviate to A (for if $\pi_{C|B} \geq \pi_{D|B}$, then we must have $\pi_{A|C} > \pi_{B|C}$). In the latter case firm 2 would have an incentive to deviate to D (for we always have $\pi_{D|D} > \pi_{B|D}$).

- In column C, firm 1 prefers A to C and D (for we have $\pi_{A|C} > \pi_{C|C} = \pi_{D|C}$). Depending on parameter values, firm 1 may prefer A to B, or B to A. However, by symmetry of the game, neither $(y_1, y_2) = (A, C)$ nor $(y_1, y_2) = (B, C)$ can be an equilibrium since we showed above that $(y_1, y_2) = (C, A)$ and $(y_1, y_2) = (C, B)$ are not.

- In column D, firm 1 is indifferent between C and D and prefers each one of these choices to A and to B (for we have $\pi_{C|D} = \pi_{D|D} > \pi_{A|D} > \pi_{B|D}$). Since the game is symmetric and we have found that $(y_1, y_2) = (D, C)$ is not an equilibrium, nor is $(y_1, y_2) = (C, D)$. Moreover, again by symmetry, since firm 1 weakly prefers D in column D, firm 2 weakly prefers D in row D. Hence $(y_1, y_2) = (D, D)$ is an equilibrium.

Next suppose $\frac{t}{p} \in \left(\frac{3(1-\gamma_B)}{2(3-\gamma_B)}, \frac{1}{3}\right)$, and again study one column of the game matrix at a time, while using the results in Lemmas S1 and S3. Note that $\frac{t}{p} < \frac{1}{3}$ implies $\frac{t}{p} < \frac{\sqrt{1-\gamma_B}}{2}$.

- In column A, firm 1 prefers D to A, B and C (for we have $\pi_{D|A} > \pi_{C|A}$ and $\pi_{D|A} = \pi_{A|A} + \pi_{B|A}$). However, $(y_1, y_2) = (D, A)$ cannot be an equilibrium, as firm 2 would have an incentive to deviate to D (for we have $\pi_{D|D} > \pi_{A|D}$).

- In column B, firm 1 again prefers D to A, B and C (for we have $\pi_{D|B} > \pi_{C|B}$ and $\pi_{D|B} = \pi_{A|B} + \pi_{B|B}$). But $(y_1, y_2) = (D, B)$ cannot be an equilibrium, as firm 2 would have an incentive to deviate to D (for we have $\pi_{D|D} > \pi_{B|D}$).

- In column C, firm 1 prefers A to B, C and D (for we have $\pi_{A|C} > \pi_{D|C} = \pi_{C|C}$ and $\pi_{A|C} > \pi_{B|C}$). However, by symmetry of the game, $(y_1, y_2) = (A, C)$ cannot be an equilibrium since we showed above that $(y_1, y_2) = (C, A)$ is not.

- In column D, firm 1 is indifferent between C and D and prefers each one of these choices to A and to B (for we have $\pi_{C|D} = \pi_{D|D} > \pi_{A|D} > \pi_{B|D}$). By symmetry, if firm 1 plays D, then firm 2 does not have an incentive to deviate. Hence $(y_1, y_2) = (D, D)$ is an equilibrium. However, again by symmetry of the game, $(y_1, y_2) = (C, D)$ cannot be an equilibrium since we showed above that $(y_1, y_2) = (D, C)$ is not.

Now suppose $\frac{t}{p} \in \left(\varphi(\gamma_B), \frac{3\gamma_B}{2(2+\gamma_B)}\right)$, and again study one column of the game matrix at a time, while using the results in Lemmas S1 and S5.

- In column A, firm 1 prefers D to A, B and C (for we have $\pi_{D|A} > \pi_{C|A}$ and $\pi_{D|A} = \pi_{A|A} + \pi_{B|A}$). However, $(y_1, y_2) = (D, A)$ cannot be an equilibrium, as firm 2 would have an incentive to deviate to D (for we have $\pi_{D|D} > \pi_{A|D}$).

- In column B, firm 1 again prefers D to A, B and C (for we have $\pi_{D|B} > \pi_{C|B}$ and $\pi_{D|B} = \pi_{A|B} + \pi_{B|B}$). But $(y_1, y_2) = (D, B)$ cannot be an equilibrium, as firm 2 would have an incentive to deviate to D (for we have $\pi_{D|D} > \pi_{B|D}$).

- In column C, firm 1 prefers B to A, C and D (for we have $\pi_{B|C} > \pi_{A|C} > \pi_{C|C} = \pi_{D|C}$). However, by symmetry of the game, $(y_1, y_2) = (B, C)$ cannot be an equilibrium since we showed above that $(y_1, y_2) = (C, B)$ is not.

- In column D, firm 1 is indifferent between C and D and prefers each one of these choices to A and to B (for we have $\pi_{C|D} = \pi_{D|D} > \pi_{A|D} > \pi_{B|D}$). By symmetry, if firm 1 plays D, then firm 2 does not have an incentive to deviate. Hence $(y_1, y_2) = (D, D)$ is an equilibrium. However, again by symmetry of the game, $(y_1, y_2) = (C, D)$ cannot be an equilibrium since we showed above that $(y_1, y_2) = (D, C)$ is not.

Finally suppose $\frac{t}{p} \in \left(\varphi(\gamma_B), \frac{1}{4}\right)$, and again study one column of the game matrix at a time, while using the results in Lemmas S1 and S6.

- In column A, firm 1 prefers D to A, B and C (for we have $\pi_{D|A} > \pi_{C|A}$ and $\pi_{D|A} = \pi_{A|A} + \pi_{B|A}$). However, $(y_1, y_2) = (D, A)$ cannot be an equilibrium, as firm 2 would have an incentive to deviate to D (for we have $\pi_{D|D} > \pi_{A|D}$).

- In column B, firm 1 again prefers D to A, B and C (for we have $\pi_{D|B} > \pi_{C|B}$ and $\pi_{D|B} = \pi_{A|B} + \pi_{B|B}$). But $(y_1, y_2) = (D, B)$ cannot be an equilibrium, as firm 2 would have an incentive to deviate to D (for we have $\pi_{D|D} > \pi_{B|D}$).

- In column C, firm 1 prefers A to B, C and D (for we have $\pi_{A|C} > \pi_{B|C}$ and $\pi_{A|C} > \pi_{C|C} = \pi_{D|C}$). However, by symmetry of the game, $(y_1, y_2) = (A, C)$ cannot be an equilibrium since we showed above that $(y_1, y_2) = (C, A)$ is not.

- In column D, firm 1 is indifferent between C and D and prefers each one of these choices to A and to B (for we have $\pi_{C|D} = \pi_{D|D} > \pi_{A|D} > \pi_{B|D}$). By symmetry, if firm 1 plays D, then firm 2 does not have an incentive to deviate. Hence $(y_1, y_2) = (D, D)$ is an equilibrium. However, again by symmetry of the game, $(y_1, y_2) = (C, D)$ cannot be an equilibrium since we showed above that $(y_1, y_2) = (D, C)$ is not.

□

2.3. Proof of Proposition 3

First suppose that $\frac{t}{p} \in \Omega_I \cup \Omega_{III}$. Then, by Proposition 1, $(y_1^*, y_2^*) \in \{(C, C), (C, D), (D, C), (D, D)\}$. Therefore the firms address the same segments of workers and, by the analysis in subsection 3.1.2 in the paper, $w_1^* = w_2^* = p - t$ and $\pi_1^* = \pi_2^* = \frac{t}{2}$. The claims in part (i) of Proposition 3 follow immediately from these expressions.

Next suppose that $\frac{t}{p} \in \Omega_{II}$. Then, given that a payoff-dominated equilibrium is not played, it follows from Propositions 1 and 2 that $(y_1^*, y_2^*) \in \{(A, C), (C, A)\}$. That is, one firm discriminates in hiring against the minority group ($y_j = A$) and the other firm does not discriminate at all ($y_{-j} = C$). If $\frac{t}{p} > \frac{3(1-\gamma_B)}{2(3-\gamma_B)}$, then it follows from subsection 3.1.4 and Lemma 1 in the paper that the equilibrium wages are $w_j^* = \frac{p}{2}$ and $w_{-j}^* = t$, and the equilibrium profits are

$$\pi_j^* = \frac{(1-\gamma_B)p^2}{8t} \quad \text{and} \quad \pi_{-j}^* = \frac{(p-t)\left[4t - (1-\gamma_B)p\right]}{4t}.$$

The claims in part (ii) of Proposition 3 follow immediately from these expressions (differentiating yields $\frac{\partial \pi_{-j}^*}{\partial t} > 0 \Leftrightarrow \frac{t}{p} < \frac{\sqrt{1-\gamma_B}}{2}$, which is implied by $\frac{t}{p} \in \Omega_{II}$). If $\frac{t}{p} < \frac{3(1-\gamma_B)}{2(3-\gamma_B)}$, then it follows from subsection 3.1.4 and Lemma 1 in the paper that the equilibrium wages are $w_j^* = p - \frac{(3-\gamma_B)t}{3(1-\gamma_B)}$ and $w_{-j}^* = p - \frac{(3+\gamma_B)t}{3(1-\gamma_B)}$, and the equilibrium profits are

$$\pi_j^* = \frac{t\,(3-\gamma_B)^2}{18(1-\gamma_B)} \quad \text{and} \quad \pi_{-j}^* = \frac{t\,(3+\gamma_B)^2}{18(1-\gamma_B)}.$$

The claims in part (iii) of Proposition 3 follow immediately from these expressions. □

2.4. Proof of Proposition 5

The structure of the proof is very similar to the one in the proof of Proposition 1, except that here there is no D action.

Proof of Proposition 5, part (i). First suppose $\frac{t}{p} \in \left[\frac{1}{2}, \frac{2}{3}\right)$. Consider the game matrix in Figure 1 in Lagerlöf (2018). Let us study one column of the matrix at a time, while using the results in Lemmas S1 and S2. Since we now have $S \in \{A, B, C\}$, we ignore the D column and the D row.

- In column A, firm 1 prefers C to A and B (for we have $\pi_{C|A} > \pi_{A|A}$ and $\pi_{C|A} > \pi_{B|A}$). However, $(y_1, y_2) = (C, A)$ cannot be an equilibrium, as firm 2 would have an incentive to deviate to C (for we have $\pi_{C|C} > \pi_{A|C}$).
- In column B, firm 1 prefers C to A and B (for we have $\pi_{C|B} > \pi_{A|B} > \pi_{B|B}$). But $(y_1, y_2) = (C, B)$ cannot be an equilibrium, as firm 2 would have an incentive to deviate to C (for we have $\pi_{C|C} > \pi_{B|C}$).
- In column C, firm 1 prefers C to A and B (for we have $\pi_{C|C} > \pi_{A|C} > \pi_{B|C}$). By symmetry (in particular, by the inequalities stated in the previous sentence), if firm 1 chooses C, then firm 2 has an incentive to also choose C. Hence $(y_1, y_2) = (C, C)$ is an equilibrium.

Next suppose $\frac{t}{p} \in \left(\frac{\sqrt{1-\gamma_B}}{2}, \frac{1}{2}\right)$, and again study one column of the game matrix at a time, while using the results in Lemmas S1 and S3. Note that $\frac{t}{p} > \frac{\sqrt{1-\gamma_B}}{2}$ implies $\frac{t}{p} > \frac{1}{3}$.

- In column A, firm 1 prefers C to A and B (for we have $\pi_{C|A} > \pi_{D|A} = \pi_{A|A} + \pi_{B|A}$). However, $(y_1, y_2) = (C, A)$ cannot be an equilibrium, as firm 2 would have an incentive to deviate to C (for we have $\pi_{C|C} > \pi_{A|C}$).
- In column B, firm 1 again prefers C to A and B (for we have $\pi_{C|B} > \pi_{D|B} = \pi_{A|B} + \pi_{B|B}$). But $(y_1, y_2) = (C, B)$ cannot be an equilibrium, as firm 2 would have an an incentive to deviate to C (for we have $\pi_{C|C} > \pi_{A|C} > \pi_{B|C}$).
- In column C, firm 1 prefers C to A and B (for we have $\pi_{C|C} > \pi_{A|C} > \pi_{B|C}$). By symmetry (in particular, by the inequalities stated in the previous sentence), if firm 1 chooses C, then firm 2 has an incentive to also choose C. Hence $(y_1, y_2) = (C, C)$ is an equilibrium.

Proof of Proposition 5, part (ii). First suppose $\frac{t}{p} \in \left(\max\left\{\frac{3(1-\gamma_B)}{2(3-\gamma_B)}, \frac{1}{3}\right\}, \frac{\sqrt{1-\gamma_B}}{2}\right)$. Let us study one column of the matrix in Figure 1 in Lagerlöf (2018) at a time, while using the results in Lemmas S1 and S3.

- In column A, firm 1 prefers C to A and B (for we have $\pi_{C|A} > \pi_{D|A} = \pi_{A|A} + \pi_{B|A}$). Given that firm 1 chooses C, firm 2 does not have an incentive to deviate to C or B (for we have $\pi_{A|C} > \pi_{D|C} = \pi_{C|C}$ and $\pi_{A|C} > \pi_{B|C}$). Hence $(y_1, y_2) = (C, A)$ is an equilibrium.
- In column B, firm 1 again prefers C to A and B (for we have $\pi_{C|B} > \pi_{D|B} = \pi_{A|B} + \pi_{B|B}$). But $(y_1, y_2) = (C, B)$ cannot be an equilibrium, as firm 2 would have an an incentive to deviate to A (for we have $\pi_{A|C} > \pi_{B|C}$).

- In column C, by symmetry (cf. the above arguments about columns A and B) the following must hold: $(y_1, y_2) = (A, C)$ is an equilibrium, but $(y_1, y_2) = (B, C)$ is not. Moreover, $(y_1, y_2) = (C, C)$ is not an equilibrium, as firm 1 would have an incentive to deviate to A (for we have $\pi_{A|C} > \pi_{D|C} = \pi_{C|C}$).

Next suppose $\frac{t}{p} \in \left(\frac{9(1-\gamma_B)}{21-13\gamma_B}, \frac{3(1-\gamma_B)}{2(3-\gamma_B)}\right)$, and again study one column of the game matrix at a time, using the results in Lemmas S1 and S4. Note that $\frac{t}{p} > \frac{9(1-\gamma_B)}{21-13\gamma_B}$ implies $\frac{t}{p} > \frac{3\sqrt{\gamma_B(1-\gamma_B)}}{2(3-\gamma_B)}$ and that $\frac{t}{p} \in \left(\frac{9(1-\gamma_B)}{21-13\gamma_B}, \frac{3(1-\gamma_B)}{2(3-\gamma_B)}\right)$ implies $\frac{t}{p} > \frac{1}{3}$.

- In column A, firm 1 prefers C to A and B (for we have $\pi_{C|A} > \pi_{D|A} = \pi_{A|A} + \pi_{B|A}$). Given that firm 1 chooses C, firm 2 does not have an incentive to deviate to C or B (for we have $\pi_{A|C} > \pi_{C|C}$ and $\pi_{A|C} > \pi_{B|C}$). Hence $(y_1, y_2) = (C, A)$ is an equilibrium.
- In column B, firm 1 again prefers C to A and B (for we have $\pi_{C|B} > \pi_{D|B} = \pi_{A|B} + \pi_{B|B}$). But $(y_1, y_2) = (C, B)$ cannot be an equilibrium, as firm 2 would have an incentive to deviate to A (for we have $\pi_{A|C} > \pi_{B|C}$).
- In column C, by symmetry (cf. the above arguments about columns A and B) the following must hold: $(y_1, y_2) = (A, C)$ is an equilibrium, but $(y_1, y_2) = (B, C)$ is not. Moreover, $(y_1, y_2) = (C, C)$ is not an equilibrium, as firm 1 would have an incentive to deviate to A (for we have $\pi_{A|C} > \pi_{C|C}$).

Now suppose $\frac{t}{p} \in \left(\max\left\{\varphi(\gamma_B), \frac{3\gamma_B}{2(2+\gamma_B)}, \frac{1}{4}\right\}, \min\left\{\frac{3(1-\gamma_B)}{2(3-\gamma_B)}, \frac{9(1-\gamma_B)}{21-13\gamma_B}\right\}\right)$. Let us study one column of the matrix in Figure 1 in Lagerlöf (2018) at a time, while using the results in Lemmas S1 and S4.

- In column A, firm 1 prefers C to A and B (for we have $\pi_{C|A} > \pi_{A|A}$ and $\pi_{C|A} > \pi_{B|A}$). Given that firm 1 chooses C, firm 2 never has an incentive to deviate to C (for we have $\pi_{A|C} > \pi_{C|C}$) and firm 2 has no incentive to deviate to B if and only if $\frac{t}{p} \geq \frac{3\sqrt{\gamma_B(1-\gamma_B)}}{2(3-\gamma_B)}$ (by part c) of Lemma S4). Hence $(y_1, y_2) = (C, A)$ is an equilibrium if and only if $\frac{t}{p} \geq \frac{3\sqrt{\gamma_B(1-\gamma_B)}}{2(3-\gamma_B)}$.
- In column B, firm 1 prefers C to A and B (for we have $\pi_{C|B} > \pi_{A|B}$ and $\pi_{C|B} > \pi_{B|B}$). Given that firm 1 chooses C, firm 2's best deviation cannot be C (for we have $\pi_{A|C} > \pi_{C|C}$) and firm 2 has no incentive to deviate to A if and only if $\frac{t}{p} \leq \frac{3\sqrt{\gamma_B(1-\gamma_B)}}{2(3-\gamma_B)}$ (by part c) of Lemma S4). Hence $(y_1, y_2) = (C, B)$ is an equilibrium if and only if $\frac{t}{p} \leq \frac{3\sqrt{\gamma_B(1-\gamma_B)}}{2(3-\gamma_B)}$.
- In column C, by symmetry (cf. the above arguments about columns A and B) the following must hold: $(y_1, y_2) = (A, C)$ is an equilibrium if and only if $\frac{t}{p} \geq \frac{3\sqrt{\gamma_B(1-\gamma_B)}}{2(3-\gamma_B)}$; and $(y_1, y_2) = (B, C)$ is an equilibrium if and only if $\frac{t}{p} \leq \frac{3\sqrt{\gamma_B(1-\gamma_B)}}{2(3-\gamma_B)}$. Moreover, $(y_1, y_2) = (C, C)$ is not an equilibrium, as firm 1 would have an incentive to deviate to A (for we have $\pi_{A|C} > \pi_{C|C}$).

Next suppose $\frac{t}{p} \in \left(\frac{3(1-\gamma_B)}{2(3-\gamma_B)}, \frac{1}{3}\right)$, and again study one column of the game matrix at a time, while using the results in Lemmas S1 and S3. Note that $\frac{t}{p} < \frac{1}{3}$ implies $\frac{t}{p} < \frac{\sqrt{1-\gamma_B}}{2}$. Also, $\frac{t}{p} \in \left(\frac{3(1-\gamma_B)}{2(3-\gamma_B)}, \frac{1}{3}\right)$ implies that $\gamma_B > \frac{3}{7}$.

- In column A, firm 1 prefers C to A and B (for we have $\pi_{C|A} > \pi_{B|A} > \pi_{A|A}$). Given that firm 1 chooses C, firm 2 does not have an incentive to deviate to B or C (for we have $\pi_{A|C} > \pi_{B|C}$ and $\pi_{A|C} > \pi_{D|C} = \pi_{C|C}$). Hence $(y_1, y_2) = (C, A)$ is an equilibrium.

- In column B, firm 1 prefers C to A and B (for we have $\pi_{C|B} > \pi_{A|B} > \pi_{B|B}$). But $(y_1, y_2) = (C, B)$ cannot be an equilibrium, as firm 2 would have an incentive to deviate to A (for we have $\pi_{A|C} > \pi_{B|C}$).

- In column C, firm 1 prefers A to B and C (for we have $\pi_{A|C} > \pi_{B|C}$ and $\pi_{A|C} > \pi_{D|C} = \pi_{C|C}$). Given that firm 1 chooses A, firm 2 does not have an incentive to deviate to A or B (for we have $\pi_{C|A} > \pi_{B|A} > \pi_{A|A}$). Hence $(y_1, y_2) = (A, C)$ is an equilibrium.

Now suppose $\frac{t}{p} \in \left(\varphi(\gamma_B), \frac{3\gamma_B}{2(2+\gamma_B)}\right]$, and again study one column of the game matrix at a time, while using the results in Lemmas S1 and S5.

- In column A, firm 1 prefers C to A and B (for we have $\pi_{C|A} > \pi_{B|A}$ and $\pi_{C|A} > \pi_{A|A}$). However, $(y_1, y_2) = (C, A)$ cannot be an equilibrium, as firm 2 would have an incentive to deviate to B (for we have $\pi_{B|C} > \pi_{A|C}$).

- In column B, firm 1 again prefers C to A and B (for we have $\pi_{C|B} > \pi_{B|B}$ and $\pi_{C|B} > \pi_{A|B}$). Given that firm 1 chooses C, firm 2 does not have an incentive to deviate to A or C (for we have $\pi_{B|C} > \pi_{A|C} > \pi_{C|C}$). Hence $(y_1, y_2) = (C, B)$ is an equilibrium.

- In column C, by symmetry (cf. the above arguments about columns A and B) the following must hold: $(y_1, y_2) = (B, C)$ is an equilibrium, but $(y_1, y_2) = (A, C)$ is not. Moreover, $(y_1, y_2) = (C, C)$ is not an equilibrium, as firm 1 would have an incentive to deviate to B (for we have $\pi_{B|C} > \pi_{C|C}$).

Finally suppose $\frac{t}{p} \in \left(\varphi(\gamma_B), \frac{1}{4}\right]$, and again study one column of the game matrix at a time, while using the results in Lemmas S1 and S6.

- In column A, firm 1 prefers C to A and B (for we have $\pi_{C|A} > \pi_{B|A}$ and $\pi_{C|A} > \pi_{A|A}$). Given that firm 1 chooses C, firm 2 does not have an incentive to deviate to B or C (for we have $\pi_{A|C} > \pi_{B|C}$ and $\pi_{A|C} > \pi_{C|C}$). Hence $(y_1, y_2) = (C, A)$ is an equilibrium.

- In column B, firm 1 prefers A and C to B (for we have $\pi_{A|B} = \pi_{C|B} > \pi_{B|B}$). But $(y_1, y_2) = (C, B)$ cannot be an equilibrium, as firm 2 would have an incentive to deviate to A (for we have $\pi_{A|C} > \pi_{B|C}$). Nor can $(y_1, y_2) = (A, B)$ be an equilibrium, as firm 2 would have an incentive to deviate to C (for we have $\pi_{C|A} > \pi_{B|A}$).

- In column C, by symmetry (cf. the above arguments about columns A and B) the following must hold: $(y_1, y_2) = (A, C)$ is an equilibrium, but $(y_1, y_2) = (B, C)$ is not. Moreover, $(y_1, y_2) = (C, C)$ is not an equilibrium, as firm 1 would have an incentive to deviate to A (for we have $\pi_{A|C} > \pi_{C|C}$).

□

References

Lagerlöf, Johan N. M. (2018) "Strategic Gains from Discrimination," August, Mimeo, University of Copenhagen.