Surfing Incognito: Welfare Effects of Anonymous Shopping

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Abstract

This paper studies consumers' incentives to hide their purchase histories when the seller's prices depend on previous behavior. Through distinct channels, hiding both hinders and facilitates trade. Indeed, the social optimum involves hiding to some extent, yet not fully. Two opposing effects determine whether a consumer hides too much or too little: the first-period social gains are only partially internalized, and there is a private (socially irrelevant) second-period gain due to price differences. If the discount factor is large, the second effect dominates and there is socially excessive hiding. This result is reversed if the discount factor is small.

Keywords: behavior-based price discrimination, dynamic pricing, consumer protection, customer recognition, privacy

JEL classification: D42, D80, L12, L40

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1 Introduction

In the last couple of decades it has become increasingly common that consumers make retail purchases on the internet. While online shopping often is convenient for consumers, it also makes it relatively easy for sellers to keep track of individual customers' purchasing decisions and thereby learn about their willingness to pay for the good. Using that knowledge, the sellers can charge personalized prices that leave certain consumers with a smaller surplus than otherwise. In a Washington Post article, Lowrey (2010) vividly describes this phenomenon and how it upsets consumers:

> *Retailers read the cookies kept on your browser or glean information from your past purchase history when you are logged into a site. That gives them a sense of what you search for and buy, how much you paid for it, and whether you might be willing and able to spend more. They alter their prices or offers accordingly. Consumers [...] tend to go apoplectic. But the practice is perfectly legal, and increasingly common—pervasive, even, for some products.*

In the economics literature, the seller practice described in the quotation has been referred to as dynamic pricing, history-based pricing, or behavior-based price discrimination (BBPD). A seller who practices BBPD does not necessarily have to do this in the blunt manner suggested in the quotation, directly presenting different consumers with different price tags. Often more subtle approaches are available. For example, a seller can distribute a specific discount coupon only to certain consumers, thereby effectively offering a personalized price to them. Introductory offers that entitle new customers to pay a lower price than returning customers pay are another example of BBPD. Yet another way in which a seller can, in a more subtle way, practice BBPD is by so-called price steering: on her website (for example), a seller presents available options (say, more- or less-expensive versions of the product) differently to different customers (see, e.g., Hannak et al., 2014).

BBPD has been studied theoretically both in monopoly and oligopoly settings. One insight from this literature is that a firm's opportunity to practice BBPD is not necessarily harmful to consumer welfare, as price discrimination tends to lead to more trade than otherwise and this can benefit also the consumers.[1] However, in specific situations and if she can, a consumer clearly has an individual incentive to hide her purchase history from the seller. Indeed, if being a returning customer is interpreted as being a high-valuation customer, then you are better off pretending to be a new customer. In practice, there are several possibilities for consumers to hide their purchase histories by using various anonymizing technologies: they can refrain from joining loyalty programs or set their browsers to reject cookies; they can choose to end a newspaper subscription and then start a new one, instead of renewing the old subscription; or they "can use a variety of credit cards or more exotic anonymous payment technologies to make purchases anonymous or difficult to trace" (Acquisti and Varian, 2005, p. 367). These

[1] Another reason why the consumers can gain from the firm's opportunity to practice BBPD is that, due to the firm's inability to precommit to future prices, it is forced to lower its first-period price in order to sell anything then (a "Coase-conjecture effect").

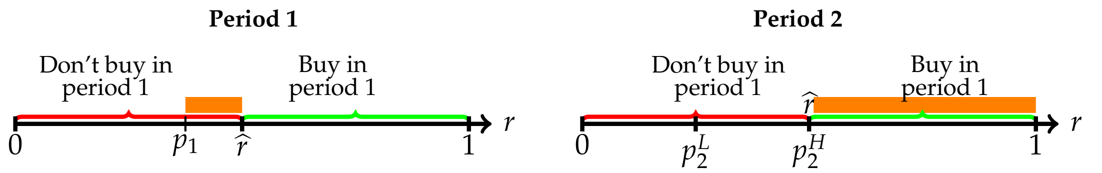


Figure 1: Equilibrium prices and cutoff value in a standard BBPD model. From a total surplus perspective, a consumer's incentive to hide is too weak in period 1 and too strong in period 2.

kinds of defense measures might come at a cost (a financial expense or nuisance) but they are certainly available.

The present paper is an attempt, within an equilibrium framework where a firm engages in BBPD, to study the incentives of consumers to hide their purchase histories with the help of anonymizing technologies. Among other things I ask whether, at the equilibrium and given other imperfections in the market, consumers use these technologies too little or too much. That is, from a social welfare perspective, does the market tend to generate too strong or too weak incentives for consumers to hide their purchase histories?

To help us think about whether (or under what circumstances) the incentives are likely to be too strong or too weak, refer to Figure 1. This figure summarizes the results of a standard two-period monopoly BBPD model with a continuum of consumers, each having a valuation r (the same across the two periods and drawn from a uniform distribution on the unit interval). The firm has no production costs; hence, the social optimum involves all consumers purchasing the good. In period 1, the firm sets a single price p_1, knowing only the distribution of valuations; the consumers then choose whether to purchase. In period 2, the firm can distinguish between buying and non-buying consumers and thus set two prices, p_2^L and p_2^H. At the equilibrium, consumers with valuations above a cutoff $\widehat{r}$ purchase in period 1. Moreover, the period 2 equilibrium price for the returning customers is higher than that of the new customers ($p_2^H > p_2^L$).[2] Finally, the period 1 equilibrium price is strictly lower than the cutoff ($p_1 < \widehat{r}$); this is because a consumer requires a positive first-period surplus in order to be willing to purchase in period 1 (for $p_2^H > p_2^L$).

Given the above framework, consider now the possibility that a consumer has the opportunity to hide her purchase decision in period 1 (thus being eligible to purchase at the price p_2^L in period 2, also if being a returning customer). Suppose also that the hiding decision is made at an ex ante stage, prior to learning the valuation r. Will her expected benefit from hiding be smaller or greater than society's expected benefit (measured as total surplus)? By inspecting Figure 1, we can identify three effects (or externalities) that determine whether a consumer with realized valuation r has too weak or too strong incentives, and these effects point in different directions:

[2] At the equilibrium, p_2^H equals the cutoff $\widehat{r}$, consistent with the figure. This price ensures that the firm sells to all the returning consumers, which is optimal to do whenever $\widehat{r} \geq 1/2$ (a condition that holds at the equilibrium).

1. For a consumer with $r \in (p_1, \hat{r})$, hiding one's purchase history will lead to *more trade*, as the consumer's acquired status as a "new" customer makes it worthwhile for her to buy the good in period 1, in a situation where she would not have bought it if lacking that status. However, when evaluating the benefit from this trade, the consumer considers only the net surplus that is generated (valuation less price, $r - p_1$). In contrast, society cares about the full surplus r (as the price is a pure transfer from the consumer to the firm). This effect thus suggests that the consumer has a too weak incentive to hide her purchase history.

2. For a consumer with $r \in (\hat{r}, 1]$, hiding one's purchase history enables the consumer, in period 2, to *buy the good at a lower price* than she otherwise would have paid, thus saving the amount $p_2^H - p_2^L$. The consumer cares about this benefit, while society does not (as the price is a pure transfer). Therefore, this effect suggests that the consumer has a too strong incentive to hide her purchase history.

3. Finally, an atomistic consumer's choice to hide her purchase history will not have any *impact on the equilibrium prices*. However, if many consumers in the market make that choice, the prices will indeed readjust to a new equilibrium. The direction of this effect is less clear than the direction of effects 1 and 2 above. But a plausible scenario would be that more hiding leads to a smaller difference between the second-period prices: the firm is less able price discriminate. This in turn might lead to less trade. If so, also this effect suggests that the consumer has a too strong incentive to hide her purchase history.

The first effect, which creates an incentive to hide too little, matters for the consumer in the period in which she makes her hiding decision. The second and third effects, which create an incentive to hide too much, matter only in the following period. This suggests that, if the consumer cares sufficiently much about the future, then the second and third effects might dominate and the consumer thus invests too much in anonymizing technologies. In the model that I set up and study, I show that this is indeed the case. I also show that, if the consumer instead is sufficiently myopic (i.e., she assigns a small weight to her second-period payoff), then we can reverse this result: the consumer invests too little in anonymizing technologies.

The formal framework that I develop is close to the standard BBPD model described above, except that I add the opportunity for the consumers to hide their purchase histories. Thus, a monopoly firm produces and sells a nondurable good in each of two time periods. Each consumer's valuation for the good is the same across the two periods and drawn from a uniform distribution. The valuation is initially the consumer's private information; the firm knows only the distribution of valuations in the market. However, by observing the first-period consumption choice, the firm can make a noisy inference about a consumer's valuation and then use this information when choosing the second-period price. In particular, a consumer's choice to purchase the good in the first period suggests that her valuation is relatively high, which creates an incentive for the firm to raise that consumer's second-period price. To protect herself from this, the consumer has the opportunity to hide her first-period purchase. This is modeled by letting the consumer choose a probability (i.e., any real number between zero and one) with which the information that she purchased the good is not available to the firm in the second period. Put differently, if the consumer chooses a particular hiding probability, then with that probability she will after her purchase look like a consumer who did not buy; with the complementary probability, she will indeed be identified by the firm as a consumer who purchased in period 1 and thus have to pay a higher price in period 2 (as in the standard setting). The choice of the hiding probability is made at an ex ante stage, before the consumer has learned her own valuation. This model feature captures the idea that the consumer adopts a long-term approach for dealing with privacy issues, for example, by choosing a setting on her computer that she sticks with through a large number of browsing sessions. I discuss this assumption at greater length in connection with the model description in Section 2. In the concluding section, I also briefly discuss the technical consequences of making the alternative assumption that the consumer's hiding decision is made *after* she has learned her valuation.

In Section 3, I begin the analysis by studying a version of the model in which the hiding probability is given exogenously and the same for all consumers. I first solve for the equilibrium (for given parameter values, this is unique). As in the standard model without the opportunity to hide one's purchase history, the equilibrium is characterized by a cutoff value of a consumer's valuation—above which she purchases in the first period, and below which she does not. Given this model with an exogenous hiding probability, I take an initial look at social welfare. In particular, I show that the welfare-maximizing value of the hiding probability is strictly interior. That is, if a total-surplus maximizing social planner could choose the hiding probability, the consumers would hide their first-period purchases to at least some extent, yet not fully. The reason why a strictly positive degree of hiding is socially optimal is that hiding generates gains from trade in the first period. Moreover, the social cost of hiding in terms of hindering second-period price discrimination is small, provided that the degree of hiding is sufficiently small.

In Section 4, I endogenize the hiding probability. I confine the analysis to equilibria where all consumers choose the same probability. I characterize and show existence of such an equilibrium. I then turn to the question whether the equilibrium value of the hiding probability is larger or smaller than the value that maximizes total surplus in the market. I first show that for the case where the common discount factor equals one—so the weights assigned to the first- and second-period payoffs are the same—the equilibrium involves too much hiding. The intuition for this result can be understood in terms of the discussion in the beginning of this introduction. By hiding her purchase history, a consumer can in the second period buy the good at a lower price than otherwise; this individual gain does not enter total surplus, as it is a pure transfer (effect 2). Moreover, the act of hiding makes it harder for the seller to practice (trade-enhancing) price discrimination in the second period (effect 3). Both those effects suggest that the consumer hides too much. Another effect of hiding one's purchase history is that it makes it possible to exploit first-period gains from trade to a greater extent; this suggests that the consumer hides too little, as she does not internalize the full benefit of the extra trade (effect 1). When the discount factor equals one, the weight on the second-period payoff is large enough for effects 2 and 3 to dominate.

A more general analysis, for the case where the discount factor is strictly below unity, is harder. Nevertheless, with the help of a numerical analysis, I can first show that the result discussed above—that the consumer hides too much—holds also for a range of discount factor values below one. Second, I provide examples where the discount factor is so low that the result above is reversed and, instead, the consumer hides too little. Intuitively, if the discount factor is sufficiently low, what matters for welfare is the first period and then the consumer does not internalize all the gains from trade that her hiding enables.

Section 5 studies an alternative model specification, in which the consumers learn whether their anonymization attempts have been successful only at a later stage. In such an environment, effect 1 discussed above cannot occur; thus, the possibility of undersupply of anonymization disappears. Section 6 discusses policy implications of the results, and Section 7 concludes. Appendix A shows that the main model used in the paper, with a continuous hiding choice, is equivalent to a certain alternative model with a binary hiding choice. Finally, Appendix B provides proofs of the results that are not proven in the main text.

1.1 Literature Review

The present paper is, of course, closely related to the literature on behavior-based price discrimination. Examples of works in this literature include Chen (1997), who studies a two-period duopoly model with switching costs. At the equilibrium, each firm offers a relatively low introductory price, meaning that it effectively pays customers to switch from the rival. Fudenberg and Tirole (2000) set up a two-period Hotelling duopoly model where each firm, in the second period, can distinguish between its own returning customers and those of the rival and thus charge the two consumer groups different prices. Chen and Zhang (2009) study a problem similar to that of Fudenberg and Tirole but assume that the consumers have access to a richer set of instruments when they strategically try to avoid high prices; in particular, a consumer can, besides buying from the rival, instead choose to wait with purchasing. Villas-Boas (2004) develops a model of BBPD with overlapping generations of consumers (living for two periods) and an infinitely lived monopoly firm. He shows that the equilibrium involves price cycles; in particular, the price to the new customers alternates between a relatively low and a relatively high level. Gehrig, Shy, and Stenbacka (2011) investigate under what circumstances an incumbent monopoly firm's opportunity to do BBPD can act as an entry barrier. They formulate a simple two-period model where a potential rival, which does not have access to information about the purchase history of the incumbent's customers, chooses whether or not to enter the market. Esteves (2009a) studies the role of informative advertising in a duopoly market where firms can practice BBPD (although with naive consumers). For surveys of the literature on BBPD, see Armstrong (2006), Fudenberg and Villas-Boas (2006), and Esteves (2009b).

The work that is closest to the present one is Conitzer, Taylor, and Wagman (2012), who also model consumers' opportunity to hide their purchase histories in a BBPD context. Their model is qualitatively quite similar to mine,[3] although their focus is primarily on other questions (namely, comparative statics on the hiding cost). However, also Conitzer et al. identify a

[3]The most important difference is that, in Conitzer et al.'s model, the consumer's decision whether to hide her purchase history is made after she has learned about her valuation. Also, in their model, that decision is binary.

negative externality and show that there can be too much privacy—consumers face a prisoners' dilemma. The present paper adds to Conitzer et al.'s insights by pointing out that there also is a positive externality associated with the atomistic consumer's choice whether to hide her purchase history. Moreover, I show that this positive externality dominates the negative one, which leads to too little privacy, if the discount factor is low enough (for large discount factors, the negative externality dominates). Furthermore, my result that the social optimum involves hiding to some extent, although not fully, cannot be found in Conitzer et al. This result obtains because both a situation where (i) no consumer is hiding and (ii) all consumers are hiding would fail to exploit gains from trade—in case (i) due to the fact that unnecessarily many consumers with valuations above the price do not purchase in period 1, and in case (ii) due to the firm's inability to practice price discrimination in period 2. Also Acquisti and Varian (2005) offer a very useful discussion of consumers' opportunity to hide their purchase histories. In Section 7 of their paper, the authors briefly consider a model where the firm cannot commit to second-period prices and where consumers have the option of hiding. Acquisti and Varian note that this "imposes a cost on the seller, in that it will not be able to implement a price-conditioning solution" (p. 378). However, the modeling choices (e.g., binary valuations and binary hiding choice) and the focus of the analysis mean that these authors fail to identify the tradeoff in the present paper or address the efficiency questions that are studied here.

On a more general level, the present paper adds to the literature on efficiency and the regulation of privacy, although in the specific setting of behavior-based price discrimination in a monopoly market. University of Chicago scholars, notably Stigler (1980) and Posner (1981), have argued that privacy is harmful to efficiency. The reason is that privacy can, by hindering information flows, prevent gains from trade from being realized. For example, privacy can lead to informational asymmetries or discourage productive investments. The present paper contributes to this discussion by showing that, in an environment with BBPD, some degree of privacy can in fact generate more gains from trade (as it enables more consumers to purchase in the first period); moreover, I show that—because of this effect—privacy can indeed be underprovided from a social welfare point of view. A similar point has recently been made in the law literature by MacCarthy (2011) and Fairfield and Engel (2015), and in the economics literature by Choi, Jeon and Kim (2019), Bergemann, Bonatti and Gan (2022), and Acemoglu et al. (2022). For example, Acemoglu et al. model negative externalities of information sharing with an online platform (and thus *positive* externalities of privacy): an individual's data (say, about her preferences) are partially informative about others' data (because different individuals' preferences are correlated); thus, the market for information will provide too much data. However, Acemoglu et al. use (in their own words) a reduced-form approach when modeling the platform's use of data. Therefore, the present paper complements their analysis by studying a specific economic environment where the negative externality of information arises naturally.[4] Other more recent studies of the effects of privacy on allocative efficiency include Hermalin and Katz (2006), Taylor (2004), and Calzolari and Pavan (2006). See also the surveys of the economics of privacy provided by Hui and Png (2006) and Acquisti, Taylor,

[4]Indeed, in their concluding section Acemoglu et al. (2022) write: "Distinguishing the uses of personal data for prices discrimination, advertising, and designing of new products and services could lead to additional novel insights".

and Wagman (2016).

Finally, Johnson (2013) develops a model of a market where firms do informative advertising and consumers have the opportunity to block the advertisements. The feature of his model that consumers can choose the extent to which they protect their privacy is reminiscent of my framework. Moreover, like me, Johnson investigates under what circumstances the consumers over- and underinvest in privacy protection. However, the tradeoff that the consumers in Johnson's setting face is different from the one of the present model. In particular, protecting one's privacy cannot lead to gains from trade in his setting, whereas in mine it can.

2 Model

The following model of behavior-based price discrimination builds on the framework used by Armstrong (2006, Section 2) and Fudenberg and Villas-Boas (2006, Section 2.1), although I extend it by allowing each consumer to take a costly action that hides her purchase history from the seller.

There are two time periods, 1 and 2. In each period, a profit-maximizing and risk-neutral monopoly firm produces and sells a nondurable good. The production technology is characterized by constant returns to scale and the per-unit cost is normalized to zero. When making decisions in period 1, the firm discounts second-period profits with the discount factor $\delta \in (0, 1]$. The consumers form a continuum and differ from each other in terms of r, the gross utility from consuming one unit of the good. In particular, the per-period consumption utility, if p is the price of the good, equals $r - p$ if buying and zero if not buying. The r values are independent and uniformly distributed on the interval $[0, 1]$, and the total mass of consumers equals one. A given consumer's valuation r is the same across the two periods. Moreover, while the consumer knows her own r when making her first-period purchase decision, the valuation of an individual consumer cannot be observed by the firm. However, unless the consumer uses an anonymizing technology, the firm can keep track of individual consumers' purchase decisions. When making first-period decisions, consumers use the same discount factor as the firm, $\delta \in (0, 1]$.

How does the anonymizing technology work? For any given consumer, let $t \in \{L, H\}$ be an indicator variable that is determined as follows. If the consumer does not buy the good in period 1, then $t = L$ for sure. If the consumer indeed buys the good in period 1, then $t = L$ with probability λ and $t = H$ probability $1 - \lambda$, for some individual-specific $\lambda \in [0, 1]$. When interacting with the consumers in period 2, the firm can observe each individual consumer's value of t; it has no other information about whether that consumer actually bought in period 1 or not. That is, we can think of t as a marker that is initially attached to any consumer who purchases the good in period 1, but which is then removed with probability λ. In Section 3, I will assume that the "hiding" or "incognito" probability $\lambda \in [0, 1]$ is given exogenously (and is the same for all consumers). In Section 4, I will extend the analysis by letting λ be chosen by the consumer. There is a cost associated with choosing any $\lambda > 0$, which is denoted by $C(\lambda)$ and is subtracted from the consumption utility. The cost function is twice continuously differentiable and it satisfies $C(0) = C'(0) = 0$, $C'(\lambda) > 0$, and $C''(\lambda) > 0$ for all $\lambda \in (0, 1]$.

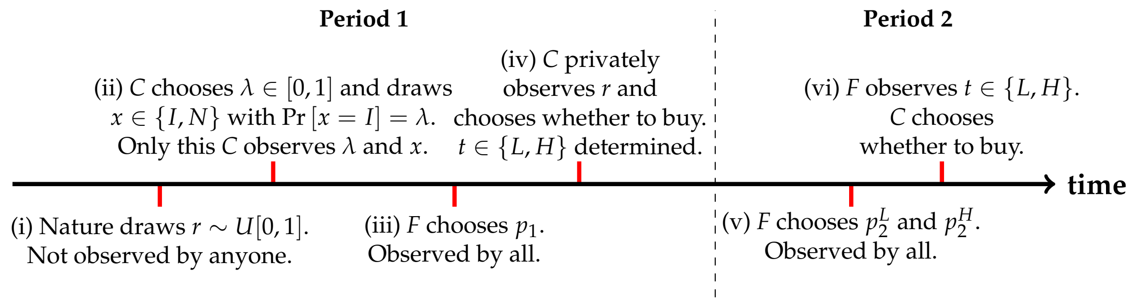


Figure 2: Sequence of events. Abbreviations: C = consumer, F = firm.

The informational assumptions stated above imply that the firm's second-period price can be made contingent on $t \in \{L, H\}$. The consumers understand that the firm may charge different second-period prices depending on if the consumer purchased in the first period or not and on the realization of t. They take this into account when deciding whether to purchase in period 1.

For the full model with endogenous λ, the timing of events is as follows—see also Figure 2. (i) Nature draws each consumer's valuation r. The realization of r is not, at this stage, observed by the firm or by any consumer. (ii) Each consumer chooses her own individual $\lambda \in [0, 1]$, at the cost $C(\lambda)$. Nature then with probability λ assigns an incognito status ($x = I$), and with probability $1 - \lambda$ a non-incognito status ($x = N$), to her. The consumer herself observes the realization of x. However, neither the choice of λ nor the realization of x is observed by the firm or by the other consumers. (iii) The firm chooses its first-period price $p_1 \geq 0$, which is observed by the consumers. (iv) Each consumer privately learns her own valuation r and then decides whether to make a first-period purchase or not. If she does buy and if $x = N$, then her indicator variable t equals H; otherwise, $t = L$. (v) We now move into period 2 and the firm chooses two second-period prices: $p_2^L \geq 0$ and $p_2^H \geq 0$. The price p_2^t must be paid by those consumers with $t \in \{L, H\}$. (vi) Consumers observe p_2^L and p_2^H and then choose whether to buy or not.

The solution concept that I employ is that of perfect Bayesian equilibrium. All players must make optimal choices at all information sets given their beliefs, and the beliefs are formed with the help of Bayes' rule when that is defined. This solution concept also requires that all consumers with access to the same information have the same beliefs.

The timing of the model specified above implies that the incognito probability λ is chosen by the consumer at an ex ante stage, before she has learned about her own valuation. This model feature captures the idea that the consumer adopts a long-term approach for dealing with certain kinds of privacy issues. For example, the consumer might today choose a particular setting on her computer and then, to save on hassle costs, stick with this for a long time and throughout many browsing and purchase situations. Alternatively, the choice of λ could represent the adoption of a simple behavioral rule or heuristic that the consumer uses in a wide range of situations, and which is updated only occasionally. If we nevertheless believe

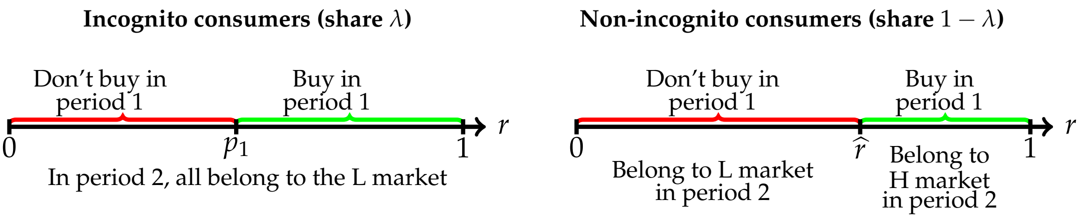


Figure 3: Period 1 behavior of incognito and non-incognito consumers.

that consumers adjust their behavior in all new situations they face, we can think of the timing of the game as an analytical shortcut. That is, the timing is a simple way of capturing the broad tradeoffs we want to study.

The model also assumes that a consumer has a continuous choice how much to hide. This is meant to capture the idea that an online shopper often can choose to put more or less time and/or effort into an attempt to keep her identity secret. For example, this person can decide to take a relatively quick look under "Settings" in her web browser to see if she can find an appropriate box to tick. Alternatively, she can choose to spend a relatively large amount of time and effort on reading a blog post or a book, or even signing up for a course where she can learn the relevant skills—thereby ensuring a higher likelihood of successful hiding. To assume a continuous hiding variable is also natural given the objective of studying the possibility of social over- or undersupply of hiding, as the continuous action makes it easier to study if the private and social benefits differ from each other on the margin. However, a reader who finds the assumption of a continuous hiding choice unsatisfactory can think of an alternative story where all consumers make a binary choice whether to hide. A choice to hide is successful for sure and leads to a cost k. The magnitude of the cost is individual-specific and drawn (independently of r) from a cumulative distribution function G. At the equilibrium of such a model, there would be partial hiding in the aggregate—as those with low enough cost would hide and those with with a high enough cost would not hide. If the function G is chosen so that its inverse, G^{-1}, is identical to the marginal cost function in the continuous-choice model, C', then the two models are equivalent—see Appendix A.

3 Exogenous Fraction of Incognito Consumers

Before solving the full model as described in Section 2, it will be useful to study a setting where the incognito probability λ is exogenous and satisfies $\lambda \in [0, 1)$.[5] This version of the model yields interesting insights in itself and it will also help us to later, in Section 4, solve the full model where the incognito probability is endogenous.

In the second period there are effectively two separate markets: a "low-valuation" market

[5] The case $\lambda = 1$, which is excluded here, means that the firm cannot keep track of any consumer's first-period decision whether to purchase the good. This version of the model is straightforward to solve. In the second period there is effectively only one market, and at the equilibrium we have $p_1 = \widehat{r} = p_2^L = 1/2$.

with consumers who pay p_2^L (as they either did not purchase in the first period or they did but have an incognito status) and a "high-valuation" market with consumers who pay p_2^H. Any equilibrium must be characterized by an endogenous threshold $\widehat{r} \in (0,1)$ with the property that, in the first period, a consumer without an incognito status (so with $x = N$) buys if $r > \widehat{r}$ and does not buy if $r < \widehat{r}$. In particular, any such consumer with valuation r has a (weak) incentive to buy in period 1 if, and only if,

$$r - p_1 + \delta \max\left\{0, r - p_2^H\right\} \geq \delta \max\left\{0, r - p_2^L\right\}. \tag{1}$$

The left-hand side of (1) is the consumer's utility if buying in period 1 (thus having to pay the second-period price p_2^H). The right-hand side is her utility if not buying in period 1 (which means that the second-period price is p_2^L). When solving for the equilibria of the model, we can exploit the fact that, for a consumer with $r = \widehat{r}$, inequality (1) must hold with equality.

The incognito consumers (i.e., those with $x = I$) always pay the second-period price p_2^L, regardless of whether they bought in the first period or not. They therefore optimally decide to purchase the good in the first period if, and only if, $r \geq p_1$. The first-period behavior of the incognito and the non-incognito consumers is summarized in Figure 3.

3.1 Equilibrium Behavior in Period 2

Consider, in turn, the firm's second-period profit-maximization problem in the high- and the low-valuation markets. Let q_2^H denote the demand that the firm faces in the high-valuation market. All consumers in this market lack an incognito status and they have valuations that are uniformly distributed on $[\widehat{r}, 1]$; cf. Figure 3. We therefore have

$$q_2^H = \begin{cases} (1-\lambda)(1-\widehat{r}) & \text{if } p_2^H \in [0, \widehat{r}] \\ (1-\lambda)\left(1-p_2^H\right) & \text{if } p_2^H \in [\widehat{r}, 1]. \end{cases} \tag{2}$$

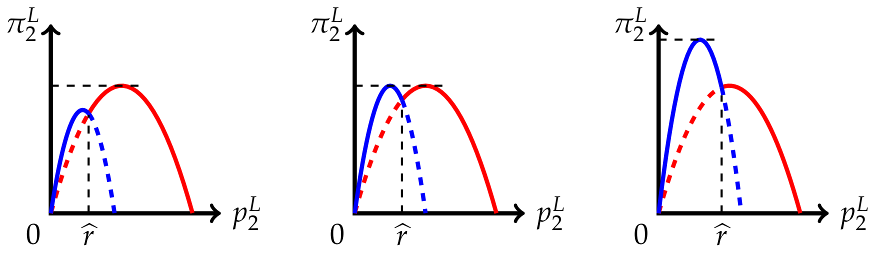
It is straightforward to see that the profits in the high-valuation market, $\pi_2^H = p_2^H q_2^H$, are maximized at $p_2^H = \max\left\{\frac{1}{2}, \widehat{r}\right\}$.

Next, consider the demand that the firm faces in the low-valuation market, denoted by q_2^L. This market consists of all consumers with an incognito status, uniformly distributed on $[0,1]$, and of the consumers without an incognito status who did not buy in period 1, uniformly distributed on $[0, \widehat{r}]$; cf. again Figure 3. We thus get that

$$q_2^L = \begin{cases} \widehat{r} - p_2^L + \lambda(1-\widehat{r}) & \text{if } p_2^L \in [0, \widehat{r}] \\ \lambda\left(1-p_2^L\right) & \text{if } p_2^L \in [\widehat{r}, 1]. \end{cases} \tag{3}$$

That is, for relatively low values of p_2^L, there are both incognito and non-incognito consumers who find it worthwhile to purchase the good, while for higher values of p_2^L only incognito consumers do.

The firm's profits in the low-valuation market equal $\pi_2^L = q_2^L p_2^L$. This profit function is



(a) $\widehat{r} < \frac{\sqrt{\lambda}}{1+\sqrt{\lambda}}$ **(b)** $\widehat{r} = \frac{\sqrt{\lambda}}{1+\sqrt{\lambda}}$ **(c)** $\widehat{r} > \frac{\sqrt{\lambda}}{1+\sqrt{\lambda}}$

Figure 4: Profit maximization in the second-period low-valuation market. The (blue-colored) curve to the left in each diagram shows the firm's profit for $p_2^L \in [0, \widehat{r}]$, whereas the (red-colored) curve to the right shows the profit for $p_2^L \in [\widehat{r}, 1]$. Cf. eqs. (3) and (4).

continuous in p_2^L (although with a kink at $p_2^L = \widehat{r}$). It is not, however, in general quasiconcave. Indeed, from the three panels of Figure 4 it is clear that the profit function may have two local optima: one where the price p_2^L is relatively low, which means that the firm sells to both incognito and non-incognito consumers; and one local optimum where the price is relatively high, which means that only (some) incognito consumer purchase the good. Which one of the local optima that is the global one (or if they both are global optima) depends on the relative magnitude of λ and $\widehat{r}$).

Lemma 1. *The price p_2^L that maximizes the profits $\pi_2^L = q_2^L p_2^L$ is given by*

$$p_2^L = \begin{cases} \frac{\lambda+(1-\lambda)\widehat{r}}{2} & \text{if } \widehat{r} \in \left[\frac{\sqrt{\lambda}}{1+\sqrt{\lambda}}, 1\right] \\ \frac{1}{2} & \text{if } \widehat{r} \in \left[0, \frac{\sqrt{\lambda}}{1+\sqrt{\lambda}}\right] \end{cases}. \tag{4}$$

The proof of Lemma 1, as well as the results stated in the remainder of the paper, can be found in Appendix B.

It is useful to note that, as one would expect, the results above imply that purchasing in period 1 and lacking an incognito status leads to a weakly higher second-period price, $p_2^H \geq p_2^L$ (with the inequality being strict unless $\widehat{r}$ is quite low—lower than one-half and in the range where the second line of (4) applies).

3.2 Equilibrium Behavior in Period 1

Let us look for an equilibrium where $\widehat{r} \geq \frac{1}{2}$ (in the proof of Proposition 1, it is shown that other equilibria do not exist). In such an equilibrium, the second-period prices equal $p_2^H = \widehat{r}$ and

$$p_2^L = \frac{\lambda + (1-\lambda)\widehat{r}}{2} \tag{5}$$

(see subsection 3.1). We also know that the threshold $\widehat{r}$ must satisfy inequality (1) with equality, when (1) is evaluated at those two second-period prices:

$$\widehat{r} - p_1 + \delta\left(\widehat{r} - \widehat{r}\right) = \delta\left[\widehat{r} - \frac{\lambda + (1-\lambda)\,\widehat{r}}{2}\right] \Leftrightarrow p_1 = \widehat{r} - \frac{\delta\left[(1+\lambda)\,\widehat{r} - \lambda\right]}{2}. \tag{6}$$

Equation (6) gives us a relationship between the two endogenous variables $\widehat{r}$ and p_1. Anticipating this relationship and the optimal second-period prices, the firm chooses its first-period price p_1 so as to maximize the following overall profits:

$$\Pi = \pi_1 + \delta(\pi_2^L + \pi_2^H) = q_1 p_1 + \delta\frac{\left[\lambda + (1-\lambda)\,\widehat{r}\right]^2}{4} + \delta\,(1-\lambda)\,(1-\widehat{r})\,\widehat{r}, \tag{7}$$

where

$$q_1 = (1-\lambda)\,(1-\widehat{r}) + \lambda\,(1-p_1) \tag{8}$$

is the first-period demand. Rather than maximizing (7) with respect to p_1 (subject to (6) and (8)), we can equivalently maximize it with respect to $\widehat{r}$ (subject to (6) and (8)). Let $\widehat{\Pi}$ denote the reduced-form profit function that we obtain by eliminating q_1 and p_1 from (7) with the help of (6) and (8). The function $\widehat{\Pi}$ is strictly concave in $\widehat{r}$. Therefore, the optimal $\widehat{r}$ satisfies the first-order condition $\partial\widehat{\Pi}/\partial\widehat{r} = 0$ or, equivalently,

$$\widehat{r} = \frac{2 - \delta\left[(1+\lambda)\left(1+\lambda-\delta\lambda^2\right) - (1-\lambda)\,(2+\lambda)\right]}{4 - \delta\left[(1+\lambda)^2\,(2-\delta\lambda) - (1-\lambda)\,(3+\lambda)\right]}. \tag{9}$$

For the above value of $\widehat{r}$ to indeed be part of an equilibrium, we must have $\widehat{r} \in \left[\frac{1}{2}, 1\right)$; this condition can be shown to always hold. Thus, there is an equilibrium where $\widehat{r}$ is given by (9). The equilibrium values of the three prices are in turn obtained from (5), (6), and $p_2^H = \widehat{r}$.

3.3 Summing Up

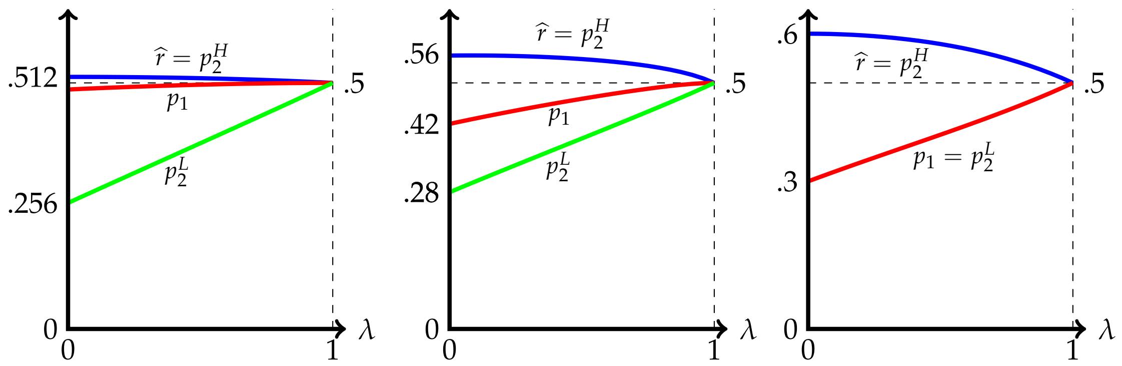
Proposition 1. *There is a unique equilibrium of the model with exogenous λ. In this equilibrium, the relationship $\widehat{r} - p_1 = \delta\left(p_2^H - p_2^L\right)$ always holds. Moreover, p_2^L, p_1, and $\widehat{r}$ are given by (5), (6), (9), respectively, and $p_2^H = \widehat{r}$. In particular, $p_2^L \le p_1 \le \frac{1}{2} \le p_2^H = \widehat{r}$.*

The results reported in Proposition 1 are illustrated in Figure 5. It is useful to also write up the expressions for the equilibrium prices and the cutoff value that one obtains when neither the firm nor the consumers discount at all, i.e., when $\delta = 1$. For this parameter value, the expressions become quite simple.

Example 1. *Suppose $\delta = 1$. Then the equilibrium prices and the cutoff value are given by*

$$p_2^L = p_1 = \frac{(3-\lambda)(1+\lambda)}{2(5-\lambda^2)} \quad \textit{and} \quad p_2^H = \widehat{r} = \frac{3-\lambda^2}{5-\lambda^2}. \tag{10}$$

One can check that the first expression (i.e., for p_2^L and p_1) is strictly increasing, and the second



(a) Example: $\delta = 0.1$. **(b)** Example: $\delta = 0.5$. **(c)** Example: $\delta = 1$.

Figure 5: Illustration of the results in Proposition 1.

one (for p_2^H and $\widehat{r}$) is strictly decreasing (see also panel (c) of Figure 5).[6] I will return to this example in Section 4.

3.4 Welfare with an Exogenous Fraction of Incognito Consumers

What is the effect of an exogenous change in λ on social welfare? To begin with, let our measure of social welfare be total surplus. Total surplus in the present model can be written as

$$W(\lambda) \stackrel{\text{def}}{=} \int_{\widehat{r}}^{1} r dr + \lambda \int_{p_1}^{\widehat{r}} r dr + \delta \int_{p_2^L}^{1} r dr. \tag{11}$$

The two first terms in (11) represent the surplus generated in period 1. In that period, all consumers with valuation $r \geq \widehat{r}$, regardless of their incognito status, purchase the good, which yields the surplus captured by the first term. Moreover, incognito consumers with valuations $r \in [p_1, \widehat{r}]$ also buy the good in period 1, yielding the second term. In period 2, all consumers with valuations $r \in [p_2^L, 1]$ buy the good, which yields the surplus captured by the last term.

Let $\widehat{\lambda}_W$ be the fraction of incognito consumers that maximizes the total surplus, as stated in (11).[7] To be able to say something about the value of $\widehat{\lambda}_W$, first note that $W(0) > W(1)$ (this is shown in the proof of Proposition 2). That is, total surplus is strictly larger with no incognito consumers than with only incognito consumers. Intuitively, in the latter case price discrimination is not feasible and thus all consumers, in both periods, must pay the price one-half. In contrast, without any incognito consumers the firm can charge a separate second-period price (namely, $p_2^L < \frac{1}{2}$) for consumers with a relatively low valuation, which increases the amount of trade.[8]

[6] The fact that $p_2^L = p_1$ for $\delta = 1$, which may look surprising, is due to the equilibrium relationship $\widehat{r} - p_1 = \delta\left(p_2^H - p_2^L\right)$ stated in Proposition 1, which in turn comes from the requirement that inequality (1) holds with equality. The latter requires that, for $\delta = 1$, the two price differences $p_2^H - p_1$ and $p_2^H - p_2^L$ equal each other.

[7] Formally, $\widehat{\lambda}_W \in \arg\max_{\lambda \in [0,1]} W(\lambda)$.

[8] There is also a first-period effect that works in the opposite direction, as $\widehat{r} > \frac{1}{2}$, although that is apparently not strong enough. Contributing to the result that $W(0) > W(1)$ is also the Coase-conjecture effect that is mentioned

Next, differentiate the total surplus function with respect to λ:

$$\frac{\partial W}{\partial \lambda} = -\widehat{r}\frac{\partial \widehat{r}}{\partial \lambda} + \lambda \left[\widehat{r}\frac{\partial \widehat{r}}{\partial \lambda} - p_1 \frac{\partial p_1}{\partial \lambda}\right] + \int_{p_1}^{\widehat{r}} r dr - \delta p_2^L \frac{\partial p_2^L}{\partial \lambda}. \tag{12}$$

The term with an integral sign in (12) represents the change in first-period surplus for the extra marginal incognito consumers who, thanks to their newly acquired incognito status, can consume the good in period 1. This term is clearly positive for all $\lambda < 1$ and in particular for $\lambda = 0$ (for λ close to one, however, the term must be close to zero, as then p_1 and $\widehat{r}$ are close to each other). The preceding term (the one with square brackets) represents the change in first-period surplus for the infra marginal incognito consumers who already had an incognito status but now face adjustments in $\widehat{r}$ and p_1. This is an indirect effect and it disappears for $\lambda = 0$. Also the two remaining terms capture indirect effects that are due to adjustments in one of the prices and in the cutoff value $\widehat{r}$. These effects are harder to understand intuitively, but for $\lambda = 0$ one can show that they cannot overturn the positive effect coming from the term with the integral sign (see the proof of Proposition 2). All in all, this means that, at $\lambda = 0$, we have $\partial W / \partial \lambda > 0$. This result in combination with our observation from above that $W(0) > W(1)$ imply that the fraction of incognito consumers that maximizes total surplus must be strictly between zero and one.

Proposition 2. *Consider the model with an exogenous λ. The fraction of incognito surfers that maximizes total surplus lies strictly between zero and unity, $\widehat{\lambda}_W \in (0, 1)$.*

Intuitively, to let all consumers have an incognito status is suboptimal as this hinders price discrimination, and price discrimination generates gains from trade. On the other hand, an increase in λ enables more consumers with valuations between $\widehat{r}$ and p_1 to purchase the good in the first period, which also generates gains from trade; moreover, the difference between $\widehat{r}$ and p_1 tends to be large for low values of λ, which makes this effect particularly important for $\lambda = 0$. Thus, increasing λ at least somewhat, starting from zero, always pays off.

4 Endogenous Fraction of Incognito Consumers

Now turn to the full model described in Section 2, where the incognito status is endogenous. At stage (ii) of the full model, each consumer chooses a level of λ that maximizes her expected utility (not yet knowing her valuation r) and expecting all other consumers to choose, say, $\lambda = \widetilde{\lambda}$. Because each consumer is infinitesimally small, her choice of λ has no impact on the prices or the cutoff value. Hence, only the direct effect on the consumer's utility, and of course the effect on the cost $C(\lambda)$, matter for the choice of λ.

The consumer's expected utility can be written as $EU(\lambda, \widetilde{\lambda}) = S(\lambda, \widetilde{\lambda}) - C(\lambda)$, where the

in footnote 1.

gross consumer surplus $S(\lambda, \widetilde{\lambda})$ is defined as

$$S(\lambda, \widetilde{\lambda}) \stackrel{\text{def}}{=} \int_{\widehat{r}}^{1} (r - p_1)\, dr + \lambda \int_{p_1}^{\widehat{r}} (r - p_1)\, dr + \delta \left[\int_{p_2^L}^{\widehat{r}} \left(r - p_2^L \right) dr + \int_{\widehat{r}}^{1} \left[r - (1-\lambda)\, p_2^H - \lambda p_2^L \right] dr \right] \quad (13)$$

and where all prices and the cutoff value are evaluated at $\lambda = \widetilde{\lambda}$. Differentiating $EU(\lambda, \widetilde{\lambda})$ with respect to λ yields

$$\frac{\partial EU}{\partial \lambda} = \int_{p_1}^{\widehat{r}} (r - p_1)\, dr + \delta \int_{\widehat{r}}^{1} \left(p_2^H - p_2^L \right) dr - C'(\lambda). \quad (14)$$

Thus, a consumer's marginal benefit from increasing λ has two components. In the first period the consumer increases her likelihood of earning a surplus whenever $r \in (p_1, \widehat{r})$; this effect is captured by the first term in (14). In the second period the consumer increases her likelihood of being eligible to pay p_2^L rather than p_2^H, when $r \in (\widehat{r}, 1]$; this is the second term in (14).

Given that the consumers are ex ante identical, it is natural to focus attention on symmetric equilibria, where all consumers choose $\lambda = \lambda^*$. In any such equilibrium $\partial EU / \partial \lambda = 0$ must hold when (14) is evaluated at $\lambda = \widetilde{\lambda} = \lambda^*$.

Proposition 3. *There exists a symmetric equilibrium of the game with endogenous λ. The fraction of incognito surfers in this equilibrium, λ^*, satisfies $\lambda^* \in (0,1)$ and is implicitly defined by*

$$\int_{p_1^*}^{\widehat{r}^*} (1 - r) dr = C'(\lambda^*), \quad (15)$$

where p_1^* and $\widehat{r}^*$ are given by p_1 and $\widehat{r}$ as stated in Proposition 1 but evaluated at $\lambda = \lambda^*$.

What are the welfare properties of the equilibrium characterized in Proposition 3?[9] Is there too much or too little incognito surfing? Consider, to begin with, a total-surplus standard; that is, let social welfare be defined as $W(\lambda) - C(\lambda)$, where $W(\lambda)$ is given by (11). The social marginal benefit, given the total-surplus standard, is stated in (12). This marginal benefit consists of a direct social welfare effect (namely, $\int_{p_1}^{\widehat{r}} r dr$) and an indirect social welfare effect (the remaining terms in (12)).

Let the *direct external effect*, $\Delta_D(\lambda)$, be defined as the extent to which the atomistic consumer's direct private marginal benefit from a larger λ is greater than the direct social welfare effect; that is,

$$\Delta_D(\lambda) \stackrel{\text{def}}{=} \int_{p_1}^{\widehat{r}} (1 - r) dr - \int_{p_1}^{\widehat{r}} r dr. \quad (16)$$

Similarly, let the *indirect external effect*, $\Delta_I(\lambda)$, be defined as the extent to which the indirect

[9] One can relatively easily verify that for $\delta = 1$, and within the family of symmetric equilibria, the equilibrium is guaranteed to be unique (because for $\delta = 1$ the left-hand-side of (15) is downward-sloping in the incognito probability). I have no reason to believe that the equilibrium is not unique also for other values of δ. However, the algebra for the general case becomes quite intractable and I must therefore refrain from making such general uniqueness claims.

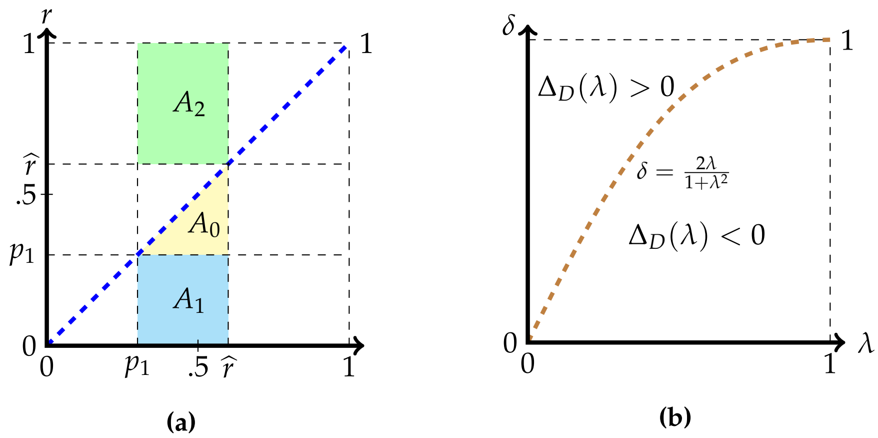


Figure 6: Social versus individual incentives to choose incognito: the direct effect. **(a)** Society cares about $A_0 + A_1$, but the atomistic consumer cares about $A_0 + A_2$. **(b)** The sign of the direct external effect, $\Delta_D(\lambda) = A_2 - A_1$.

private marginal benefit from a larger λ is greater than the indirect social welfare effect:

$$\Delta_I(\lambda) \stackrel{\text{def}}{=} \widehat{r}\frac{\partial \widehat{r}}{\partial \lambda} - \lambda\left[\widehat{r}\frac{\partial \widehat{r}}{\partial \lambda} - p_1\frac{\partial p_1}{\partial \lambda}\right] + \delta p_2^L \frac{\partial p_2^L}{\partial \lambda} \tag{17}$$

(note that the indirect private marginal benefit is zero, as the consumer is atomistic). Finally, the *total external effect* is the sum of the direct and the indirect effects, $\Delta(\lambda) \stackrel{\text{def}}{=} \Delta_D(\lambda) + \Delta_I(\lambda)$. Given these definitions, I say that there is too much incognito surfing if $\Delta(\lambda^*) > 0$ and there is too little if $\Delta(\lambda^*) < 0$.

In order to understand under what circumstances we have too much or too little incognito surfing, consider first the direct external effect, $\Delta_D(\lambda)$. As already explained, this effect measures the extent to which the private (direct) marginal benefit from a larger λ is greater than the social direct marginal benefit from a larger λ. The latter marginal benefit equals the sum of the valuations of the additional consumers who, thanks to being able to surf incognito, find it worthwhile to purchase the good in the first period now when doing this has no impact on the second-period price; it is shown in Figure 6, panel (a), as the area $A_0 + A_1$. In contrast, the private marginal benefit corresponds to the area $A_0 + A_2$ in the same figure. The area A_0 is the sum of the *net* valuations of the additional consumers who purchase the good thanks to the incognito status (the consumers do not benefit from A_1 as this amount is paid to the firm). The area A_2 is the discounted value of the amount of money the consumer can save in period 2 thanks the incognito status, which entitles her to pay p_2^L instead of p_2^H for the good (these savings do not affect total surplus as they are just a transfer from the firm to the consumer). This discounted amount of money can be expressed as the area A_2 in the figure thanks to the equilibrium relationship $\widehat{r} - p_1 = \delta(p_2^H - p_2^L)$: at the equilibrium, $\widehat{r}$ is such that a consumer with the valuation $r = \widehat{r}$ is indifferent between between purchasing in the first period and not doing that—see (1).

We thus have $\Delta_D(\lambda) = A_2 - A_1$. Moreover, it is clear from Figure 6, panel (a), that $A_2 > A_1$ if and only if $1 - \widehat{r} > p_1$. The latter inequality can be solved for δ, which yields the following result:

$$\Delta_D(\lambda) > 0 \Leftrightarrow \delta > \frac{2\lambda}{1+\lambda^2}. \tag{18}$$

That is, the direct external effect is positive if and only if the consumers care sufficiently much about the second period. (See Figure 6, panel (b), for an illustration.) In other words, if the consumers are sufficiently forward-looking, then the direct external effect, all else equal, works in the direction of too much incognito surfing. Intuitively, in the first period the consumers benefit too little from an incognito status (the private benefit is A_0 but the social benefit is $A_0 + A_1$), whereas in the second period the consumers benefit too much (the private benefit is A_2 but the social benefit is zero). Thus, if the second period matters sufficiently much relative to the first period, the consumers might have a too strong incentive to invest in λ.

Indeed, the graph in Figure 6, panel (b), shows that if the discount factor is sufficiently close to one, then the direct external effect is positive for all values of λ. Similarly, if the discount factor is sufficiently close to zero, then the direct external effect is negative for all values of λ. Of course, also the indirect external effect matters for whether there is too much or too little incognito surfing. However, it seems plausible that the indirect external effect might be of second-order importance (after all, this effect influences welfare only through the equilibrium values of the prices and the cutoff $\widehat{r}$, not directly through λ). For the case with a large discount factor, this reasoning turns out to be correct. That is, as stated in Proposition 4 below, if the discount factor equals unity, there is always too much incognito surfing.

Proposition 4. *Suppose $\delta = 1$. Then, relative to a total-surplus maximizing benchmark, the equilibrium yields too much incognito surfing: $\Delta(\lambda^*) > 0$.*

For the case where the discount factor is below unity, it is more difficult to obtain analytical results. However, let us assume the following functional form for the cost function:

$$C(\lambda) = \frac{c}{2}\lambda^2, \qquad c > 0. \tag{19}$$

We can then perform the welfare comparison using numerical methods. Also, let λ_W^* denote the value of λ that maximizes $W(\lambda) - C(\lambda)$. The numerical analysis reveals that—as hypothesized above—we can indeed obtain the result that there is too little incognito surfing ($\lambda^* < \lambda_W^*$), provided that we pick a value of the discount factor that is low enough. Results of the simulation exercise are shown in Figure 7. Panel (a) considers a case where the discount factor is relatively large, $\delta = 0.9$. We see that then, in keeping with Proposition 4, the equilibrium yields more incognito surfing than is socially desirable. Panel (b) considers examples with $\delta = 0.15$ and $\delta = 0.1$. For $\delta = 0.15$, we still obtain the result that there is too much incognito surfing, However, for $\delta = 0.1$, the graphs show that the equilibrium yields less incognito surfing than is socially desirable (for a range of different values of the cost parameter c).

We summarize the main insight from the simulation exercise as follows:[10]

[10]Details about the simulation exercise can be found in the Supplementary Material, Lagerlöf (2022). This document and the Matlab code are available at www.johanlagerlof.com.

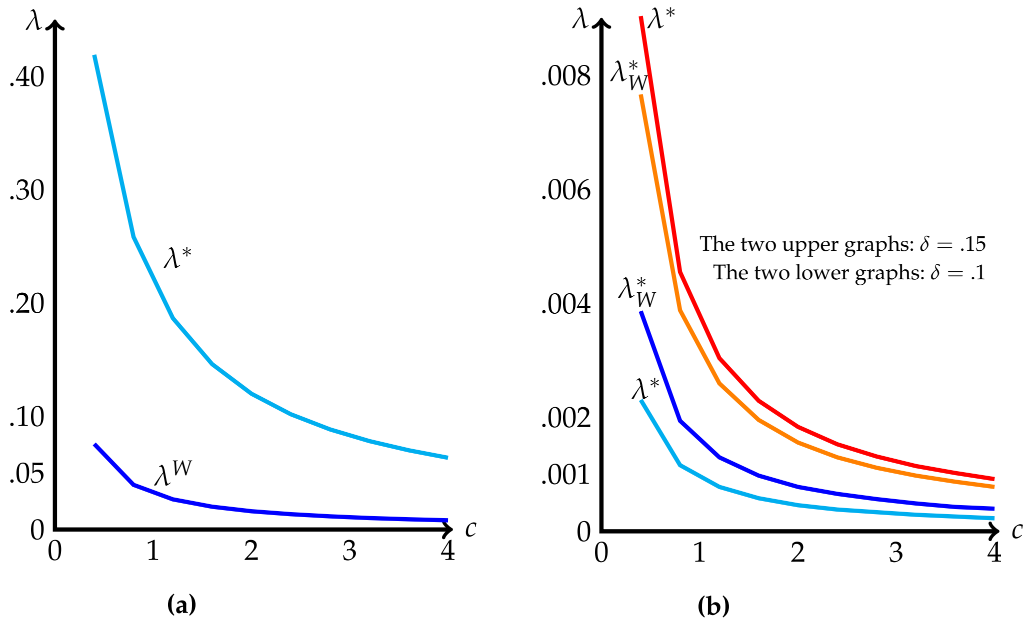


Figure 7: Socially optimal vs. equilibrium values of λ. **(a)** The case $\delta = 0.9$. There is too much incognito surfing at the equilibrium. **(b)** The cases $\delta \in \{0.1, 0.15\}$. Equilibrium undersupply of λ is possible for $\delta = 0.1$.

Result 1. *Let the cost function $C(\lambda)$ be given by (19). Then, by letting δ be sufficiently small, we can construct numerical examples where, relative to a total-surplus maximizing benchmark, the market outcome yields too little incognito surfing: $\lambda^* < \lambda_W^*$.*

Summing up, the intuition for the results reported in Proposition 4 and Result 1 is that the consumers benefit from an incognito status both in period 1 and in period 2. However, relative to the social benefit, the consumers' benefit is too small in period 1 and too large in period 2. Therefore, if they discount heavily (low δ), they tend to have a socially too weak incentive to invest in an incognito status. In contrast, if they assign a sufficiently large weight to the second-period payoffs, the consumers tend to have a too strong incentive.

Finally in this section, consider the question whether there is too much or too little incognito surfing given a consumer-surplus standard. Clearly, with this standard, of the three welfare effects discussed in the introduction, the two first ones do not matter; this is because now the consumer and society cares to the same extent about the *direct* effect of changing λ. Whether there is over- or underprovision of anonymity therefore depends on the sign of the *indirect* effect, which the consumer ignores. The intuition discussed in the introduction suggests that the indirect effect is negative: a larger λ should make it harder for the firm to price discriminate, which can be expected to hurt trade and consumer welfare. This indeed turns out to be the case. The indirect effect is defined as the derivative of $S(\lambda, \widetilde{\lambda})$, as stated in (13), with respect to $\widetilde{\lambda}$. We can write (see the proof of Proposition 5):

$$\frac{\partial S(\lambda,\widetilde{\lambda})}{\partial\widetilde{\lambda}} = -\left[1-\widehat{r}+\lambda\left(\widehat{r}-p_1\right)\right]\frac{\partial p_1}{\partial\widetilde{\lambda}} - \delta\left(\widehat{r}-p_2^L\right)\frac{\partial p_2^L}{\partial\widetilde{\lambda}} - \delta\left[(1-\lambda)\frac{\partial p_2^H}{\partial\widetilde{\lambda}}+\lambda\frac{\partial p_2^L}{\partial\widetilde{\lambda}}\right]\left(\widehat{r}-p_1\right). \tag{20}$$

In the above expression, the first two terms are negative as long as p_1 and p_2^L are increasing in $\widetilde{\lambda}$ (which they at least typically are; cf. Figure 5). The last term is negative if the expression in square brackets is positive. For $\delta = 1$ one can show that it is and that $\partial S(\lambda,\widetilde{\lambda})/\partial\widetilde{\lambda}$ overall, evaluated at an equilibrium, is negative:

Proposition 5. *Suppose $\delta = 1$. Then, relative to a consumer-surplus maximizing benchmark, the equilibrium yields too much incognito surfing: $\frac{\partial S(\lambda,\widetilde{\lambda})}{\partial\widetilde{\lambda}}\,|_{\widetilde{\lambda}=\lambda} < 0$ for all $\lambda \in (0,1]$.*

For values of the discount factor strictly below one, the algebra again becomes cumbersome. However, one can show that evaluated at $\delta = 0$, $\partial S(\lambda,\widetilde{\lambda})/\partial\widetilde{\lambda}$ equals zero. Moreover, numerical simulations suggest that $\partial S(\lambda,\widetilde{\lambda})/\partial\widetilde{\lambda} < 0$ for all $\delta \in (0,1)$ and all $\lambda \in (0,1)$. We can thus—tentatively—conclude that if one employs a consumer-surplus standard, then incognito surfing cannot be undersupplied. Again, this is what we expected intuitively, as the reason identified in the introduction for why a consumer would hide too little (i.e., effect 1) cannot matter with a consumer-surplus standard.

5 Consumers Not Knowing Their Incognito Status

The model studied in the previous sections assumes that a consumer makes her period 1 purchase decision *after* having learned if she has an incognito status (i.e., *after* having observed $x \in \{I, N\}$). The key implication of this assumption is that it enables the consumer to make a purchase that she knows for sure is secret, which makes her willing to buy at any price p_1 below her valuation r. This, in turn, can help increase trade—and thus social welfare; moreover, society's gain from the trade (which equals r) will be larger than that of the individual (which equals $r - p_1$). Therefore, the model feature in question appears to be important for the possibility of anonymity being underprovided from a social welfare point of view. To investigate more carefully whether this is so, I will here study a setup where the consumer observes $x \in \{I, N\}$ only after she has made her period 1 purchase decision (but before she takes any action in period 2). All other aspects of the model described in Section 2 are unchanged.

In this modified version of the model, any consumer with valuation r has a (weak) incentive to buy in period 1 if, and only if,

$$r - p_1 + \delta\left[\lambda \max\left\{0, r-p_2^L\right\} + (1-\lambda)\max\left\{0, r-p_2^H\right\}\right] \geq \delta \max\left\{0, r-p_2^L\right\}. \tag{21}$$

The difference compared to the corresponding inequality in the original model, (1), is the expression is square brackets on the left-hand side of (21). This is here stated in expected terms, as the consumer does not yet know which second-period price she will have to pay if buying in the first period. The second-period demand functions are the same as in our previous analysis, as the consumer at this point in time has learned which price she must pay; thus, also the optimal second-period prices are the same as before and given by (4) and by $p_2^H = \max\left\{\frac{1}{2}, \widehat{r}\right\}$. This means that, in an equilibrium with $\widehat{r} \geq \frac{1}{2}$,[11] we have $p_2^H = \widehat{r}$ and $p_2^L = [\lambda + (1-\lambda)\widehat{r}]/2$. By plugging these prices into (21), replacing the inequality with an equality, and rearranging, we obtain the following relationship between $\widehat{r}$ and p_1:

$$p_1 = \widehat{r} - \frac{\delta(1-\lambda)\left[(1+\lambda)\widehat{r} - \lambda\right]}{2} \tag{22}$$

As in our previous analysis, write the firm's expected overall profits from the perspective of period 1 as $\Pi = \pi_1 + \delta(\pi_2^L + \pi_2^H)$. The second term, $\delta(\pi_2^L + \pi_2^H)$, is the same as in (7), whereas the first term is here replaced by $\pi_1 = (1-\widehat{r})p_1$. Plugging (22) into our new expression for Π and then maximizing this with respect to $\widehat{r}$ yields

$$\widehat{r} = \frac{2 + \delta(1-\lambda)^2}{4 + \delta(1-\lambda)^2}, \tag{23}$$

which indeed satisfies the requirement that $\widehat{r} \in \left[\frac{1}{2}, 1\right)$.

Proposition 6. *Consider the model where the consumers do not learn their incognito status in period 1 and where λ is exogenous. There is a unique equilibrium of this model. In this equilibrium, the relationship $\widehat{r} - p_1 = \delta(1-\lambda)\left(p_2^H - p_2^L\right)$ always holds. Moreover, p_2^L, p_1, and $\widehat{r}$ are given by (5), (22), (23), respectively, and $p_2^H = \widehat{r}$. In particular, $p_2^L \leq p_1 \leq \frac{1}{2} \leq p_2^H = \widehat{r}$.*

Total surplus in this alternative model can be written as

$$W(\lambda) \stackrel{\text{def}}{=} \int_{\widehat{r}}^{1} r dr + \delta \int_{p_2^L}^{1} r dr. \tag{24}$$

Relative to the total surplus expression in (11), here one of the terms representing the first-period surplus does not appear. That term in (11) represented the surplus for those consumers who *knew* their first-period purchase was hidden and who had drawn a valuation between $\widehat{r}$ and p_1. No such consumers exist here, as the incognito status is observed only at a later stage.

Differentiating $W(\lambda)$ with respect to λ yields

$$\frac{\partial W}{\partial \lambda} = -\widehat{r}\frac{\partial \widehat{r}}{\partial \lambda} - \delta p_2^L \frac{\partial p_2^L}{\partial \lambda} = \frac{4\delta(1-\lambda)\left[2 + \delta(1-\lambda)^2\right]}{\left[4 + \delta(1-\lambda)^2\right]^3} - \frac{\delta\left[2(1+\lambda) + \delta(1-\lambda)^2\right]\left[4 - \delta(1-\lambda)^2\right]}{2\left[4 + \delta(1-\lambda)^2\right]^3}. \tag{25}$$

One can verify that the expression in (25) is strictly positive at $\lambda = 0$ and strictly negative at $\lambda = 1$. This gives us the following result.

Proposition 7. *Consider the model where the consumers do not learn their incognito status in period 1 and where λ is exogenous. The fraction of incognito surfers that maximizes total surplus lies strictly between zero and unity, $\widehat{\lambda}_W \in (0, 1)$.*

Now consider the version of the alternative model where λ is endogenous. The consumer's expected utility can in this model be written as $EU(\lambda, \widetilde{\lambda}) = S(\lambda, \widetilde{\lambda}) - C(\lambda)$, where the gross

[11] In the proof of Proposition 6, stated below, it is shown that an equilibrium with $\widehat{r} < \frac{1}{2}$ does not exist.

consumer surplus $S(\lambda, \widetilde{\lambda})$ is defined as

$$S(\lambda, \widetilde{\lambda}) \stackrel{\text{def}}{=} \int_{\widehat{r}}^{1} (r - p_1)\, dr + \delta \left[\int_{p_2^L}^{\widehat{r}} \left(r - p_2^L\right) dr + \int_{\widehat{r}}^{1} \left[r - (1-\lambda)\, p_2^H - \lambda p_2^L \right] dr \right], \tag{26}$$

where all prices and the cutoff value are evaluated at $\lambda = \widetilde{\lambda}$. Differentiating $S(\lambda, \widetilde{\lambda})$ with respect to λ yields

$$\frac{\partial S(\lambda, \widetilde{\lambda})}{\partial \lambda} = \delta \int_{\widehat{r}}^{1} \left(p_2^H - p_2^L\right) dr = \frac{\delta(1-\widetilde{\lambda}) \left[2 + \delta(1-\widetilde{\lambda})\right]}{\left[4 + \delta(1-\widetilde{\lambda})^2\right]^2}. \tag{27}$$

Given that the consumers are ex ante identical, it is natural to focus attention on symmetric equilibria, where all consumers choose $\lambda = \lambda^*$. In any such equilibrium $\partial EU/\partial \lambda = 0$ must hold, when (27) is evaluated at $\lambda = \widetilde{\lambda} = \lambda^*$.

Proposition 8. *Consider the model where the consumers do not learn their incognito status in period 1 and where λ is endogenous. There exists a symmetric equilibrium of that model (which within the family of symmetric equilibria is unique). The fraction of incognito surfers in this equilibrium, λ^*, satisfies $\lambda^* \in (0, 1)$ and is implicitly defined by*

$$\frac{(\widehat{r} - p_1^*)\,(1 - \widehat{r})}{1 - \lambda^*} = C'(\lambda^*), \tag{28}$$

where p_1^* and $\widehat{r}^*$ are given by p_1 and $\widehat{r}$ as stated in Proposition 6 but evaluated at $\lambda = \lambda^*$.

Letting social welfare be defined as $W(\lambda) - C(\lambda)$, where $W(\lambda)$ is given by (24), we note that there is too much incognito surfing from a social-welfare perspective if, and only if, $\frac{\partial W}{\partial \lambda} < \frac{\partial S}{\partial \lambda}$ at $\lambda = \lambda^*$. Similarly, defining $S^A(\lambda) \stackrel{\text{def}}{=} S(\lambda, \lambda)$, there is too much incognito surfing according to a consumer-surplus standard if, and only if, $\frac{\partial S^A}{\partial \lambda} < \frac{\partial S}{\partial \lambda}$ at $\lambda = \lambda^*$.

Proposition 9. *Consider the model where the consumers do not learn their incognito status in period 1 and where λ is endogenous.*

- ***(a)*** *Relative to a total-surplus maximizing benchmark, the equilibrium yields too much incognito surfing.*
- ***(b)*** *Relative to a consumer-surplus maximizing benchmark, the equilibrium yields too much incognito surfing.*

6 Policy Discussion

A common, at least implicit, view among consumer groups is that consumers protect their identity to a too small extent when they shop online. This view can be inferred from, for example, the discussion of the "Do Not Track" system, which originally was proposed in 2007 by some consumer groups (see Schwartz et al. (2007)). However, as discussed in the introduction, the previous theoretical literature on behavior-based price discrimination and privacy suggests the opposite. Conitzer et al. (2012) identify a *negative* externality associated with consumers' hiding activity, which would mean that there is *too much* anonymization. One contribution of the present paper is to point out that, in some economic environments, there is also a *positive* externality associated with consumers' hiding activity, and that this can outweigh the negative one. The positive externality arises because consumers do not internalize all the social gains from trade that arise when they, thanks to their anonymous status, find it worthwhile to make a purchase. As a consequence, the theoretical prediction is consistent with the apparent view among consumer groups that consumers hide too little from a social welfare point of view.

What are the policy consequences of the results derived in the present paper, and what are the policy instruments that the logic applies to? Two, qualitatively different, kinds of policy instruments come to mind. The first one would be a tax or subsidy directly associated with the activity of consumer hiding, in combination with a lump-sum transfer to or from the consumer. The second kind of policy would be one that affects the consumers' cost of hiding—if we write the cost function in the model as $\kappa C(\lambda)$, instead of just $C(\lambda)$, we could think of this policy as altering the parameter κ. The results in the paper about under- and oversupply of anonymization (Propositions 4, 5, and 9, and Result 1) have, strictly speaking, bearing only on the first kind of policy. However, the results reported in Propositions 2 and 7 suggest that, say, lowering κ down to zero or raising it infinitely, would be socially sub-optimal, as some interior level of hiding is always desirable in the setup that is studied.

The reason why, for example, Result 1 does not imply that a lowering of κ is desirable is that the consumers' incentives to hide would be misaligned with society's interests also after such a change. For any given $\kappa C(\lambda)$—before or after a change in κ—the consumer's marginal benefit from an increase in λ is lower than society's. However, if a policymaker had the ability to appropriately distort a consumer's hiding incentives—say, with a sufficiently well-targeted and hiding-contingent transfer—it would be socially desirable. Of course, the practical implementation of such a policy would be associated with the same kind of difficulties as many similar policies discussed in public economics and related fields. For example, the informational demands on the policymaker are enormous. Nevertheless, the present analysis can, I believe, be useful for policymakers in that is shows that not even the desired direction in which to change the consumers' hiding incentives is clear. This is a conclusion that is more cautionary than it is actionable, but also such conclusions can be helpful.

The results also give policymakers some broad guidance about what circumstances under which it is desirable to try to strengthen, as opposed to weaken, consumers' incentives to hide. First, the positive externality—which in the present setting appears to be the single effect that pushes the overall outcome in the direction of too little hiding—can only arise with a total surplus standard. This suggests that a policymaker who cares only about consumer surplus should try to weaken consumers' incentives to hide. Second, the positive externality is present only in the first one of the two periods that are modeled here. Therefore, a policymaker who puts a great weight on welfare in future periods, relative to the present, should again try to weaken consumers' incentives to hide.

Third, the analysis in Section 5 suggests that the positive externality arises only if the consumer learns that she can shop anonymously, after having taken the hiding action. To assess whether this is the case, a policymaker can ask if it seems plausible that the hiding actions lead to less cautionary purchasing behavior on the part of the consumer. If the answer is yes, it is more likely that there is undersupply of hiding—and, therefore, the policymaker should try to strengthen consumers' incentives to hide. For example, if the hiding action consists of canceling a subscription to a newspaper for the purpose of qualifying for an introductory offer, then the policymaker can ask if it likely that this behavior leads to more sales of subscriptions than without such an opportunity to hide.

7 Concluding Remarks

I have studied the incentives of consumers to hide their purchases, in an environment in which a monopoly firm practices behavior-based price discrimination. The analysis yielded two main results. First, in the version of the model where the fraction of hiding consumers is exogenous (and hiding is costless), the total-surplus maximizing level of this fraction is strictly interior. The reason is that both (i) a fraction of zero and (ii) a fraction of one would fail to exploit gains from trade—in case (i) due to the fact that unnecessarily many consumers with valuations above the price do not purchase in period 1, and in case (ii) due to the firm's inability to practice price discrimination in period 2. Second, in the version of the model where the choice of hiding is endogenous (and comes at a cost), the market outcome yields, from a social welfare point of view, too much hiding if the discount factor is large and too little if the discount factor is small. This is because the sign of the first-period externality differs from that of the second-period externality.

Throughout the analysis I have maintained the assumption that the hiding probability is chosen at an ex ante stage. I suggested in Section 2 that this model feature naturally captures the idea that a consumer adopts a simple rule or heuristic that she uses in a wide range of situations, updating it only occasionally. It would nevertheless be interesting to explore the alternative setting where the consumer makes her choice ex post. However, one analytical difficulty with such an extension is the fact that the endogenous hiding probability will depend on the consumer's valuation.[12] This, in turn, makes the firm's objective function at the stage where it chooses $\widehat{r}$ (or, equivalently, p_1) a polynomial of a higher degree than two (in the current model, it is quadratic). As a consequence, solving for the equilibrium value of $\widehat{r}$ analytically would become much less straightforward and in some cases impossible.

Another extension that might provide further insights would be to allow the firm to take some (costly) action that makes it harder for consumers to hide their purchasing history (cf. Johnson's (2013) model, in which firms can choose the number of advertisements that the consumers are exposed to and try to protect themselves from). Finally, it would be interesting to study the effects on the present paper's results of a change in the degree of competition in the market.

[12] More precisely, for consumers with $r \in (\widehat{r}, 1]$, the hiding probability will *not* depend on r, as the benefit from hiding equals the difference between the two second-period prices (cf. Figure 1). However, for consumers with $r \in (p_1, \widehat{r})$, the chosen hiding probability will indeed depend on the own valuation, as the benefit from hiding equals $r - p_1$.

Appendix A: Binary Hiding Decision

This appendix presents an alternative modeling framework, in which the consumers' hiding choice is not continuous but binary. It is shown that this model is equivalent to the continuous-choice model in Section 2.

Thus, consider a model that is identical to the one in Section 2, except that here all consumers make a binary choice whether to hide. A choice to hide is successful for sure and leads to a cost $k \in [0, \overline{k}]$, where $\overline{k} > 0$ is finite and sufficiently large (alternatively, $\overline{k} = \infty$). The magnitude of the cost k is individual-specific and drawn (independently of r) from a cumulative distribution function G. Each consumer learns perfectly and privately about her own k prior to making her hiding decision. We assume that G is continuously differentiable and strictly increasing on $(0, \overline{k})$, and we denote the associated density by g.

In this alternative model, we let λ denote the *fraction* of consumers who choose to hide (as opposed to the consumers' common likelihood of successful hiding). Clearly, for a fixed value of λ, the analysis of the alternative model and of the original model are identical, and thus the equilibrium prices and the equilibrium cutoff value of r are also the same across the two models.

What are the incentives to hide in the alternative model, compared to the ones in the original model? Let $\widetilde{\lambda}$ denote the fraction of consumers who are expected to hide. Given $\widetilde{\lambda}$, an individual consumer will herself have a weak incentive to hide if, and only if, her cost k does not exceed the benefit from hiding, which can be written as $\Delta_S\left(\widetilde{\lambda}\right) \stackrel{\text{def}}{=} S\left(1, \widetilde{\lambda}\right) - S\left(0, \widetilde{\lambda}\right)$. Here, $S\left(\lambda, \widetilde{\lambda}\right)$ is identical to the consumer surplus function stated in (13). This means that there exists a unique $k' \in \left(0, \overline{k}\right)$, defined by $k' = \Delta_S\left(\widetilde{\lambda}\right)$, such that all consumers with $k < k'$ choose to hide, and all consumers with $k > k'$ choose not to hide. At an equilibrium, the consumers' beliefs are correct ($\widetilde{\lambda} = \lambda^*$), and the equilibrium cutoff value of k, denoted by k^*, is thus characterized by

$$k^* = \Delta_S(\lambda^*).$$

The equilibrium cutoff value k^*, in turn, determines the equilibrium fraction of consumers who hide as $\lambda^* = G(k^*)$. Combining the two equilibrium conditions, we can write

$$\lambda^* = G[\Delta_S(\lambda^*)] \Leftrightarrow \Delta_S(\lambda^*) = G^{-1}(\lambda^*), \tag{A1}$$

where G^{-1} is the inverse of G.

Note that, since $S\left(\lambda, \widetilde{\lambda}\right)$ is an affine function of $\widetilde{\lambda}$, we can write $\Delta_S\left(\widetilde{\lambda}\right) = \partial S\left(\lambda, \widetilde{\lambda}\right)/\partial\widetilde{\lambda}$, which is the marginal benefit of hiding in the original model.[13] Thus, comparing (A1) and (15), we see that, if $G^{-1} \equiv C'$, the equilibrium condition in the model with a binary hiding decision is identical to the one in the original model. If we indeed assume that $G^{-1} \equiv C'$, also the aggregate equilibrium costs of hiding are identical in the two models. In the model with a binary hiding decision, these costs can be written as

$$\begin{aligned}
\int_0^{k^*} kg(k)dk &= \int_0^{G^{-1}(\lambda^*)} kg(k)dk = \lambda^* G^{-1}(\lambda^*) - \int_0^{G^{-1}(\lambda^*)} G(k)dk \\
&= \int_0^{G^{-1}(\lambda^*)} [\lambda^* - G(k)]\, dk \\
&= \int_0^{\lambda^*} G^{-1}(k)dk = \int_0^{\lambda^*} C'(k)dk = C(\lambda^*),
\end{aligned} \tag{A2}$$

where the second equality is obtained by integration by parts, the fourth equality holds as G^{-1} is the inverse of G (cf. Figure 8), and the fifth equality holds due to the assumed identity $G^{-1} \equiv C'$. The last term in (A2), $C(\lambda^*)$, indeed represents the aggregate equilibrium costs in the original model.

[13] In addition, from the proof of Proposition 3, we know that $\partial S\left(\lambda, \widetilde{\lambda}\right)/\partial\widetilde{\lambda} = \int_{p_1^*}^{\widehat{r}^*}(1-r)dr$.

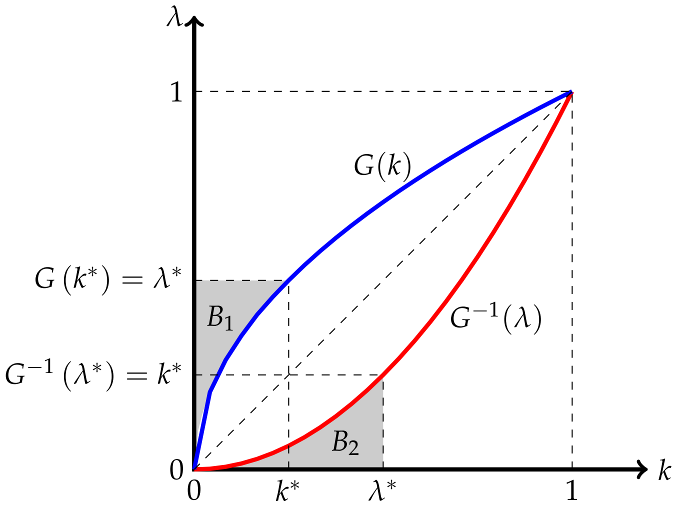


Figure 8: Illustration of a step in the proof that equilibrium hiding costs are identical in the two models discussed in Appendix A. Given that G and G^{-1} are each other's inverse, the areas B_1 and B_2 must be identical.

Appendix B: Proofs

Proof of Lemma 1. Using the information in the text, we can write total second-period profits in the low-valuation market as

$$\pi_2^L\left(p_2^L\right) = \begin{cases} [\lambda + (1-\lambda)\widehat{r} - p_2^L]p_2^L & \text{if } p_2^L \in [0, \widehat{r}] \\ \lambda\left(1 - p_2^L\right)p_2^L & \text{if } p_2^L \in [\widehat{r}, 1]. \end{cases}$$

This profit expression is continuous in p_2^L, but it is not necessarily concave or quasiconcave. However, the expression is clearly concave (and quadratic) in p_2^L in each of the two ranges $[0, \widehat{r}]$ and $[\widehat{r}, 1]$. The solution to the problem of maximizing π_2^L with respect to p_2^L subject to $p_2^L \in [0, \widehat{r}]$ can therefore easily be found with the help of a first-order condition. The solution is given by

$$\widehat{p}_2^L = \begin{cases} \frac{\lambda + (1-\lambda)\widehat{r}}{2} & \text{if } \widehat{r} \geq \frac{\lambda}{1+\lambda} \\ \widehat{r} & \text{if } \widehat{r} \leq \frac{\lambda}{1+\lambda}. \end{cases} \tag{A3}$$

Similarly, the solution to the problem of maximizing π_2^L with respect to p_2^L subject to $p_2^L \in [\widehat{r}, 1]$ is given by

$$\widetilde{p}_2^L = \begin{cases} \widehat{r} & \text{if } \widehat{r} \geq \frac{1}{2} \\ \frac{1}{2} & \text{if } \widehat{r} \leq \frac{1}{2}. \end{cases} \tag{A4}$$

Notice that the cutoff point in (A3) is strictly smaller than the one in (A4): $\frac{\lambda}{1+\lambda} < \frac{1}{2}$. Therefore, if $\widehat{r} \leq \frac{\lambda}{1+\lambda}$, we should compare the profits at $\widetilde{p}_2^L = \frac{1}{2}$ and at $\widehat{p}_2^L = \widehat{r}$ in order to find the global optimum for that region. With a bit of algebra one can verify that this optimum is at $\widetilde{p}_2^L = \frac{1}{2}$, which is indeed consistent with equation (4). Similarly, if $\widehat{r} > \frac{1}{2}$, we should compare the profits at $\widehat{p}_2^L = \frac{\lambda+(1-\lambda)\widehat{r}}{2}$ and at $\widetilde{p}_2^L = \widehat{r}$ in order to find the global optimum for that region. Again, with a bit of algebra one can check that this optimum is at $\widehat{p}_2^L = \frac{\lambda+(1-\lambda)\widehat{r}}{2}$, which is consistent with equation (4). The comparison that remains is the one for $\widehat{r} \in \left[\frac{\lambda}{1+\lambda}, \frac{1}{2}\right]$, in which we must compare the profits at $\widetilde{p}_2^L = \frac{1}{2}$ and at $\widehat{p}_2^L = \frac{\lambda+(1-\lambda)\widehat{r}}{2}$. The profits at the latter price equal $\pi_2^L\left(\widehat{p}_2^L\right) = \left[\lambda + (1-\lambda)\widehat{r}\right]^2/4$, whereas the profits at $\widetilde{p}_2^L = \frac{1}{2}$ equal $\pi_2^L\left(\widetilde{p}_2^L\right) = \frac{\lambda}{4}$. We can now compare these profit levels:

$$\pi_2^L\left(\widehat{p}_2^L\right) > \pi_2^L\left(\widetilde{p}_2^L\right) \Leftrightarrow \frac{\left[\lambda + (1-\lambda)\widehat{r}\right]^2}{4} > \frac{\lambda}{4} \Leftrightarrow \widehat{r} > \frac{\sqrt{\lambda}}{1+\sqrt{\lambda}}.$$

We can conclude that the optimal price is indeed as stated in Lemma 1. □

Proof of Proposition 1. In order to derive the optimal behavior in period 1 and to identify all possible equilibria of the game, we need to investigate three cases:

$$\text{(i) } \widehat{r} \geq \frac{1}{2}; \qquad \text{(ii) } \widehat{r} \in \left(\frac{\sqrt{\lambda}}{1+\sqrt{\lambda}}, \frac{1}{2} \right); \qquad \text{(iii) } \widehat{r} \leq \frac{\sqrt{\lambda}}{1+\sqrt{\lambda}}.$$

Refer to an equilibrium that arises under case (i) as a type (i) equilibrium, and analogously for cases (ii) and (iii). In the main text it was shown that there exists a unique type (i) equilibrium, and this equilibrium was solved for. It remains to show that a type (ii) or a type (iii) equilibrium never exists.

First consider the possible existence of a type (ii) equilibrium. In such an equilibrium, the second-period prices are given by eq. (5) and $p_2^H = \frac{1}{2}$ (see subsection 3.1). In particular, consumers with a valuation $r \in \left(\widehat{r}, \frac{1}{2}\right)$ and without an incognito status buy in the first period but do not buy in period 2. Thus, using (1), we have that the threshold $\widehat{r}$ satisfies $\widehat{r} - p_1 + 0 = \delta\left(\widehat{r} - p_2^L\right)$, which again yields (6). The firm's overall profits equal

$$\Pi = q_1 p_1 + \delta \frac{[\lambda + (1-\lambda)\widehat{r}]^2}{4} + \frac{\delta(1-\lambda)}{4}, \tag{A5}$$

with p_1 and q_1 given by (6) and (8), respectively. Denote by $\widetilde{\Pi}(\widehat{r})$ the reduced-form profit function that we obtain by eliminating p_1 from (A5) with the help of (6) and (8). The optimal $\widehat{r}$ satisfies the first-order condition $\partial\widetilde{\Pi}/\partial\widehat{r} = 0$ or, equivalently,

$$\widehat{r} = \frac{2 - \delta(1+\lambda)\left(1+\lambda-\delta\lambda^2\right) + \delta\lambda(1-\lambda)}{4 - \delta(1+\lambda)^2(2-\delta\lambda) - \delta(1-\lambda)^2}. \tag{A6}$$

For this value of $\widehat{r}$ to indeed be part of an equilibrium, we must have $\widehat{r} < \frac{1}{2}$. However, some algebra verifies that this inequality is always violated.

Next consider the possible existence of a type (iii) equilibrium. In such an equilibrium, $p_2^H = p_2^L = \frac{1}{2}$ (this follows from $p_2^H = \max\left\{\frac{1}{2}, \widehat{r}\right\}$ and Lemma 1). The threshold $\widehat{r}$ must satisfy (1) with equality when evaluated at $p_2^H = p_2^L = \frac{1}{2}$, which implies that $\widehat{r} = p_1$. That is, the consumers choose to purchase in period 1 if, and only if, their valuation exceeds the first-period price. This means that there is effectively no interaction between the two periods and the firm's problem of choosing p_1 is tantamount to the problem of choosing p_1 in a one-period model. Hence, $p_1 = \frac{1}{2}$ $(= \widehat{r})$. However, $\widehat{r} = \frac{1}{2}$ contradicts the initial assumption that we have case (iii). It follows that a type (iii) equilibrium cannot exist.

The statements about existence, uniqueness, and characterization of the type (i) equilibrium are proven in the main text. The claim that $\widehat{r} - p_1 = \delta\left(p_2^H - p_2^L\right)$ follows from (1) holding with an equality. It remains to verify the claims about the relationships between the cutoff value and the prices. The claim that $p_1 \leq \widehat{r}$ follows immediately from (6) and the fact that $\widehat{r} \geq \frac{1}{2}$. Next, to prove the claim that $p_2^L \leq p_1$, note that (1) holding with an equality yields

$$p_1 - \delta p_2^L = (1-\delta)\widehat{r} \Leftrightarrow p_1 - p_2^L = (1-\delta)\left(\widehat{r} - p_2^L\right) \geq 0, \tag{A7}$$

where the inequality follows from $\widehat{r} \geq \frac{1}{2}$ and $p_2^L \leq \frac{1}{2}$ (that the latter must hold can be seen from (5)).

Finally, to prove the relationship $p_1 \leq \frac{1}{2}$, use (6) to write

$$p_1 = \widehat{r} - \frac{\delta\left[(1+\lambda)\widehat{r} - \lambda\right]}{2} \leq \frac{1}{2} \Leftrightarrow \widehat{r} \leq \frac{1-\delta\lambda}{2-\delta(1+\lambda)}.$$

By plugging in the expression for $\widehat{r}$ stated in (9) into the last inequality and then rewriting, one obtains the equivalent inequality $\widehat{Q}(\delta,\lambda) \geq 0$, where

$$\begin{aligned}\widehat{Q}(\delta,\lambda) &\stackrel{\text{def}}{=} (1-\delta\lambda)\left[4 - \delta\left[(1+\lambda)^2(2-\delta\lambda) - (1-\lambda)(3+\lambda)\right]\right] \\ &\quad - [2-\delta(1+\lambda)]\left[2 - \delta\left[(1+\lambda)\left(1+\lambda-\delta\lambda^2\right) - (1-\lambda)(2+\lambda)\right]\right].\end{aligned}$$

By rearranging the above expression, one can factor out the positive constant δ and thus write $\widehat{Q}(\delta,\lambda) \geq 0 \Leftrightarrow Q(\delta,\lambda) \geq 0$, where

$$\begin{aligned} Q(\delta,\lambda) \overset{\text{def}}{=} & \ 2(1-\lambda) + [2-\delta(1+\lambda)] \left[(1+\lambda)\left(1+\lambda-\delta\lambda^2\right) - (1-\lambda)(2+\lambda)\right] \\ & - (1-\delta\lambda)\left[(1+\lambda)^2(2-\delta\lambda) - (1-\lambda)(3+\lambda)\right] \end{aligned}$$

The function $Q(\delta,\lambda)$ is in fact linear in δ (note that the quadratic terms add up to zero). Moreover, one can verify that $Q(0,\lambda) = (1-\lambda)^2$ and $Q(1,\lambda) = 2(1-\lambda)^2$. It follows that $Q(\delta,\lambda) \geq 0$ for all $\lambda \in [0,1]$ (and for all $\lambda \in [0,1)$, the inequality holds strictly).

The remaining equilibrium relationships are proven in the main text or are straightforward. □

Proof of Proposition 2. In order to show that $\widehat{\lambda}_W$ is interior, first note the following relationship:

$$\frac{12+16\delta+3\delta^2}{8(4+\delta)} = W(0) > W(1) = \frac{3(1+\delta)}{8}. \tag{A8}$$

That is, total surplus is strictly larger with no incognito surfers than with only incognito surfers. The expression for $W(0)$ in (A8) was obtained from (11) and by noting that, evaluated at $\lambda = 0$, $\widehat{r} = (2+\delta)/(4+\delta)$ and $p_2^L = (2+\delta)/[2(4+\delta)]$. Similarly, the expression for $W(1)$ was obtained from (11) and by noting that, evaluated at $\lambda = 1$, $\widehat{r} = p_1 = p_2^L = 1/2$.

Given (A8) and the arguments already provided in the main text, it remains to show that $\lim_{\lambda\to 0} \partial W(\lambda)/\partial\lambda > 0$ for all $\delta \in (0,1]$. To do that, first write the expression for $\widehat{r}$ stated in (9) as follows: $\widehat{r} = N(\lambda,\delta)/D(\lambda,\delta)$, where

$$N(\lambda,\delta) \overset{\text{def}}{=} 2 - \delta(1+\lambda)\left(1+\lambda-\delta\lambda^2\right) + \delta(1-\lambda)(2+\lambda),$$

$$D(\lambda,\delta) \overset{\text{def}}{=} 4 - \delta(1+\lambda)^2(2-\delta\lambda) + \delta(1-\lambda)(3+\lambda).$$

Differentiating yields

$$\frac{\partial N(\lambda,\delta)}{\partial\lambda} = -\delta\left[\left(1+\lambda-\delta\lambda^2\right) + (1+\lambda)(1-2\delta\lambda)\right] + \delta[-(2+\lambda)+(1-\lambda)],$$

$$\frac{\partial D(\lambda,\delta)}{\partial\lambda} = -\delta\left[2(1+\lambda)(2-\delta\lambda) - \delta(1+\lambda)^2\right] + \delta[-(3+\lambda)+(1-\lambda)].$$

Taking limits, we have

$$\lim_{\lambda\to 0} N(\lambda,\delta) = 2-\delta+2\delta, \qquad \lim_{\lambda\to 0} D(\lambda,\delta) = 4-2\delta+3\delta,$$

$$\lim_{\lambda\to 0} \frac{\partial N(\lambda,\delta)}{\partial\lambda} = -2\delta-\delta, \qquad \lim_{\lambda\to 0} \frac{\partial D(\lambda,\delta)}{\partial\lambda} = -\delta(4-\delta) - 2\delta.$$

We can now write

$$\begin{aligned} \lim_{\lambda\to 0} \frac{\partial\widehat{r}}{\partial\lambda} &= \lim_{\lambda\to 0} \frac{\frac{\partial N(\lambda)}{\partial\lambda} D(\lambda,\delta) - N(\lambda,\delta)\frac{\partial D(\lambda)}{\partial\lambda}}{[D(\lambda,\delta)]^2} \\ &= \frac{(-2\delta-\delta)(4-2\delta+3\delta) - (2-\delta+2\delta)[-\delta(4-\delta)-2\delta]}{(4-2\delta+3\beta)^2} \\ &= \frac{-3\delta(4+\delta) + \delta(2+\delta)(6-\delta)}{(4+\delta)^2} = \frac{\delta^2(1-\delta)}{(4+\delta)^2}. \end{aligned}$$

We also have

$$\lim_{\lambda\to 0} \widehat{r} = \frac{2-\delta+2\delta}{4-2\delta+3\delta} = \frac{2+\delta}{4+\delta}.$$

Next, from (5) we have $p_2^L = \frac{1}{2}[\lambda + (1-\lambda)\widehat{r}]$, which yields

$$\frac{\partial p_2^L}{\partial\lambda} = \frac{1}{2}\left[1 - \widehat{r} + (1-\lambda)\frac{\partial\widehat{r}}{\partial\lambda}\right].$$

Thus, $\lim_{\lambda\to 0} p_2^L = \frac{1}{2}\lim_{\lambda\to 0}\widehat{r}$. Moreover, we can write $\int_{p_1}^{\widehat{r}} rdr = \frac{1}{2}\left(\widehat{r}^2 - p_1^2\right)$. Hence,

$$\lim_{\lambda\to 0}\int_{p_1}^{\widehat{r}} rdr = \frac{\left[\lim_{\lambda\to 0}\widehat{r}\right]^2 - \left[\frac{2-\delta}{2}\lim_{\lambda\to 0}\widehat{r}\right]^2}{2} = \frac{\delta(4-\delta)}{8}\left[\lim_{\lambda\to 0}\widehat{r}\right]^2,$$

where (6) was used to obtain $\lim_{\lambda\to 0} p_1$. By using (12) and the above results, we can write

$$\begin{aligned}
\lim_{\lambda\to 0}\frac{\partial W}{\partial\lambda} &= -\lim_{\lambda\to 0}\widehat{r}\lim_{\lambda\to 0}\frac{\partial\widehat{r}}{\partial\lambda} + \lim_{\lambda\to 0}\int_{p_1}^{\widehat{r}} rdr - \delta\lim_{\lambda\to 0} p_2^L\lim_{\lambda\to 0}\frac{\partial p_2^L}{\partial\lambda} \\
&= \left[\lim_{\lambda\to 0}\widehat{r}\right]\left[-\lim_{\lambda\to 0}\frac{\partial\widehat{r}}{\partial\lambda} + \frac{\delta(4-\delta)}{8}\lim_{\lambda\to 0}\widehat{r} - \frac{\delta}{2}\lim_{\lambda\to 0}\frac{\partial p_2^L}{\partial\lambda}\right] \\
&= \left[\lim_{\lambda\to 0}\widehat{r}\right]\left[-\lim_{\lambda\to 0}\frac{\partial\widehat{r}}{\partial\lambda} + \frac{\delta(4-\delta)}{8}\lim_{\lambda\to 0}\widehat{r} - \frac{\delta}{4}\left(1-\lim_{\lambda\to 0}\widehat{r} + \lim_{\lambda\to 0}\frac{\partial\widehat{r}}{\partial\lambda}\right)\right] \\
&= \left[\lim_{\lambda\to 0}\widehat{r}\right]\left[-\frac{4+\delta}{4}\lim_{\lambda\to 0}\frac{\partial\widehat{r}}{\partial\lambda} + \frac{\delta(6-\delta)}{8}\lim_{\lambda\to 0}\widehat{r} - \frac{\delta}{4}\right] \\
&= \left[\lim_{\lambda\to 0}\widehat{r}\right]\left[\frac{-2\delta^2(1-\delta) + \delta(6-\delta)(2+\delta) - 2\delta(4+\delta)}{8\,(4+\delta)}\right] = \left[\lim_{\lambda\to 0}\widehat{r}\right]\left[\frac{\delta(4+\delta^2)}{8\,(4+\delta)}\right] > 0.
\end{aligned}$$

The last inequality holds for all $\delta \in (0,1]$, as $\lim_{\lambda\to 0}\widehat{r} > 0$ and the numerator of the ratio is also positive. □

Proof of Proposition 3. First note that we cannot have a symmetric equilibrium with $\lambda^* = 0$ or $\lambda^* = 1$. For if $\lambda^* = 1$, then all the prices are the same and thus the two first terms in (14) vanish, whereas the third term (i.e., the marginal cost) is strictly positive; hence, $\partial EU/\partial\lambda < 0$ at $\lambda = 1$ and the consumer would have an incentive to choose $\lambda < 1$. Similarly, if $\lambda^* = 0$, then the sum of the two first terms in (14) is strictly positive, but the marginal cost is zero; as a consequence, $\partial EU/\partial\lambda > 0$ at $\lambda = 0$ and the consumer would want to choose $\lambda > 0$. In order to show existence of a symmetric equilibrium with $\lambda^* \in (0,1)$, it suffices to note that the equation $\int_{p_1}^{\widehat{r}}(r-p_1)\,dr + \delta\int_{\widehat{r}}^1\left(p_2^H - p_2^L\right)dr = C'(\lambda^*)$ must have at least one root $\lambda^* \in (0,1)$. For the right-hand side is, by assumption, increasing in λ^* and it equals zero at $\lambda^* = 0$; moreover, the left-hand side is strictly positive evaluated at $\lambda^* = 0$ and zero evaluated at $\lambda^* = 1$. A symmetric equilibrium is characterized by the equation just stated, and the left-hand side of this can be rewritten as

$$\begin{aligned}
\int_{p_1}^{\widehat{r}}(r-p_1)\,dr + \delta\int_{\widehat{r}}^1\left(p_2^H - p_2^L\right)dr &= \int_{p_1}^{\widehat{r}}(r-p_1)\,dr + (\widehat{r}-p_1)(1-\widehat{r}) \\
&= \frac{1}{2}(\widehat{r}-p_1)(2-\widehat{r}-p_1) = \int_{p_1}^{\widehat{r}}(1-r)\,dr,
\end{aligned}$$

where the first equality is due to the relationship $\widehat{r} - p_1 = \delta\left(p_2^H - p_2^L\right)$ stated in Proposition 1. □

Proof of Proposition 4. Under the assumption that $\delta = 1$, we have $p_2^L = p_1 = (3-\lambda)(1+\lambda)\,/\,\left[2\left(5-\lambda^2\right)\right]$ and $p_2^H = \widehat{r} = \left(3-\lambda^2\right)/\left(5-\lambda^2\right)$ (see Example 1). Differentiating these expressions yields

$$\frac{\partial p_1}{\partial\lambda} = \frac{5-2\lambda+\lambda^2}{\left(5-\lambda^2\right)^2} \quad\text{and}\quad \frac{\partial\widehat{r}}{\partial\lambda} = -\frac{4\lambda}{\left(5-\lambda^2\right)^2}. \tag{A9}$$

This means that the direct welfare effect can be written as

$$\Delta_D(\lambda) = \int_{p_1}^{\widehat{r}}(1-2r)\,dr = r(1-r)\,|_{p_1}^{\widehat{r}} = \widehat{r}(1-\widehat{r}) - p_1(1-p_1) = (\widehat{r}-p_1)(1-\widehat{r}-p_1) = \frac{(3+\lambda)(1-\lambda)}{2\left(5-\lambda^2\right)}.$$

Similarly, the indirect welfare effect can be written as

$$\Delta_I(\lambda) = (1-\lambda)\,\widehat{r}\frac{\partial\widehat{r}}{\partial\lambda} + (1+\lambda)\,p_1\frac{\partial p_1}{\partial\lambda} = \frac{\left(5-\lambda^2\right)\left(3-\lambda+5\lambda^2+\lambda^3\right)}{2\left(5-\lambda^2\right)^3}.$$

From inspection, it is clear that $\Delta_I(\lambda) > 0$ for all $\lambda \in [0,1]$ and $\Delta_D(\lambda) \geq 0$ for all $\lambda \in [0,1]$ (with an equality only for $\lambda = 1$). $\square$

Proof of Proposition 5. Differentiating $S(\lambda, \widetilde{\lambda})$, as stated in (13), with respect to $\widetilde{\lambda}$ yields

$$\begin{aligned}
\frac{\partial S(\lambda,\widetilde{\lambda})}{\partial \widetilde{\lambda}} &= -\left(\widehat{r}-p_1\right)\frac{\partial \widehat{r}}{\partial \widetilde{\lambda}} + \lambda\left(\widehat{r}-p_1\right)\frac{\partial \widehat{r}}{\partial \widetilde{\lambda}} + \delta\left(\widehat{r}-p_2^L\right)\frac{\partial \widehat{r}}{\partial \widetilde{\lambda}} - \delta\left[\widehat{r}-(1-\lambda)p_2^H-\lambda p_2^L\right]\frac{\partial \widehat{r}}{\partial \widetilde{\lambda}} \\
&\quad - \frac{\partial p_1}{\partial \widetilde{\lambda}}\int_{\widehat{r}}^{1} dr - \lambda \frac{\partial p_1}{\partial \widetilde{\lambda}}\int_{p_1}^{\widehat{r}} - \delta \frac{\partial p_2^L}{\partial \widetilde{\lambda}}\int_{p_2^L}^{\widehat{r}} - \delta\left[(1-\lambda)\frac{\partial p_2^H}{\partial \widetilde{\lambda}} + \lambda\frac{\partial p_2^L}{\partial \widetilde{\lambda}}\right]\int_{\widehat{r}}^{1} dr \\
&= (1-\lambda)\left[-\left(\widehat{r}-p_1\right)+\delta\left(p_2^H-p_2^L\right)\right]\frac{\partial \widehat{r}}{\partial \widetilde{\lambda}} \\
&\quad - \left[(1-\widehat{r})+\lambda\left(\widehat{r}-p_1\right)\right]\frac{\partial p_1}{\partial \widetilde{\lambda}} - \delta\left(\widehat{r}-p_2^L\right)\frac{\partial p_2^L}{\partial \widetilde{\lambda}} - \delta\left(1-\widehat{r}\right)\left[(1-\lambda)\frac{\partial p_2^H}{\partial \widetilde{\lambda}} + \lambda\frac{\partial p_2^L}{\partial \widetilde{\lambda}}\right].
\end{aligned} \tag{A10}$$

However, the first line after the last equality equals zero (due to the relationship $\widehat{r} - p_1 = \delta\left(p_2^H - p_2^L\right)$ stated in Proposition 1), which gives us the expression for $\partial S(\lambda, \widetilde{\lambda})/\partial \widetilde{\lambda}$ in (20).

With $\delta = 1$, the equilibrium prices and cutoff value are as stated in Example 1 (see also the proof of Proposition 4). Moreover, $p_1 = p_2^L$ and $\widehat{r} = p_2^H$ hold. By (A9), we have $\frac{\partial \widehat{r}}{\partial \lambda} = \frac{\partial p_2^L}{\partial \lambda} > 0$ for all $\lambda \in [0,1]$ and $\frac{\partial p_1}{\partial \lambda} = \frac{\partial p_2^H}{\partial \lambda} > 0$ for all $\lambda \in (0,1]$. We can also compute

$$(1-\lambda)\frac{\partial p_2^H}{\partial \lambda} + \lambda\frac{\partial p_2^L}{\partial \lambda} = \frac{\lambda(1+\lambda)^2}{(5-\lambda)^2},$$

which is strictly positive for all $\lambda \in (0,1]$. $\square$

Proof of Proposition 6. The proof is a very close analogue to the proof of Proposition 1. Again, we need to investigate also case (ii) and case (iii), as stated in the beginning of that proof. For case (ii), although condition (1) is here replaced by condition (21), the second-period prices are also here given by (5) and $p_2^H = \frac{1}{2}$. The rest of case (ii) and all of case (iii) are identical, or close to identical, to the analysis in the proof of Proposition 1. The arguments about the relationships between the prices and cutoff value are also again valid in this setting, with the exception of the proofs that $p_2^L \le p_1$ and $p_1 \le \frac{1}{2}$. To prove the first relationship in this setting, note that (21) holding with equality yields

$$p_1 - p_2^L = \left[1-\delta\left(1-\lambda\right)\right]\left(\widehat{r}-p_2^L\right) \ge 0,$$

where the inequality follows from $\widehat{r} \ge \frac{1}{2}$ and $p_2^L \le \frac{1}{2}$ (that the latter must hold can be seen from (5)). Next, to prove the relationship $p_1 \le \frac{1}{2}$, use (22) to write

$$p_1 = \widehat{r} - \frac{\delta(1-\lambda)\left[(1+\lambda)\widehat{r}-\lambda\right]}{2} \le \frac{1}{2} \Leftrightarrow \widehat{r} \le \frac{1-\delta\lambda(1-\lambda)}{2-\delta(1-\lambda^2)}.$$

By plugging in the expression for $\widehat{r}$ stated in (23) into the last inequality and then simplifying, one obtains $(2-\lambda)(1-\lambda) \ge 0$, which holds for all λ (and it holds strictly for all $\lambda < 1$). $\square$

Proof of Proposition 7. Evaluating the derivative in (25) at $\lambda = 0$ and $\lambda = 1$ yields

$$\frac{\partial W}{\partial \lambda}\Big|_{\lambda=0} = \frac{\delta(2+\delta)}{2(4+\delta)^2}, \qquad \frac{\partial W}{\partial \lambda}\Big|_{\lambda=1} = -\frac{\delta}{8}, \tag{A11}$$

which proves the result. $\square$

Proof of Proposition 8. To show existence of a symmetric equilibrium with $\lambda^* \in (0,1)$, which in the family of symmetric equilibria is unique, it suffices to show that the equation $Z(\lambda) = C'(\lambda)$, where

$$Z(\lambda) \stackrel{\text{def}}{=} \frac{\delta(1-\widetilde{\lambda})\left[2+\delta(1-\widetilde{\lambda})\right]}{\left[4+\delta(1-\widetilde{\lambda})^2\right]^2}$$

(cf. (27)), has exactly one root in $\lambda \in [0,1]$ and that this root is in the interior of the interval. This, turn, follows from the fact that $Z(1) = C'(0) = 0$, $Z(0) > 0$, $C'(\lambda) > 0$ for all $\lambda \in (0,1]$, and

$$Z'(\lambda) = -2\left[4+\delta(1-\lambda)^2\right]\left[4+\delta(1-\lambda)-3\delta(1-\lambda)^2-\delta^2(1-\lambda)^3\right] < 0 \quad \text{for all } \lambda \in [0,1].$$

The characterization of the equilibrium stated in (28) follows from (27) and the relationship $\widehat{r} - p_1 = \delta(1-\lambda)\left(p_2^H - p_2^L\right)$ stated in Proposition 6. □

Proof of Proposition 9. First consider part (a) of the proposition. By using (A11) and (27), we can write

$$\begin{aligned}
&\frac{\partial W(\lambda)}{\partial \lambda} < \frac{\partial S(\lambda,\lambda)}{\partial \lambda} \Leftrightarrow \\
&\frac{8\delta(1-\lambda)\left[2+\delta(1-\lambda)^2\right]}{2\left[4+\delta(1-\lambda)^2\right]^3} - \frac{\delta\left[2(1+\lambda)+\delta(1-\lambda)^2\right]\left[4-\delta(1-\lambda)^2\right]}{2\left[4+\delta(1-\lambda)^2\right]^3} < \frac{2\delta(1-\lambda)\left[2+\delta(1-\lambda)\right]}{2\left[4+\delta(1-\lambda)^2\right]^2} \Leftrightarrow \\
&8(1-\lambda)\left[2+\delta(1-\lambda)^2\right] - \left[2(1+\lambda)+\delta(1-\lambda)^2\right]\left[4-\delta(1-\lambda)^2\right] < 2(1-\lambda)\left[2+\delta(1-\lambda)\right]\left[4+\delta(1-\lambda)^2\right] \\
&\Leftrightarrow \left[2(1+\lambda)+\delta(1-\lambda)^2\right]\left[4-\delta(1-\lambda)^2\right] > 2(1-\lambda)\left\{\left[2+\delta(1-\lambda)^2\right]\left[4-2-\delta(1-\lambda)\right]-4-2\delta(1-\lambda)\right\} \\
&= -2\delta(1-\lambda)^2\left[2(1+\lambda)+\delta(1-\lambda)^2\right],
\end{aligned}$$

which holds for all $\lambda \in (0,1)$ and all $\delta > 0$.

Next consider part (b). We need to show that the indirect effect on the consumer surplus function of increasing λ, which only society cares about, is negative. The indirect effect can be written as

$$\begin{aligned}
\frac{\partial S(\lambda,\widetilde{\lambda})}{\partial \widetilde{\lambda}} &= -\left(\widehat{r}-p_1\right)\frac{\partial \widehat{r}}{\partial \widetilde{\lambda}} + \delta\left(\widehat{r}-p_2^L\right)\frac{\partial \widehat{r}}{\partial \widetilde{\lambda}} - \delta\left[\widehat{r}-(1-\lambda)p_2^H-\lambda p_2^L\right]\frac{\partial \widehat{r}}{\partial \widetilde{\lambda}} \\
&\quad - (1-\widehat{r})\frac{\partial p_1}{\partial \widetilde{\lambda}} - \delta\left(\widehat{r}-p_2^L\right)\frac{\partial p_2^L}{\partial \widetilde{\lambda}} - \delta(1-\widehat{r})\left[(1-\lambda)\frac{\partial p_2^H}{\partial \widetilde{\lambda}} + \lambda\frac{\partial p_2^L}{\partial \widetilde{\lambda}}\right] \\
&= \left[-\left(\widehat{r}-p_1\right) + \delta(1-\lambda)\left(p_2^H - p_2^L\right)\right]\frac{\partial \widehat{r}}{\partial \widetilde{\lambda}} \\
&\quad - (1-\widehat{r})\frac{\partial p_1}{\partial \widetilde{\lambda}} - \delta\left(\widehat{r}-p_2^L\right)\frac{\partial p_2^L}{\partial \widetilde{\lambda}} - \delta(1-\widehat{r})\left[(1-\lambda)\frac{\partial \widehat{r}}{\partial \widetilde{\lambda}} + \lambda\frac{\partial p_2^L}{\partial \widetilde{\lambda}}\right]
\end{aligned}$$

or, evaluating at the common equilibrium value of λ,

$$\frac{\partial S(\lambda,\widetilde{\lambda})}{\partial \widetilde{\lambda}}\Big|_{\widetilde{\lambda}=\lambda} = -(1-\widehat{r})\frac{\partial p_1}{\partial \lambda} - \delta\left(\widehat{r}-p_2^L\right)\frac{\partial p_2^L}{\partial \lambda} - \delta(1-\widehat{r})\left[(1-\lambda)\frac{\partial \widehat{r}}{\partial \lambda} + \lambda\frac{\partial p_2^L}{\partial \lambda}\right], \tag{A12}$$

where the last equality uses the relationship $\widehat{r} - p_1 = \delta(1-\lambda)\left(p_2^H - p_2^L\right)$, which holds by Proposition 6. The expression for $\widehat{r}$ is given by (23). Further, compute the following expressions for the equilibrium prices and the associated derivatives with respect to λ:

$$p_2^L = \frac{\lambda + (1-\lambda)\widehat{r}}{2} = \frac{2(1+\lambda)+\delta(1-\lambda)^2}{2\left[4+\delta(1-\lambda)^2\right]}, \quad p_1 = \widehat{r} - \frac{\delta\left[(1+\lambda)\widehat{r}-\lambda\right]}{2} = \frac{4-\delta(1-\lambda)^3}{2\left[4+\delta(1-\lambda)^2\right]},$$

$$\frac{\partial \widehat{r}}{\partial \lambda} = -\frac{4\delta(1-\lambda)}{\left[4+\delta(1-\lambda)^2\right]^2} < 0, \quad \frac{\partial p_2^L}{\partial \lambda} = \frac{4-\delta(1-\lambda)^2}{\left[4+\delta(1-\lambda)^2\right]^2} > 0, \quad \frac{\partial p_1}{\partial \lambda} = \frac{\delta(1-\lambda)\left[8+12(1-\lambda)+\delta(1-\lambda)^3\right]}{2\left[4+\delta(1-\lambda)^2\right]^2} > 0. \tag{A13}$$

It follows from (A13) that all terms in (A12), except for the penultimate one involving $\frac{\partial \widehat{r}}{\partial \lambda}$, are negative. Moreover, the sum of the first (negative) term and the single positive term is negative:

$$-(1-\widehat{r})\frac{\partial p_1}{\partial \lambda} - \delta(1-\widehat{r})(1-\lambda)\frac{\partial \widehat{r}}{\partial \lambda} = -\frac{(1-\widehat{r})\,\delta(1-\lambda)\left[8+4(1-\lambda)+\delta(1-\lambda)^3\right]}{2\left[4+\delta(1-\lambda)^2\right]^2},$$

which completes the proof. □

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