Supplementary Material to "Surfing Incognito: Welfare Effects of Anonymous Shopping"

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1. Introduction

In this Supplementary Material, which is not meant to be published, I provide some proofs that were omitted from Lagerlöf (2019). In particular, I here show the calculations that are needed to do the simulations in Figures 7a and 7b of that paper.

2. Proofs of Results Not Proven in the Paper

2.1. Deriving the Atomistic Consumer's Marginal Benefit from Surfing Incognito

In this subsection, I first derive an explicit expression for the left-hand side of eq. (17) in the paper, which characterizes the equilibrium hiding probability λ^*. From (10) in the paper we have that

$$\widehat{r} = \frac{2 - \delta(1+\lambda)\left(1+\lambda-\delta\lambda^2\right) + \beta(1-\lambda)(2+\lambda)}{4 - \delta(1+\lambda)^2(2-\delta\lambda) + \beta(1-\lambda)(3+\lambda)}.$$

Setting $\beta = \delta$, this simplifies to

$$\begin{aligned} \widehat{r} &= \frac{2 - \delta(1+\lambda)\left(1+\lambda-\delta\lambda^2\right) + \delta(1-\lambda)(2+\lambda)}{4 - \delta(1+\lambda)^2(2-\delta\lambda) + \delta(1-\lambda)(3+\lambda)} \\ &= \frac{2 + \left(1-3\lambda-2\lambda^2\right)\delta + (1+\lambda)\lambda^2\delta^2}{4 + \left(1-6\lambda-3\lambda^2\right)\delta + \lambda(1+\lambda)^2\delta^2}. \end{aligned} \tag{S1}$$

From (7) in the paper we have

$$\widehat{r} - p_1 = \frac{\delta\left[(1+\lambda)\widehat{r} - \lambda\right]}{2}$$

and

$$\widehat{r} + p_1 = \frac{4\widehat{r} - \delta\left[(1+\lambda)\widehat{r} - \lambda\right]}{2}.$$

Given $\beta = \delta$, we can also write

$$\begin{aligned}(1+\lambda)\widehat{r}-\lambda &= \frac{2(1+\lambda)+\left(1-3\lambda-2\lambda^2\right)(1+\lambda)\delta+(1+\lambda)^2\lambda^2\delta^2}{4+\left(1-6\lambda-3\lambda^2\right)\delta+\lambda(1+\lambda)^2\delta^2} \\ &\quad -\frac{4\lambda+\left(1-6\lambda-3\lambda^2\right)\lambda\delta+\lambda^2(1+\lambda)^2\delta^2}{4+\left(1-6\lambda-3\lambda^2\right)\delta+\lambda(1+\lambda)^2\delta^2} \\ &= \frac{2(1-\lambda)+\left(1-2\lambda-\lambda^2\right)(1-\lambda)\delta}{4+\left(1-6\lambda-3\lambda^2\right)\delta+\lambda(1+\lambda)^2\delta^2} = \frac{(1-\lambda)\left[2+\left(1-2\lambda-\lambda^2\right)\delta\right]}{4+\left(1-6\lambda-3\lambda^2\right)\delta+\lambda(1+\lambda)^2\delta^2}\end{aligned}$$

and

$$\begin{aligned}4\widehat{r}-\delta\left[(1+\lambda)\widehat{r}-\lambda\right] &= \frac{4\left[2+\left(1-3\lambda-2\lambda^2\right)\delta+(1+\lambda)\lambda^2\delta^2\right]}{4+\left(1-6\lambda-3\lambda^2\right)\delta+\lambda(1+\lambda)^2\delta^2}-\frac{\delta(1-\lambda)\left[2+\left(1-2\lambda-\lambda^2\right)\delta\right]}{4+\left(1-6\lambda-3\lambda^2\right)\delta+\lambda(1+\lambda)^2\delta^2} \\ &= \frac{8+2\left(1-5\lambda-4\lambda^2\right)\delta-\left(1-3\lambda-3\lambda^2-3\lambda^3\right)\delta^2}{4+\left(1-6\lambda-3\lambda^2\right)\delta+\lambda(1+\lambda)^2\delta^2}.\end{aligned}$$

Furthermore,

$$\begin{aligned}2-\left(\widehat{r}+p_1\right) &= 2-\frac{4\widehat{r}-\delta\left[(1+\lambda)\widehat{r}-\lambda\right]}{2} \\ &= 2-\frac{8+2\left(1-5\lambda-4\lambda^2\right)\delta-\left(1-3\lambda-3\lambda^2-3\lambda^3\right)\delta^2}{2\left[4+\left(1-6\lambda-3\lambda^2\right)\delta+\lambda(1+\lambda)^2\delta^2\right]} \\ &= \frac{4\left[4+\left(1-6\lambda-3\lambda^2\right)\delta+\lambda(1+\lambda)^2\delta^2\right]-\left[8+2\left(1-5\lambda-4\lambda^2\right)\delta-\left(1-3\lambda-3\lambda^2-3\lambda^3\right)\delta^2\right]}{2\left[4+\left(1-6\lambda-3\lambda^2\right)\delta+\lambda(1+\lambda)^2\delta^2\right]} \\ &= \frac{8+2\left(1-7\lambda-2\lambda^2\right)\delta+\left(1+\lambda+5\lambda^2+\lambda^3\right)\delta^2}{2\left[4+\left(1-6\lambda-3\lambda^2\right)\delta+\lambda(1+\lambda)^2\delta^2\right]}.\end{aligned}$$

Using the above results, we can write

$$\begin{aligned}\int_{p_1}^{\widehat{r}}(1-r)\,dr &= \widehat{r}-p_1-\frac{\widehat{r}^2-p_1^2}{2} = \frac{\widehat{r}-p_1}{2}\left[2-\left(\widehat{r}+p_1\right)\right] \\ &= \frac{\delta(1-\lambda)\left[2+\left(1-2\lambda-\lambda^2\right)\delta\right]\left[8+2\left(1-7\lambda-2\lambda^2\right)\delta+\left(1+\lambda+5\lambda^2+\lambda^3\right)\delta^2\right]}{4\left[4+\left(1-6\lambda-3\lambda^2\right)\delta+\lambda(1+\lambda)^2\delta^2\right]^2},\end{aligned}$$

which yields the left-hand side of eq. (17) in the paper. Given the assumed functional form for the cost function, the right-hand side of that equation equals $c\lambda$. Thus, the equation that defines the equilibrium hiding probability λ is given by

$$\frac{\delta(1-\lambda)\left[2+\left(1-2\lambda-\lambda^2\right)\delta\right]\left[8+2\left(1-7\lambda-2\lambda^2\right)\delta+\left(1+\lambda+5\lambda^2+\lambda^3\right)\delta^2\right]}{4\left[4+\left(1-6\lambda-3\lambda^2\right)\delta+\lambda(1+\lambda)^2\delta^2\right]^2}-c\lambda=0. \tag{S2}$$

The root of equation (S2) can, for some assumed value of δ, be solved for with the help of numerical methods. The Matlab code that I have used for this can be found in the file "Incognito_code_Eq_v1.m" (downloadable at www.johanlagerlof.com).

2.2. Deriving Total Surplus

In order to compute the socially optimal values of the hiding probability, λ^W, I first derive an explicit expression for total surplus, $W(\lambda)-C(\lambda)$. I then, with the help of numerical methods, identify the value of λ that maximizes this expression.

From (13) in the paper, we have that total surplus can be written as

$$W(\lambda) - C(\lambda) = \int_{\widehat{r}}^{1} r dr + \lambda \int_{p_1}^{\widehat{r}} r dr + \delta \int_{p_2^L}^{1} r dr - \frac{c\lambda^2}{2}.$$

Write

$$\int_{\widehat{r}}^{1} r dr = \frac{1 - \widehat{r}^2}{2} = \frac{(1 - \widehat{r})(1 + \widehat{r})}{2}.$$

Using (S1), we can write

$$\begin{aligned} 1 - \widehat{r} &= 1 - \frac{2 + (1 - 3\lambda - 2\lambda^2)\delta + (1+\lambda)\lambda^2\delta^2}{4 + (1 - 6\lambda - 3\lambda^2)\delta + \lambda(1+\lambda)^2\delta^2} \\ &= \frac{\left[4 + (1 - 6\lambda - 3\lambda^2)\delta + \lambda(1+\lambda)^2\delta^2\right] - \left[2 + (1 - 3\lambda - 2\lambda^2)\delta + (1+\lambda)\lambda^2\delta^2\right]}{4 + (1 - 6\lambda - 3\lambda^2)\delta + \lambda(1+\lambda)^2\delta^2} \\ &= \frac{2 - (3\lambda + \lambda^2)\delta + \lambda(1+\lambda)\delta^2}{4 + (1 - 6\lambda - 3\lambda^2)\delta + \lambda(1+\lambda)^2\delta^2} = \frac{2 - \lambda(3+\lambda)\delta + \lambda(1+\lambda)\delta^2}{4 + (1 - 6\lambda - 3\lambda^2)\delta + \lambda(1+\lambda)^2\delta^2} \end{aligned}$$

and

$$\begin{aligned} 1 + \widehat{r} &= 1 + \frac{2 + (1 - 3\lambda - 2\lambda^2)\delta + (1+\lambda)\lambda^2\delta^2}{4 + (1 - 6\lambda - 3\lambda^2)\delta + \lambda(1+\lambda)^2\delta^2} \\ &= \frac{\left[4 + (1 - 6\lambda - 3\lambda^2)\delta + \lambda(1+\lambda)^2\delta^2\right] + \left[2 + (1 - 3\lambda - 2\lambda^2)\delta + (1+\lambda)\lambda^2\delta^2\right]}{4 + (1 - 6\lambda - 3\lambda^2)\delta + \lambda(1+\lambda)^2\delta^2} \\ &= \frac{6 + (2 - 9\lambda - 5\lambda^2)\delta + \lambda(1+\lambda)(1+2\lambda)\delta^2}{4 + (1 - 6\lambda - 3\lambda^2)\delta + \lambda(1+\lambda)^2\delta^2} = \frac{6 + (2+\lambda)(1-5\lambda)\delta + \lambda(1+\lambda)(1+2\lambda)\delta^2}{4 + (1 - 6\lambda - 3\lambda^2)\delta + \lambda(1+\lambda)^2\delta^2}. \end{aligned}$$

Thus,

$$\int_{\widehat{r}}^{1} r dr = \frac{\left[2 - \lambda(3+\lambda)\delta + \lambda(1+\lambda)\delta^2\right]\left[6 + (2+\lambda)(1-5\lambda)\delta + \lambda(1+\lambda)(1+2\lambda)\delta^2\right]}{2\left[4 + (1 - 6\lambda - 3\lambda^2)\delta + \lambda(1+\lambda)^2\delta^2\right]^2}.$$

We can also write

$$\begin{aligned} \int_{p_1}^{\widehat{r}} r dr &= \frac{\widehat{r}^2 - p_1^2}{2} = \frac{(\widehat{r} - p_1)(\widehat{r} + p_1)}{2} \\ &= \frac{\delta\left[(1+\lambda)\widehat{r} - \lambda\right]\left[4\widehat{r} - \delta\left[(1+\lambda)\widehat{r} - \lambda\right]\right]}{8} \\ &= \frac{\delta(1-\lambda)\left[2 + (1 - 2\lambda - \lambda^2)\delta\right]\left[8 + 2(1 - 5\lambda - 4\lambda^2)\delta - (1 - 3\lambda - 3\lambda^2 - 3\lambda^3)\delta^2\right]}{8\left[4 + (1 - 6\lambda - 3\lambda^2)\delta + \lambda(1+\lambda)^2\delta^2\right]^2}, \end{aligned}$$

where the second equality uses eq. (7) in the paper. Finally, we have

$$\int_{p_2^L}^{1} r dr = \frac{1 - \left(p_2^L\right)^2}{2} = \frac{\left(1 - p_2^L\right)\left(1 + p_2^L\right)}{2}.$$

Note that, using (6) in the paper,

$$\begin{aligned} 1 - p_2^L &= 1 - \frac{\lambda + (1-\lambda)\widehat{r}}{2} = \frac{2 - \lambda - (1-\lambda)\widehat{r}}{2} \\ &= \frac{(2-\lambda)\left[4 + (1 - 6\lambda - 3\lambda^2)\delta + \lambda(1+\lambda)^2\delta^2\right] - (1-\lambda)\left[2 + (1 - 3\lambda - 2\lambda^2)\delta + (1+\lambda)\lambda^2\delta^2\right]}{2\left[4 + (1 - 6\lambda - 3\lambda^2)\delta + \lambda(1+\lambda)^2\delta^2\right]} \\ &= \frac{2(3-\lambda) + (1 - 9\lambda - \lambda^2 + \lambda^3)\delta + 2\lambda(\lambda+1)\delta^2}{2\left[4 + (1 - 6\lambda - 3\lambda^2)\delta + \lambda(1+\lambda)^2\delta^2\right]} \end{aligned}$$

and

$$
\begin{aligned}
1+p_2^L &= 1+\frac{\lambda+(1-\lambda)\widehat{r}}{2}=\frac{2+\lambda+(1-\lambda)\widehat{r}}{2} \\
&= \frac{(2+\lambda)\left[4+\left(1-6\lambda-3\lambda^2\right)\delta+\lambda\left(1+\lambda\right)^2\delta^2\right]+(1-\lambda)\left[2+\left(1-3\lambda-2\lambda^2\right)\delta+(1+\lambda)\lambda^2\delta^2\right]}{2\left[4+\left(1-6\lambda-3\lambda^2\right)\delta+\lambda\left(1+\lambda\right)^2\delta^2\right]} \\
&= \frac{2\left(5+\lambda\right)+\left(3-15\lambda-11\lambda^2-\lambda^3\right)\delta+2\lambda\left(\lambda+1\right)\left(2\lambda+1\right)\delta^2}{2\left[4+\left(1-6\lambda-3\lambda^2\right)\delta+\lambda\left(1+\lambda\right)^2\delta^2\right]}.
\end{aligned}
$$

So,

$$
\int_{p_2^L}^{1} rdr=\frac{\left[2\left(3-\lambda\right)+\left(1-9\lambda-\lambda^2+\lambda^3\right)\delta+2\lambda\left(\lambda+1\right)\delta^2\right]\left[2\left(5+\lambda\right)+\left(3-15\lambda-11\lambda^2-\lambda^3\right)\delta+2\lambda\left(\lambda+1\right)\left(2\lambda+1\right)\delta^2\right]}{8\left[4+\left(1-6\lambda-3\lambda^2\right)\delta+\lambda\left(1+\lambda\right)^2\delta^2\right]^2}.
$$

In conclusion, we have

$$
\begin{aligned}
W\left(\lambda\right)-C\left(\lambda\right) &= \int_{\widehat{r}}^{1} rdr+\lambda\int_{p_1}^{\widehat{r}} rdr+\delta\int_{p_2^L}^{1} rdr-\frac{c\lambda^2}{2} \\
&= \frac{\left[2-\lambda\left(3+\lambda\right)\delta+\lambda\left(1+\lambda\right)\delta^2\right]\left[6+\left(2+\lambda\right)\left(1-5\lambda\right)\delta+\lambda\left(1+\lambda\right)\left(1+2\lambda\right)\delta^2\right]}{2\left[4+\left(1-6\lambda-3\lambda^2\right)\delta+\lambda\left(1+\lambda\right)^2\delta^2\right]^2} \\
&+\frac{\delta\lambda\left(1-\lambda\right)\left[2+\left(1-2\lambda-\lambda^2\right)\delta\right]\left[8+2\left(1-5\lambda-4\lambda^2\right)\delta-\left(1-3\lambda-3\lambda^2-3\lambda^3\right)\delta^2\right]}{8\left[4+\left(1-6\lambda-3\lambda^2\right)\delta+\lambda\left(1+\lambda\right)^2\delta^2\right]^2} \\
&+\frac{\delta\left[2\left(3-\lambda\right)+\left(1-9\lambda-\lambda^2+\lambda^3\right)\delta+2\lambda\left(\lambda+1\right)\delta^2\right]}{8\left[4+\left(1-6\lambda-3\lambda^2\right)\delta+\lambda\left(1+\lambda\right)^2\delta^2\right]^2} \\
&\times\left[2\left(5+\lambda\right)+\left(3-15\lambda-11\lambda^2-\lambda^3\right)\delta+2\lambda\left(\lambda+1\right)\left(2\lambda+1\right)\delta^2\right]-\frac{c\lambda^2}{2}. \qquad \text{(S3)}
\end{aligned}
$$

For any particular value of δ, we can solve for the value of λ that maximizes the expression in (S3) with the help of numerical methods. The Matlab code that I have used for this can be found in the file "Incognito_code_W_v1.m" (downloadable at `www.johanlagerlof.com`).

References

Johan N. M. Lagerlöf. Surfing Incognito: Welfare Effects of Anonymous Shopping. Mimeo, University of Copenhagen, May 2019.