

Surfing Incognito: Welfare Effects of Anonymous Shopping

Seminar at the University of Minho

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Introduction (1/5)

What is behavior-based price discrimination (BBPD)?

- You interact with a seller over time. The seller does not know your valuation but can observe your period 1 purchase decision.
- Buying is a noisy signal that your valuation is high, so it makes the seller charge higher period 2 price than if you hadn't bought.

Retailers read the cookies kept on your browser or glean information from your past purchase history when you are logged into a site. That gives them a sense of what you search for and buy, how much you paid for it, and whether you might be willing and able to spend more. They alter their prices or offers accordingly. Consumers [...] tend to go apoplectic. But the practice is perfectly legal, and increasingly common—pervasive, even, for some products. [Washington Post, Dec 2010]

How BBPD can be implemented in practice:

- Introductory offers, targeted discount coupons, etc.

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How BBPD can be implemented in practice:

- Introductory offers, targeted discount coupons, etc.

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My two BBPD projects:

- 1 A fraction of the consumers being **naive** (they don't understand that buying now will lead to a higher future price).
 - Still not finished.
- 2 The consumers trying to **hide** the fact that they are returning customers.
 - What I'll talk about today.

Graphical summary of the results of a standard BBPD model

- One firm, zero cost, two periods, consumer val's $r \sim U[0, 1]$.



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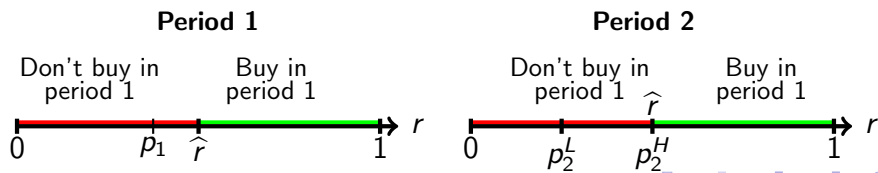
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Introduction (3/5)

- Theoretical work suggests that BBPD can be good for welfare:
 - 1 Price discrimination can increase trade and thus total surplus.
 - 2 BBPD can lead to a "Coase-conjecture effect", thus lower price.
- Still, an individual consumer should want to avoid BBPD, by...
 - setting the browser to reject cookies;
 - using different credit cards or accounts;
 - cancel and then renew a newspaper subscription;
 - refraining from joining loyalty programs; etc.
- My research question:
 - **Will consumers, given other imperfections in the market, tend to use such anonymizing technologies too little or too much (from a total surplus perspective)?**
- To help us think about this question, consider again the graphical summary of the results of the standard BBPD model.
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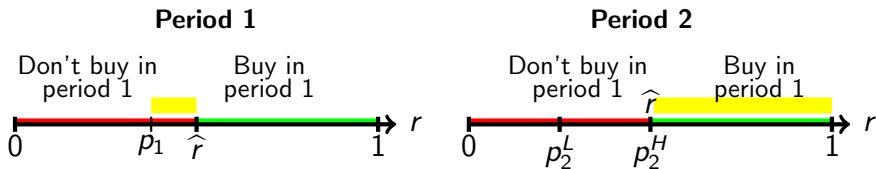
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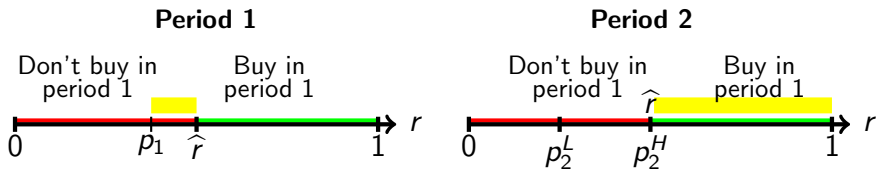
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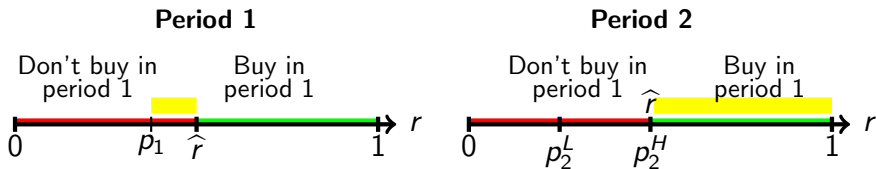
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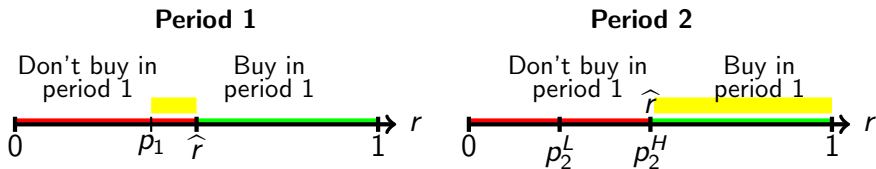
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 - Consumer does not internalize full surplus. [Too weak hiding inc]
 - 2 For $r \in (\hat{r}, 1]$, hiding leads to **lower period 2 consumer price**.
 - Socially irrelevant gain. [Too strong hiding incentive]
 - 3 A consumer ignores effect of hiding choice on period 2 prices.
 - More hiding leads to a smaller price difference, less period 2 price discrimination, and less trade. [Too strong hiding inc]
- Conjecture (and later a result):
 - **Too strong incentive to hide if, and only if, consumers are sufficiently forward-looking.**



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Two main results:

- 1 Consumers have too strong incentive to hide if, and only if, they are sufficiently forward-looking (using a total surplus standard).
- 2 In a version of the model with an exogenous level of hiding: the social optimum involves hiding to some extent, yet not fully.

Closest paper:

- Conitzer, Taylor, and Wagman (2012, Marketing Science).
 - Binary hiding choice—I study a continuous hiding choice.
 - But more importantly: they don't address my welfare questions. They do comparative statics on hiding cost. Neither one of my two main results appears in their paper.

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Model (1/2)

- Two periods (1 and 2). Monopoly **firm** selling nondurable good.
 - No production costs. Discount factor: $\beta \in [0, 1]$.
- A continuum of **consumers**.
 - Valuations $r \sim U[0, 1]$. Mass of consumers: one.
 - Per-period utility: $r - p$ if buying (p is price); otherwise 0.
 - The consumers' (common) discount factor: $\delta \in (0, 1]$.
- The firm cannot observe an individual consumer's r . However, unless the consumer uses an **anonymizing technology**, the firm can keep track of individual consumers' purchase decisions.
- Hiding choice made before knowing own r . Justifications:
 - 1 Using a **long-term approach** for dealing with some privacy issues.
 - 2 Simple **behavioral rule or heuristic** that the consumer uses in a wide range of situations, and which is updated only occasionally.
 - 3 **Analytical shortcut**: No reason to believe that the simplification leads to systematically stronger or weaker incentives to hide.

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Timing

- 1 Consumer chooses own individual $\lambda \in [0, 1]$, at cost $C(\lambda)$.
 - $\Pr[x = I] = \lambda$ (incognito), $\Pr[x = N] = 1 - \lambda$ (no incognito).
 - Consumer observes $x \in \{I, N\}$.
 - Neither λ nor x observed by the firm or by the other consumers.
- 2 Firm chooses first-period price $p_1 \geq 0$ (observed by all).
- 3 Consumer privately learns own r . Decides whether to buy or not.
 - If she buys and if $x = N$, then an indicator variable $t = H$.
 - Otherwise, $t = L$.
- 4 Firm chooses two second-period prices: $p_2^L \geq 0$ and $p_2^H \geq 0$.
 - The price p_2^t must be paid by those consumers with $t \in \{L, H\}$.
- 5 Consumers observe p_2^L and p_2^H and then choose whether to buy.

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Analysis: Exogenous hiding probability (1/3)

- Start by solving model with an **exogenous** $\lambda \in [0, 1)$.
- An eq. is characterized by a threshold $\hat{r} \in (0, 1)$ such that a non-incognito consumer buys in period 1 iff $r \geq \hat{r}$.



- A non-incognito consumer with valuation r has a (weak) incentive to buy in period 1 if, and only if,

$$r - p_1 + \delta \max \{0, r - p_2^H\} \geq \delta \max \{0, r - p_2^L\}. \quad (1)$$

- For $r = \hat{r}$, (1) holds with equality.
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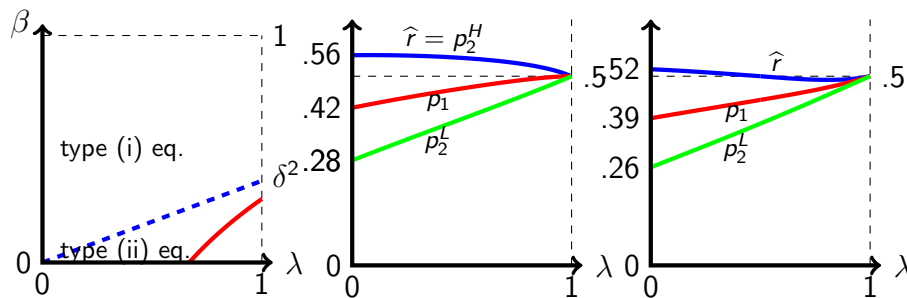


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Analysis: Exogenous hiding probability (2/3)



(a) Param'r space.

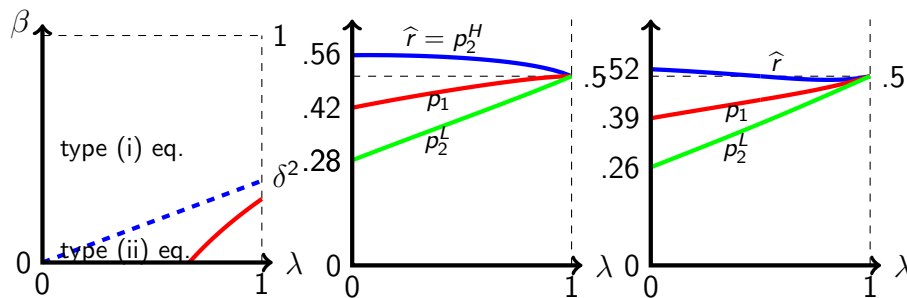
(b) $\beta = \delta = \frac{1}{2}$.

(c) $\beta = \frac{1}{8}, \delta = \frac{1}{2}$.

■ Example with $\beta = \delta = 1$ (which is case (i)):

$$p_2^L = p_1 = \frac{(3 - \lambda)(1 + \lambda)}{2(5 - \lambda^2)} \quad \text{and} \quad p_2^H = \hat{r} = \frac{3 - \lambda^2}{5 - \lambda^2}.$$

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Analysis: Exogenous hiding probability (3/3)

Welfare analysis

- Assume $\delta = \beta$.
 - \Rightarrow the equilibrium is always of type (i) and $\hat{r} \geq \frac{1}{2}$.
 - Total surplus is well defined in our intertemporal setting.
- Total surplus can be written as

$$W(\lambda) \stackrel{\text{def}}{=} \int_{\hat{r}}^1 r dr + \lambda \int_{p_1}^{\hat{r}} r dr + \delta \int_{p_2^L}^1 r dr, \quad \text{for } \lambda \in [0, 1].$$

- **Result:** The fraction of incognito surfers that maximizes total surplus lies strictly between zero and unity.
- Intuition:
 - If $\lambda \approx 1$, $\hat{r} - p_1$ is close to zero and small direct benefit of incr λ .
 - If $\lambda \approx 0$, $\hat{r} - p_1$ is large and there is a big direct benefit of incr λ .

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Analysis: Endogenous hiding probability (1/4)

- Now λ is endogenous.
- Continue assuming $\delta = \beta$.
- Each consumer chooses λ to max. her expected utility (not yet knowing valuation r) and expecting all others to choose $\lambda = \tilde{\lambda}$.
- The consumer's expected utility: $EU(\lambda) = S(\lambda) - C(\lambda)$, where

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Analysis: Endogenous hiding probability (2/4)

- The consumer's marginal utility of an increase in λ :

$$\frac{\partial EU}{\partial \lambda} = \int_{p_1}^{\hat{r}} (r - p_1) dr + \delta \int_{\hat{r}}^1 (p_2^H - p_2^L) dr - C'(\lambda).$$

- First term: Gain from extra consumption in first period.
- Second term: Gain from lower price in second period.
- Social welfare: $W(\lambda) - C(\lambda)$. Society's marginal benefit:

$$\frac{\partial W}{\partial \lambda} = -\hat{r} \frac{\partial \hat{r}}{\partial \lambda} + \lambda \left[\hat{r} \frac{\partial \hat{r}}{\partial \lambda} - p_1 \frac{\partial p_1}{\partial \lambda} \right] - \delta p_2^L \frac{\partial p_2^L}{\partial \lambda} + \int_{p_1}^{\hat{r}} r dr.$$

- Last term: Gain from extra consumption in first period.
- Remaining terms: Indirect effects via prices and cutoff value.
- Heuristic approach: To understand sign of externality, focus on **direct effects** (indirect effects dealt with by me in the algebra).

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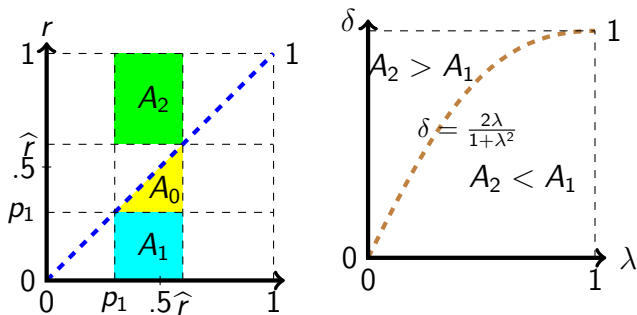
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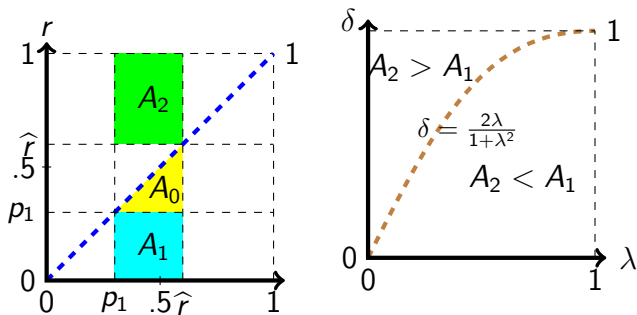
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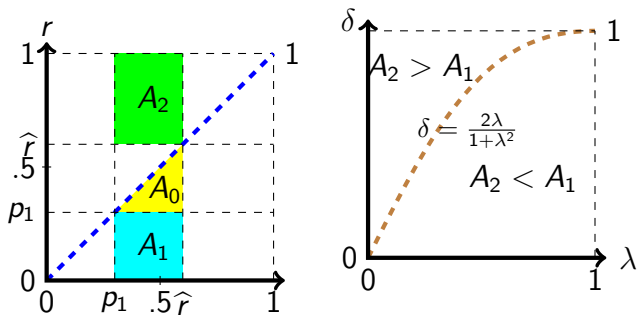
- Last term: Gain from extra consumption in first period.
- Remaining terms: Indirect effects via prices and cutoff value.
- Heuristic approach: To understand sign of externality, focus on **direct effects** (indirect effects dealt with by me in the algebra).



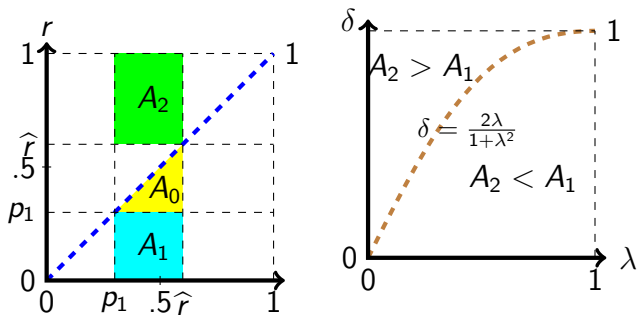
- Society's direct marginal benefit: $\int_{p_1}^{\hat{r}} r dr = A_0 + A_1$.
- Consumer's marginal benefit: $A_0 + A_2$.
 - A_2 because of this eq relationship: $\hat{r} - p_1 = \delta(p_2^H - p_2^L)$.
- So, too strong (direct) hiding inc. iff $A_2 > A_1$.
- Figure on the right suggests:
 - Too much hiding if $\delta = 1$. [Proven analytically.]
 - Too little hiding if $\delta \approx 0$. [Shown numerically. (See next slide.)]



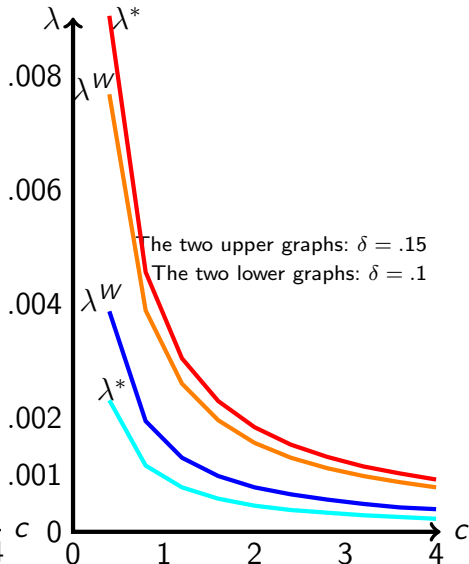
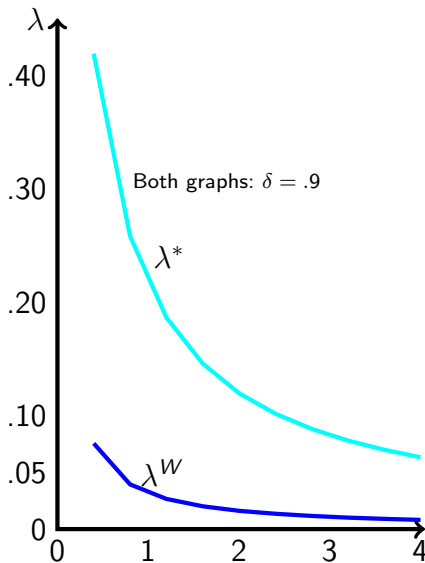
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Conclusions and possible future work (1/1)

Main results and contributions:

- The social optimum requires hiding to some extent, yet not fully.
- Consumers have a too strong incentive to hide if, and only if, they are sufficiently forward-looking (using a TS standard).
- I offer a BBPD model in which we can have $\hat{r} < \frac{1}{2}$ and where some consumers buy in period 1 but not in period 2.
- Broad take-away: Hiding both hinders and facilitates trade, through distinct channels.

Possible extensions:

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2 Counteracting actions taken by the firm.

3 Varying the degree of market power. Multiple firms.

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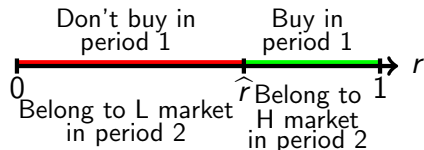
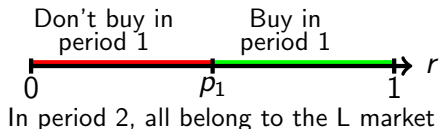
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Appendix: Analysis of exogenous hiding prob (1/3)

Incognito consumers (share λ) **Non-incognito consumers (share $1 - \lambda$)**



Second period H market

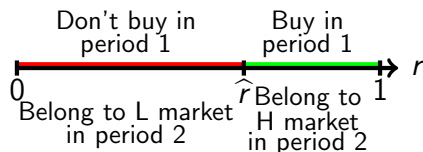
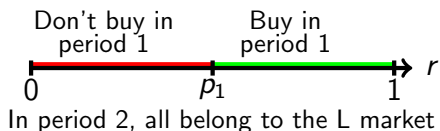
■ Demand:

$$q_2^H = \begin{cases} (1 - \lambda)(1 - p_2^H) & \text{if } p_2^H \in [\hat{r}, 1] \\ (1 - \lambda)(1 - \hat{r}) & \text{if } p_2^H \in [0, \hat{r}]. \end{cases}$$

■ Profits: $\pi_2^H = p_2^H q_2^H$. Optimal price: $p_2^H = \max\left\{\frac{1}{2}, \hat{r}\right\}$.

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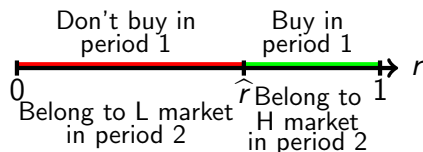
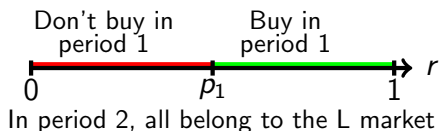
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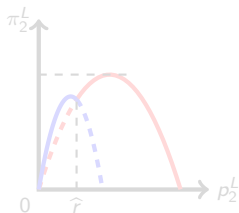
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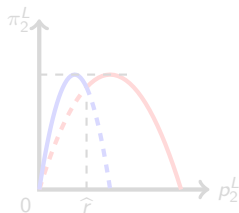
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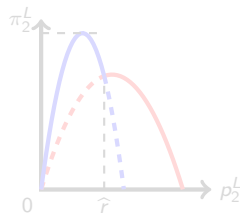
$$p_2^L = \begin{cases} \frac{\lambda + (1 - \lambda)\hat{r}}{2} & \text{if } \hat{r} \in \left[\frac{\sqrt{\lambda}}{1 + \sqrt{\lambda}}, 1 \right] \\ \frac{1}{2} & \text{if } \hat{r} \in \left[0, \frac{\sqrt{\lambda}}{1 + \sqrt{\lambda}} \right]. \end{cases}$$



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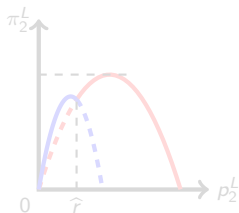
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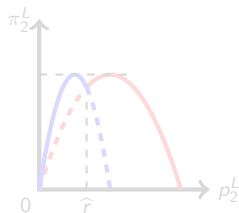
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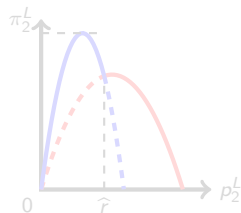
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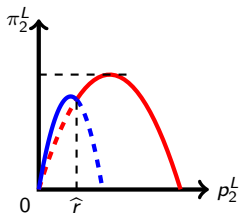
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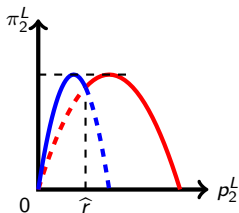
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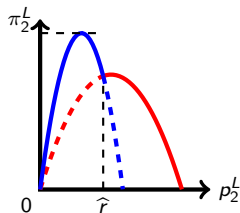
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Appendix: Analysis of exogenous hiding prob (3/3)

- We need to investigate three possibilities:

$$(i) \hat{r} \geq \frac{1}{2}; \quad (ii) \hat{r} \in \left(\frac{\sqrt{\lambda}}{1 + \sqrt{\lambda}}, \frac{1}{2} \right); \quad (iii) \hat{r} \leq \frac{\sqrt{\lambda}}{1 + \sqrt{\lambda}}.$$

- Case (iii) can't be an eq.
- But (i) and (ii) can.

- Case (i) yields eq. values:

$$\hat{r} = \frac{2 - \delta(1 + \lambda)(1 + \lambda - \delta\lambda^2) + \beta(1 - \lambda)(2 + \lambda)}{4 - \delta(1 + \lambda)^2(2 - \delta\lambda) + \beta(1 - \lambda)(3 + \lambda)}. \quad (2)$$

$$\text{and } p_2^L = \frac{1}{2} [\lambda + (1 - \lambda)\hat{r}] \text{ and } p_1 = \hat{r} - \frac{\delta}{2} [(1 + \lambda)\hat{r} - \lambda].$$

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