Bertrand under Uncertainty: Private and Common Costs

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Abstract

This paper proposes an n-firm homogeneous-good Bertrand model with private information about costs. The model allows for any non-negative correlation between the cost draws and for any demand elasticity but still yields a closed-form solution. The solution is simple, in pure strategies, and involves price dispersion. For some parameter values, a weak version of the winner's curse arises. This framework is used to study the question whether cost uncertainty softens competition. Earlier literature has shown that the answer (perhaps counter-intuitively) is "no," while assuming (i) independent cost draws and (ii) no drastic innovations. The analysis here shows that relaxing (ii) but not (i) does not alter that result. However, when the cost draws are sufficiently highly correlated and the price elasticity of demand is sufficiently low, cost uncertainty indeed softens competition.

Keywords: Bertrand competition, Hansen-Spulber model, private information, information sharing, common values, private values, winner's curse

JEL classification: D43 (Oligopoly and Other Forms of Market Imperfection), D44 (Auctions), L13 (Oligopoly and Other Imperfect Markets)

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1 Introduction

The Bertrand model of price competition predicts that price equals marginal cost and that firms earn zero profit—a result which is often referred to as the Bertrand paradox, as it suggests that the presence of only two firms is sufficient to eliminate all market power and give rise to the perfectly competitive outcome. The paradox has prompted a number of scholars to study extensions and variations of Bertrand's original model. By doing that they have identified several model features that, if added to the standard setup, resolves the paradox by providing some amount of market power to the firms. Examples of such features include product differentiation, capacity constraints, repeated interaction, and cost asymmetries between firms.

In an interesting paper, Spulber (1995) studies another, empirically very plausible, variation of the standard Bertrand model, namely to assume that each price-setting firm has private information about some characteristic of its production technology, a leading example of which is the firm's (constant) marginal cost.[1] Spulber shows that in that setting there is a unique and symmetric equilibrium price, which is increasing in the own marginal cost. Importantly, the equilibrium price lies strictly between the marginal cost and the monopoly price, which means that the firms have some market power and earn a positive profit.[2]

What exactly is the model feature that gives rise to that outcome, thus solving the Bertrand paradox? An answer that naturally comes to mind is that it is asymmetric information (or uncertainty more generally), and this is exactly what Spulber (1995) suggests (p. 10, emphasis added):

> Asymmetric information thus plays an important role in imperfect competition. In [the model] studied here, the surgical precision that is required to price slightly below [...] higher cost rivals is eliminated by the lack of exact knowledge about the characteristics of the rivals. In the short run, with market structure fixed, *asymmetric information appears to reduce competition*[...].

It is not only Spulber himself who interprets his results in this fashion, so do several other authors. For example, Spiegel and Tookes (2008, p. 33, fn. 33) write that "Spulber (1995) also shows how, in Bertrand competition, not knowing rivals' costs implies equilibrium prices that are above marginal costs (i.e., information asymmetry softens product market competition)."[3]

[1]The model that Spulber studies has also found its way into textbooks—see Wolfstetter (1999, pp. 236-37) and Belleflamme and Peitz (2015, pp. 47-49). Arozamena and Weinschelbaum (2009) study a sequential version of Spulber's model and compare with the simultaneous-move version. Lofaro (2002) obtains a closed-form solution of Spulber's model by assuming a uniform distribution, and he then compares the price competition outcomes with the quantity competition outcomes. Athey (2002, p. 198) generalizes some of Spulber's results in a number of directions, allowing for, among other things, asymmetric cost distributions. Abbink and Brandts (2007) test Spulber's model in the laboratory.

[2]The firm that draws the lowest marginal cost (and thus charges the lowest price) earns a positive profit ex post. The other firms earn a zero profit ex post, but their expected profit, at the stage before they have learned their cost parameter, is positive.

[3]See also Wolfstetter (1999), who presents two simple versions of the model, one with inelastic and one with elastic demand. He introduces the analysis of these models by stating that (p. 236): "A much simpler

But is it really asymmetric information (or uncertainty) that softens competition? When Spulber assumes uncertainty about the firms' marginal cost parameters, he implicitly also introduces the assumption that these parameters may differ from each other. This means that, in principle, the model feature that creates market power could be cost heterogeneity and not uncertainty. Moreover, it actually follows from relatively early work of Hansen (1988) that, at least for a special case of Spulber's (1995) model, it is indeed cost heterogeneity that softens product market competition. Uncertainty is, in Hansen's (1988) setting and given the presence of cost heterogeneity, not anti-competitive but pro-competitive—at least in the sense that it lowers expected price and raises expected consumer and total surplus. Uncertainty also makes expected industry profits go up, which of course can be thought of as providing the firms with more market power.

However, Hansen's analysis is carried out under at least two assumptions that seem restrictive and which, if relaxed, conceivably could reverse his results. First, Hansen's setup does not allow for the possibility that a firm makes a "drastic innovation"—that is, that a firm's cost advantage is so large that, under complete information, it can optimally behave like a monopolist. Allowing for drastic innovations would tend to lower the expected price in the complete information model, which may overturn Hansen's results. Second, and most importantly, Hansen's (as well as Spulber's) analysis of the cost uncertainty model assumes that *the cost draws are independent*. Such an assumption indeed appears to be restrictive: Empirically we would expect firms' costs to be at least to some extent positively correlated, as all firms in a market are likely to be affected by external and economy-wide shocks to wages, interest rates, the price of material and energy, etc.

To understand why we should expect Hansen's results to be reversed if we assumed a sufficiently strong positive correlation between the costs, think of the extreme case where all firms in the market share the same (constant marginal) cost. First note that if this common cost is known, we are back to the Bertrand paradox and zero market power. Next suppose the cost is *not* known but each firm observes some private signal about its true value. If we could show that, in this model, the expected equilibrium price lies strictly above the expected marginal cost (which seems plausible), then we would have obtained the result that the firms have some market power. Hence, in this scenario, Spulber (1995) and the other authors would indeed be right when claiming that asymmetric information softens competition and resolves the Bertrand paradox.

In the present paper I investigate the consequences of relaxing the two assumptions of no drastic innovations and independent cost draws. I do this by setting up a homogeneous-good Bertrand model with cost uncertainty and privately observed cost signals. This model is fairly stylized and relies on specific functional forms. Still, the model is sufficiently rich to allow for any given number of firms, any level of demand elasticity, and any level of non-negative correlation between the firms' costs—private costs, common costs, and combinations of those extremes.[4] In addition, the model allows for many different values of the firms' ex ante efficiency

[relative to the model with capacity constraints] resolution of the Bertrand paradox can be found by introducing incomplete information." Yet another paper that refers (twice—on p. 638 and p. 646) to Spulber's (1995) result as a "resolution of the Bertrand paradox" is Abbink and Brandts (2007).

[4]The case with private costs and downward-sloping demand is in turn a special case of the model studied by

level (i.e., a parameter that measures their common likelihood of drawing a relatively low cost signal). My model yields a simple, closed-form solution.[5] This solution is in pure strategies and involves price dispersion. It also enables a comparison with the outcome of the corresponding complete information model. I show that under the assumption of private costs, the possibility of drastic innovations is not enough to overturn any of Hansen's results. However, when the cost draws are sufficiently highly correlated and the price elasticity of demand is sufficiently low, cost uncertainty indeed softens competition—in the sense of, in expected terms, price and profits being higher and consumer surplus and total surplus being lower. A noteworthy and interesting corollary of the analysis is that, when the cost correlation is high enough, the firms suffer from a weak version of the winner's curse: For some signal realizations, the firm that wins the market incurs a loss. Indeed, the probability of this occurring can, for particular parameter configurations, be arbitrarily close to one.

There is a related literature that studies firms' incentive to share information about their own marginal cost parameter under Bertrand competition with *differentiated* goods. Gal-Or (1986) analyses such a model with two firms and independent cost draws. She shows that not to share information is a dominant strategy for each duopolist. Raith (1996) considers the more general case with n firms and does allow for correlated cost draws.[6] Also Raith, however, assumes differentiated goods. Moreover, neither Gal-Or (1986) nor Raith (1996) makes any comparison between market outcomes with complete and incomplete information. Other related literature includes Milgrom and Weber's (1982) seminal analysis of auctions with affiliated values. Most of their paper concerns the case where bidders are risk neutral and can purchase one unit of a good. However, Section 8 of their paper also considers the case with risk averse bidders, which is similar in spirit to a model with downward-sloping demand. Milgrom and Weber show that, "for models that include both affiliation and risk aversion, the first- and second-price auctions [which, in my setting, correspond to incomplete and complete information] cannot generally be ranked by their expected prices" (p. 1114). The present paper assumes a particular oligopoly setting and then compares the two expected prices, for various values of the parameters that measure demand elasticity and cost correlation. It also makes comparisons of other entities, such as consumer surplus and total surplus (which have no natural equivalent in the Milgrom-Weber model), across the two informational regimes.

Spulber (1995).

[5] Wolfstetter (1999), Lofaro (2002), Abbink and Brandts (2007), and Belleflamme and Peitz (2015) all derive closed-form solutions in a setting with a uniform cost distribution on $[0, 1]$ and linear demand. (In their experimental study, Abbink and Brandts (2007) actually assume a uniform distribution on $[0, 99]$; but this specification is equivalent to the one in the other three papers, although with another scaling of the units in which output and cost are measured.) Wolfstetter (1999) studies a version of the model with a uniform cost distribution on $[0, 1]$ and an inelastic demand and also derives a closed-form solution (this is effectively a standard private-value first-price auction model). The specification used in the present paper is a substantial generalization of all the above models.

[6] In that environment, Raith reports, Gal-Or's result can be reversed. As shown by Jin (2000), however, that particular finding in Raith's paper is based on an algebraic error: in fact, Gal-Or's result extends also to Raith's setting with an arbitrary number of firms. (In his survey of the information sharing literature, Vives (1999, Section 8.3.1) also reports the incorrect result.)

2 Hansen's (1988) results

This section starts out by discussing how we can disentangle the effect of uncertainty on market power from the one of cost heterogeneity, by comparing Spulber's (1995) model to a complete information Bertrand model with heterogenous costs. It also provides a review of Hansen's (1988) model and his results.

As explained in the introduction, Spulber (1995) studies a standard one-shot, homogeneous-good Bertrand model with n ex ante identical firms, but adds the assumption that each firm has private information about some characteristic of its production technology—for example, the firm's constant marginal cost. The cost draws in Spulber's model are independent. He shows that in that environment the equilibrium price lies strictly above the marginal cost. Our first goal is to understand whether it is the uncertainty as such that creates market power for the firms in Spulber's model. To that end, it is useful to note that when adding private information about the marginal cost, Spulber makes *two* assumptions:

A1. The firms' marginal cost parameters may differ from each other.

A2a. Each firm has private information about its own marginal cost parameter.

In order to assess the role of asymmetric information, we can compare the outcome of Spulber's model with the outcome of a benchmark that also makes assumption A1 but replaces A2a with[7]

A2b. The (possibly different) marginal cost parameters are common knowledge among the firms.

Assumptions A1 and A2b give rise to a standard variation of the Bertrand setup, discussed in many textbooks. The equilibrium outcome of that model is that the lowest-cost firm wins the whole market and charges a price equal to the minimum of the monopoly price and the marginal cost of the firm with the second-lowest cost draw.[8] That is, also this model with complete information but cost heterogeneity gives rise to a market price above marginal cost and a positive profit for the lowest-cost firm. A first conclusion is thus that, also within Spulber's framework, asymmetric information is not *required* for the firms (or at least one firm) to have market power. A more interesting question, however, is whether the amount of market power in the model A1+A2b is less than that in the model A1+A2a. That is, in this environment with independent costs, does asymmetric information soften competition, as Spulber (1995) and the other authors cited in the introduction suggest?

[7]In fact, Spulber (1995) does compare these two models in Section III of his paper, but not in terms of their competitiveness.

[8]In order to sustain this behavior as part of a Nash equilibrium, some textbooks assume a particular sharing rule that says that the more efficient firm gets all the demand if both firms charge the same price. Making such an assumption is not necessary, however: If the less efficient firm uses a mixed strategy, then the outcome can be sustained as part of an equilibrium under a standard sharing rule; see Blume (2003).

Thanks to work of Hansen (1988), we actually know that, at least for a special case[9] of Spulber's (1995) model, the answer to the above question is "no." Hansen's model is couched in terms of a procurement auction in which two firms bid for the right to serve a market with a downward-sloping demand, and within that framework he compares the outcomes of an open (descending) auction and a (first-price) sealed bid auction. The open auction is effectively a Bertrand game with complete information (i.e., A1+A2b), whereas the sealed bid auction is the same as Spulber's (1995) incomplete information model (i.e., A1+A2a). Hansen (1988) shows that the sealed bid auction yields a lower expected price than the open auction. It also yields a higher expected total surplus. Under a somewhat stronger assumption about the demand function, Hansen can also show that the sealed bid auction yields a higher expected profit for the firms and a higher expected consumer surplus, meaning that both consumers and firms are better off under incomplete information.

It is instructive to look at what broad arguments Hansen (1988) uses when proving the result that the sealed bid auction yields a lower expected price than the open auction. First he notes that in an open auction the equilibrium price strategies are the same regardless of whether the quantity is variable or fixed (or, in oligopoly language, whether demand is elastic or inelastic): In either case, the lowest-cost firm can win the whole market with a price that equals the marginal cost of the firm with the second-lowest cost. This yields equality (a) in (1):

$$\mathbb{E}\left[p \mid \text{open, variable}\right] \overset{(a)}{=} \mathbb{E}\left[p \mid \text{open, fixed}\right] \overset{(b)}{=} \mathbb{E}\left[p \mid \text{sealed, fixed}\right] \overset{(c)}{>} \mathbb{E}\left[p \mid \text{sealed, variable}\right]. \tag{1}$$

The argument that yields equality (a) relies critically on Hansen's assumption that the lowest-cost firm's optimal monopoly price always exceeds the marginal cost of the firm with the second-lowest cost (i.e., a "drastic innovation" must not be possible); without that assumption, the equality would be replaced by a "<"-sign and Hansen's proof would no longer be valid.

Next, Hansen invokes the revenue equivalence theorem, which in this model (and fairly generally) says that in a fixed-quantity auction the expected revenue (which equals the expected price) is the same regardless of whether the auction is sealed-bid or open.[10] This is equality (b) in (1). Finally Hansen shows that, in a sealed bid auction, the equilibrium price must be lower when the quantity is variable compared to when it is fixed (inequality (c) in (1)). The intuition for this result is straightforward. In the sealed bid auction, if the firm raises its price, it will have a higher profit if it still wins the market, but the probability of winning has decreased. The optimal price balances those two effects. However, the former (positive) effect is smaller when demand is downward-sloping, as the higher price then leads to a loss of sales. Therefore the expected price must be lower when demand is elastic.

[9] Hansen (1988) assumes that the firms have a constant returns to scale technology and that the uncertainty concerns each firm's marginal cost parameter, which is only one of the possibilities that Spulber (1995) considers. As Spulber, Hansen assumes that the cost draws are independent. Hansen also assumes a duopoly, whereas Spulber allows for an arbitrary number of firms. Finally, Hansen assumes that the support of the unknown marginal cost parameter is such that the monopoly cost always lies above the marginal cost of the second most efficient firm—an assumption that Spulber does not need to make, given the comparisons he makes in his paper.

[10] On the revenue equivalence theorem, see for example Klemperer (1999) or Krishna (2002).

Jointly, the three steps (a), (b), and (c) yield the desired result that the expected price in the sealed bid auction with a variable quantity is lower than the expected price in the open auction with a variable quantity—or, in other words, cost uncertainty in the Bertrand model intensifies competition in the sense that it lowers the expected market price. While step (c) appears to be quite robust, it has already been noted that step (a) relies critically on Hansen's assumption about the support of the cost distribution from which the firms draw their costs. If we allowed for the possibility that the winning firm has such a large cost advantage that it sometimes, under complete information, optimally charges its monopoly price, then the expected price in the open auction with a variable quantity would be lower than the expected price in the open auction with a fixed quantity. This could conceivably also reverse the result that cost uncertainty in the Bertrand model intensifies competition. In the model to be studied in Section 3, the cost advantage of the most efficient firm will indeed sometimes be so large that it corresponds to a drastic innovation.

It is also clear from the above reasoning that Hansen's proof relies on the revenue equivalence theorem (step (b)). In an environment where that theorem does not hold, the proof will not be valid and it is again conceivable that the result could be reversed. One such environment is a common value auction (or, in oligopoly language, a model with common costs). It is known that in such an auction we have $\mathbb{E}\left[p \mid \text{open, fixed}\right] < \mathbb{E}\left[p \mid \text{sealed, fixed}\right]$;[11] that is, equality (b) is replaced by a "$<$"-sign. Whether Hansen's result is reversed would then depend on the relative magnitudes of the inequalities (b) and (c), as they point in different directions. However, if we let the demand elasticity in the model with a variable quantity be very low, we should expect inequality (c) to be very close to an equality, without this affecting the reverse inequality (b). Hence, at least for a sufficiently low demand elasticity and for "sufficiently common" costs, we should expect Hansen's result to be reversed. This reasoning is indeed the logic behind the main result to be presented in the next section.

3 Price competition with and without private information

3.1 The model

There are $n \geq 2$ risk neutral and profit-maximizing firms that compete à la Bertrand in a homogeneous-good product market. The firms are ex ante identical, they choose their prices simultaneously, and they interact only once. Market demand is given by $D\left(p\right) = \left(1-p\right)^{\epsilon}$, where p denotes price and $\epsilon \geq 0$ is an exogenous parameter. This specification implies that, for a given price p, the parameter ϵ is proportional to the price elasticity of demand: the larger is ϵ, the more elastic is demand (and for $\epsilon = 0$, demand is completely inelastic).[12] The firm that charges the lowest price, denoted by $p_{\min}$, sells the quantity $D\left(p_{\min}\right)$, while the rival firms sell nothing and make a zero profit. If two or more firms have chosen the same price, they share the market equally.

Each firm has a production technology that is characterized by constant returns to scale,

[11] See Milgrom and Weber (1982) and Krishna (2002, Ch. 6)

[12] In particular, the price elasticity of demand equals $\eta \overset{\text{def}}{=} -D'(p)p/D(p) = \epsilon p/(1-p)$.

with firm i's marginal cost being denoted by c_i. In particular, I assume that[13]

$$c_i = (1-\alpha)\, s_i + \frac{\alpha}{n-1} \sum_{j \neq i} s_j, \tag{2}$$

where $\alpha \in \left[0, \frac{n-1}{n}\right]$ is a parameter and s_i is firm i's *signal.* The firms' signals are independently drawn from the cumulative distribution function

$$F(s_i) = 1 - (1 - s_i)^x, \tag{3}$$

with support $[0,1]$ and with $x > 0$ being an efficiency parameter: An increase in x makes the firms ex ante more efficient, in the sense that this makes lower signal values more likely.[14]

The above assumptions imply that, for $\alpha = 0$, the firms' cost parameters are independent; this is the standard framework with *private costs* that is used in Hansen (1988) and Spulber (1995). For $\alpha = \frac{n-1}{n}$ the cost parameters are identical (equal to $\frac{1}{n}\sum_{j=1}^{n} s_j$); this is the other polar case: a framework with *common costs.* For values of α in between those two extremes, the costs are not common, although they are still positively correlated.[15]

I will solve, and then compare, two versions of this model: one where the signals s_i (and thus also the firms' cost parameters) are common knowledge and one where each signal s_i is firm i's private information.

3.2 Incomplete information

First consider the incomplete information model. That is, assume that each signal s_i is private information of firm i, although it is common knowledge that c_i is given by (2) and that the s_i's are independently drawn from the distribution in (3). I will look for a symmetric equilibrium strategy $p^{II}(s)$ that is strictly increasing and differentiable in the signal s that the firm observes (the superscript "II" stands for incomplete information). Denote the inverse of this function by $\chi(p)$, meaning that χ is the signal value that would give rise to the price p. Suppose firm i expects all its rivals to follow the equilibrium strategy $p^{II}(\cdot)$. Then, given that firm i has drawn a signal s_i and chooses a price p_i, its expected profit can be written as

$$\mathbb{E}[\pi_i \mid s_i] = (1 - p_i)^{\varepsilon} \left(p_i - \mathbb{E}[c_i \mid \text{firm } i \text{ wins}]\right) \Pr\left[\text{firm } i \text{ wins}\right]. \tag{4}$$

[13] Vincent (1995) uses a similar formulation when studying the incentives to keep reserve prices secret in a common-value auction.

[14] We can think of each firm i in the market as having access to x research laboratories, each of which produces one independent signal draw from a uniform distribution on $[0,1]$; the firm then uses the lowest one of the x draws as its signal. This procedure is equivalent to receiving one single draw from the distribution $F(s_i)$, defined in (3) above. Obviously, however, the analysis does not rely on this interpretation or on x being an integer.

[15] The analysis of the model with incomplete information is valid for all $\alpha \in [0,1)$. However, any $\alpha \in \left(\frac{n-1}{n}, 1\right)$ would mean that, in the incomplete information model, each firm had more precise information about its rivals' costs than about its own cost—a model feature that seems odd. Moreover, with such an assumption, the computations for the complete information model would be more tedious. For simplicity I therefore assume that $\alpha \in \left[0, \frac{n-1}{n}\right]$.

The term $\mathbb{E}[c_i \mid \text{firm } i \text{ wins}]$ is firm i's expected cost, conditional on the signal s_i and on being the cheapest firm in the market. In particular, we can compute

$$\begin{aligned}\mathbb{E}[c_i \mid \text{firm } i \text{ wins}] &= \mathbb{E}[c_i \mid p_i < p_j \text{ all } j \neq i] \\ &= (1-\alpha)s_i + \alpha\left(\frac{1+x\chi(p_i)}{1+x}\right) \end{aligned} \tag{5}$$

(see Lagerlöf, 2016, for a derivation).[16] The last term in eq. (4), $\Pr[\text{firm } i \text{ wins}]$, is the probability with which firm i is indeed the cheapest firm in the market and all the rivals thus having drawn signals that exceed $\chi(p_i)$:

$$\Pr[\text{firm } i \text{ wins}] = [1 - F(\chi(p_i))]^{n-1}. \tag{6}$$

When firm i chooses p_i to maximize (4), it trades off its desire to charge a low price in order to win the market against its desire to set a relatively high price that ensures that it earns a large profit in case it does win the market. We have the following result.

Proposition 1. ***(Equilibrium)*** *The following is an equilibrium strategy:*

$$p^{II}(s_i) = A + (1-A)s_i, \qquad \textit{where} \qquad A \stackrel{def}{=} \frac{1-\alpha}{1+\epsilon+(n-1)x} + \frac{\alpha}{1+x}. \tag{7}$$

Proof. See Appendix.

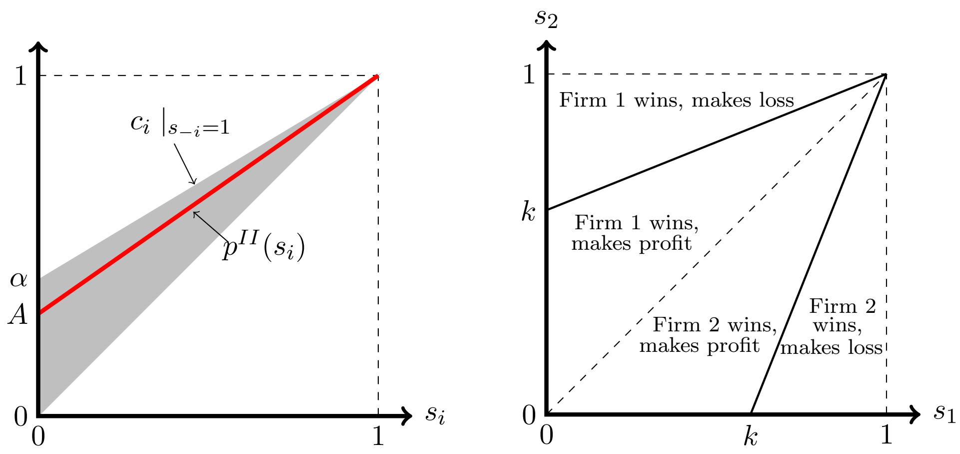
The model thus has an equilibrium that is quite simple, with each firm charging a price that is linear (or affine) in its own signal.[17] See panel (a) of Fig. 1 for an illustration. This linear equilibrium strategy satisfies $p^{II}(1) = 1$, and for all $s_i < 1$ the firm chooses a price strictly above its observed signal (so above the 45-degree line in the figure). Therefore, since the intercept of (7) is strictly decreasing in ϵ, the whole price schedule moves downwards as ϵ increases: As demand becomes more elastic, price drops.[18] Similarly, the price schedule is decreasing in x and in n: As the likelihood that the rivals have drawn low signal values increases, and as the number of firms increases, price goes down. Finally, except for the knife-edge case $(\epsilon, n) = (0, 2)$, the price schedule is strictly increasing in α: As the common costs component becomes more important, price goes up. (For the special case with an inelastic demand and a duopoly market, $(\epsilon, n) = (0, 2)$, the equilibrium price is independent of α.)

In this model a firm's ex post equilibrium profit can, if the common cost component is large enough, be strictly negative. See again panel (a) of Fig. 1 for an illustration. At the time when

[16] In order to understand the last term in (5), consider the special case $x = 1$, meaning that the signal distribution is uniform. The expected value of a rival's signal, conditional on this signal not being lower than the own signal $\chi(p_i)$, is then given by the mid-point of the interval between $\chi(p_i)$ and one, which equals $(1 + \chi(p_i))/2$.

[17] As mentioned in the Introduction, this result is a generalization of the models in the existing literature that yield a closed-form solution. By setting $(\alpha, \epsilon, x) = (0, 1, 1)$, we obtain the specifications used in Lofaro (2002), Belleflamme and Peitz (2015), and the elastic-demand model studied by Wolfstetter (1999, p. 237). By setting $(\alpha, \epsilon, x) = (0, 0, 1)$ we obtain the specification in the inelastic-demand model used by Wolfstetter (1999, p. 236).

[18] One can verify that the price elasticity of demand is indeed increasing in the parameter ϵ, also when taking into account the effect that goes through the equilibrium price. That is, $\eta[p^{II}(s_i)] = \epsilon p^{II}(s_i)/[1 - p^{II}(s_i)]$ is strictly increasing in ϵ.



(a) The equilibrium price. (b) Profit outcomes for firm that wins market.

Figure 1: Panel (a) shows the equilibrium price (thick, red line). If choosing a price in the shaded area, a firm makes a loss with positive probability. Panel (b), which assumes $n = 2$ and that inequality (8) holds, shows where in the signal space each one of the two firms wins the market and indeed makes a loss and a profit, respectively. Note: $k \stackrel{\text{def}}{=} \frac{1+x+\epsilon\alpha}{(1+x+\epsilon)(1+x)\alpha}$.

firm i chooses its price, it knows its own signal and thus the relevant location on the horizontal axis in the figure. But firm i's cost depends also on the rivals' signals, which it does not know. Therefore the cost may, for a given horizontal position, be located anywhere in the shaded area in the figure. The upper edge of the shaded area corresponds to the worst-case scenario where all other signals equal one. The lower edge of the area (i.e., the 45-degree line) represents the best-case scenario where all other signals equal firm i's signal. (One or more of the other signals can of course be lower than that—but if so, in a symmetric equilibrium, one of the rival firms wins the market and firm i does not have to incur any cost.) If the equilibrium strategy intersects the shaded area, there exist signal realizations of the rivals for which firm i makes a loss. As is evident from the figure, this happens exactly when the parameter α exceeds A, the intercept of the equilibrium strategy. Panel (b) of Fig. 1 illustrates, for the duopoly case, the fact that the winning firm tends to make an ex post loss when the rival's signal is large relative to the own signal.

The following corollary spells out under what circumstances we indeed have $\alpha > A$. It also, for the duopoly case, states the ex ante likelihood of a loss to occur.

Corollary 1. ***(Ex post loss)*** *There exist signals $(s_1, \ldots, s_n) \in [0,1]^n$ such that the winning firm earns a strictly negative profit if, and only if, the following condition holds:*

$$\alpha > A \Leftrightarrow \alpha > \frac{1+x}{1+[2+(n-1)x+\epsilon]\,x}. \tag{8}$$

Moreover, suppose $n = 2$ and that inequality (8) is satisfied. Then the ex ante probability that the firm that wins the market earns a strictly negative profit equals

$$\left[\frac{[(1+x)^2+x\epsilon]\alpha-(1+x)}{(1+x+\epsilon)(1+x)\alpha}\right]^x, \tag{9}$$

which is strictly increasing in both α and ϵ.

Proof. See Appendix.

By inspecting (8), we can conclude that an ex post loss cannot occur in this model if, for example, either (i) costs are private ($\alpha = 0$) or (ii) demand is inelastic, there is a duopoly and the signal distribution is uniform ($(\epsilon, n, x) = (0, 2, 1)$). However, such a loss is always incurred for some signal realizations if (i) costs are common ($\alpha = \frac{n-1}{n}$) or (ii) if costs are not fully private and the elasticity and/or the number of firms and/or the efficiency parameter are large enough (so if $\alpha > 0$ and at least one of n, ϵ, and x is sufficiently large).

Inspecting (9) we can also note that, assuming a duopoly, the ex ante probability that the firm that wins the market makes a loss is increasing in α and ε. In fact, for the case with common costs ($\alpha = \frac{1}{2}$), this probability can be made arbitrarily close to one by choosing ϵ large and x small.[19] The result that the winning firm makes an ex post loss is, of course, fully consistent with this firm being an expected profit maximizer—a loss is possible because the firm, when choosing its price, does not know all the components of its cost. However, the firm's interim or ex ante expected profit cannot be negative. (For a discussion of the winner's curse ex ante, interim and ex post, see Cox and Isaac (1984).)

3.3 Complete information

Consider now a Bertrand model that is identical to the one above, except that the signals $(s_1, \dots, s_n)$, and thus also the marginal cost parameters of all the firms, are common knowledge. This is a model that is often analyzed in textbooks; as already discussed in Section 2, the equilibrium outcome is that the firm with the lowest cost draw serves the whole market and charges a price that equals either the second most efficient firm's marginal cost or, if that is lower, the optimal monopoly price:

$$p^{CI} = \min\left\{c_{(2)}, \frac{1 + \epsilon c_{(1)}}{1 + \epsilon}\right\},$$

where the superscript "CI" is short for complete information and the subscript (j) indicates the jth order statistic (i.e., the jth lowest realization among the n random variables).

In general, in this model, the optimal monopoly price may indeed be lower than the nearest rival's marginal cost. However, it is clear that this cannot happen if demand is inelastic ($\epsilon = 0$). Nor can it occur for a number of other parameter configurations, as the following lemma establishes.

Lemma 1. ***(No drastic innovations)*** *Suppose the following condition holds:*

$$\alpha \geq \frac{(n-1)\epsilon}{1 + n\epsilon}. \tag{10}$$

Then the second most efficient firm's marginal cost does not exceed the most efficient firm's optimal monopoly price, which means that $p^{CI} = c_{(2)}$.

Proof. See Appendix.

[19] Given $\alpha = \frac{1}{2}$, the probability approaches $\left(\frac{x}{1+x}\right)^x$ as $\epsilon \to \infty$. The probability $\left(\frac{x}{1+x}\right)^x$, in turn, equals one if $x = 0$ and it approaches $e^{-1} \approx 0.37$ as $x \to \infty$.

3.4 Comparison

In this subsection I will compare the equilibrium outcomes of the models in subsections 3.2 and 3.3 with regard to market price, consumer surplus, profits, and total surplus. The comparisons are made from an ex ante point of view, that is, in expected terms at the stage where the firms have observed neither their own nor any of their rivals' signals.

As argued in the Introduction and in Section 2, the model studied in the present paper could conceivably overturn Hansen's (1988) results also if costs are private (i.e., with $\alpha = 0$). (If this were the case, the reason would be that the signal distribution in the present model allows for a drastic innovation.) I therefore start out by comparing the two models under the assumption of private costs. To make the algebra more manageable, I also set $\epsilon = 1$.

Proposition 2. ***(Private costs)*** *Suppose* $(\alpha, \epsilon) = (0, 1)$. *Then, with incomplete information instead of complete information:*

(i) the expected price is lower ($\mathbb{E}\left[p^{II}\right] < \mathbb{E}\left[p^{CI}\right]$*;*

(ii) the expected consumer surplus is larger ($\mathbb{E}\left[S^{II}\right] > \mathbb{E}\left[S^{CI}\right]$*;*

(iii) the expected total surplus is larger ($\mathbb{E}\left[W^{II}\right] > \mathbb{E}\left[W^{CI}\right]$*;*

(iv) and the expected industry profits are larger ($\mathbb{E}\left[\Pi^{II}\right] > \mathbb{E}\left[\Pi^{CI}\right]$*.*

Proof. See the Supplementary Material (Lagerlöf, 2016).

The relationships reported in Proposition 2 are all in line with the ones found in Hansen (1988). Thus, at least in the model studied here, Hansen's assumption about the cost distribution—implying that the lowest-cost firm's optimal monopoly price always exceeds the second-lowest cost draw—does not matter for the results. The results in Proposition 2 also hold for any arbitrary number of firms, whereas Hansen assumed a duopoly market.

We can conclude that, both in Hansen's (1988) model and in the present environment with private costs, asymmetric information is not anti-competitive but pro-competitive, at least in the sense that the firms' equilibrium mark-ups are (in expectation) smaller in an environment with asymmetric information. In addition, asymmetric information yields an outcome that is more efficient (in that expected total surplus is larger) and socially more desirable (in that both consumers and firms are better off).

Now allow also for common and partially common costs (so $\alpha \geq 0$). Consider first the relatively straightforward case where demand is inelastic. The results for this case are not novel, as they (at least for all intents and purposes) follow from the analysis in Milgrom and Weber (1982). It is nevertheless useful to establish these results in the present framework, before studying the case with elastic demand.

Proposition 3. ***(Inelastic demand)*** *Suppose demand is inelastic,* $\epsilon = 0$. *Then:*

(i) $\mathbb{E}\left[p^{II}\right] \geq \mathbb{E}\left[p^{CI}\right]$, $\mathbb{E}\left[S^{II}\right] \leq \mathbb{E}\left[S^{CI}\right]$, *and* $\mathbb{E}\left[\Pi^{II}\right] \geq \mathbb{E}\left[\Pi^{CI}\right]$, *where the inequalities hold strictly if, and only if,* $\alpha \in \left(0, \frac{n-1}{n}\right]$*;*

(ii) $\mathbb{E}\left[W^{II}\right] = \mathbb{E}\left[W^{CI}\right]$ *for all* $\alpha \in \left[0, \frac{n-1}{n}\right]$.

Proof. See Appendix.

Proposition 3 says that, with private costs, the expected price is the same with complete and with incomplete information (this is the revenue equivalence theorem). However, for any degree of common or partially common costs (so for any $\alpha > 0$), the expected price under incomplete information is strictly higher. That is, in line with Spulber's (1995) intuition as quoted in the Introduction, uncertainty here softens competition. Proposition 3 also says that the consumer is worse off and the firm is better off under incomplete information (with indifference for $\alpha = 0$). This is simply because, with inelastic demand, expected consumer surplus and profits are linear and, respectively, decreasing and increasing in the expected price. Finally, welfare is always the same with complete and with incomplete information. The reason is that the sum of expected consumer surplus and profits does not depend on the expected price, as this cancels out.

Next allow, again, for elastic demand. For this case the algebra that is required for some of the comparisons quickly becomes hard to work with. I will therefore, to begin with and for the most general version of the model, identify a condition that is *sufficient* for overturning the main results from Hansen (1988).

Proposition 4. ***(Sufficient condition)*** *Suppose that*

$$\alpha > \frac{(n-1)\,\epsilon}{1 + n\epsilon + (n-1)\,x} \stackrel{def}{=} \Psi(\epsilon, n, x). \tag{11}$$

Then $\mathbb{E}\left[p^{II}\right] > \mathbb{E}\left[p^{CI}\right]$ *and* $\mathbb{E}\left[S^{II}\right] < \mathbb{E}\left[S^{CI}\right]$. *Moreover, if both (11) and* $\epsilon \geq 1$ *hold, then* $\mathbb{E}\left[\Pi^{II}\right] > \mathbb{E}\left[\Pi^{CI}\right]$.

Proof. See Appendix.

The function $\Psi(\epsilon, n, x)$ that is defined in condition (11) is graphed in panel (a) of Fig. 2 against the parameter ϵ. For values of α above this cutoff value the expected price with incomplete information is higher than with complete information—so uncertainty softens competition. As the figure shows, this sufficient condition becomes more stringent as the elasticity parameter ϵ increases. It also becomes more stringent as n increases and as x decreases. In particular, as x approaches infinity (meaning that all firms are likely to draw very low signals), uncertainty softens competition for all $\alpha > 0$. Proposition 4 also states that if condition (11) holds, then uncertainty makes the consumers worse off; and, if in addition $\epsilon \geq 1$, uncertainty makes the firms better off. While the latter result is in line with the conclusions in Hansen (1988), the result about consumer welfare overturns his findings. To say anything about expected total surplus is hard in this general version of the model, as the algebra becomes quite untractable. However, in Result 1 below I will return to the question about expected total surplus in a somewhat more specialized version of the model.

In order to obtain a condition for expected price that is both sufficient and necessary I now study the case in which there are two firms and the signals are drawn from a uniform distribution.

Proposition 5. ***(Necessary and sufficient condition)*** *Assume a duopoly market and a uniform signal distribution* $((n, x) = (2, 1))$. *Then* $\mathbb{E}\left[p^{II}\right] \gtreqless \mathbb{E}\left[p^{CI}\right]$ *as*

$$\alpha \gtreqless \frac{2\epsilon}{2 + 3\epsilon - \epsilon^2 + \sqrt{(1+\epsilon)(4 + 8\epsilon + \epsilon^2 + \epsilon^3)}} \stackrel{def}{=} \alpha^*(\epsilon). \tag{12}$$

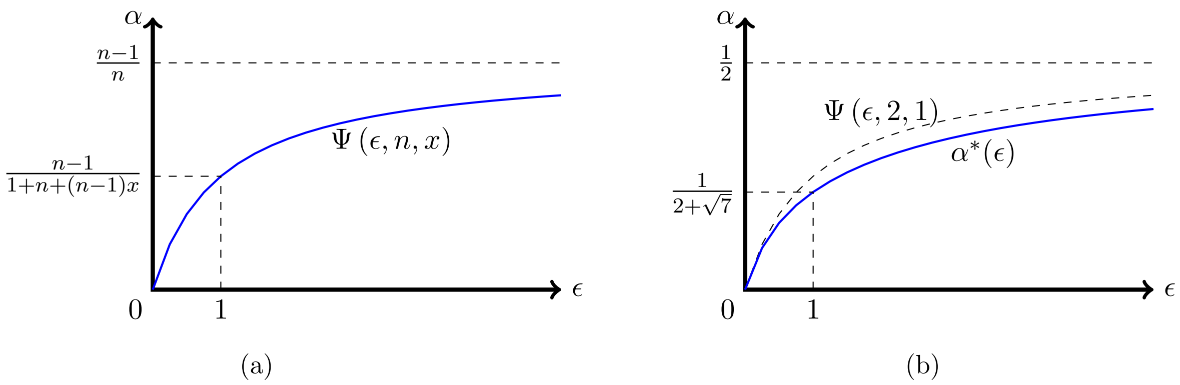


Figure 2: Panel (a) shows the necessary condition stated in Prop. 4. If $\alpha > \Psi(\epsilon, n, x)$, then expected price and expected consumer surplus is larger, respectively lower, under incomplete information. Moreover, if $\alpha > \Psi(\epsilon, n, x)$ and $\epsilon \geq 1$, then expected profits are larger under incomplete information. Panel (b) assumes $(n, x) = (2, 1)$ and shows the necessary and sufficient condition stated in Prop. 5. Expected price is higher under incomplete information if, and only if, $\alpha > \alpha^*(\epsilon)$.

Proof. See Appendix.

The cutoff value $\alpha^*(\epsilon)$ that is defined in condition (12) is graphed in panel (b) of Fig. 2 against the parameter ϵ. As is illustrated by the figure, the function $\alpha^*(\epsilon)$ is increasing in its argument and it satisfies $\alpha^*(0) = 0$ and $\lim_{\epsilon\to\infty} \alpha^*(\epsilon) = \frac{1}{2}$. For values of α above this cutoff value the expected price with complete information is lower than with incomplete information. Moreover, by the arguments in the proof of Proposition 4, $\alpha \geq \alpha^*(\epsilon)$ is also a sufficient condition for expected consumer surplus to be higher with complete information than with incomplete information. Similarly, $\alpha \geq \alpha^*(\epsilon)$ and $\epsilon \geq 1$ are jointly a sufficient condition for expected industry profits to be higher with complete information than with incomplete information.

An important question that has been left unanswered by the above analysis concerns expected welfare (i.e., expected total surplus). We have found that for sufficiently high values of α, expected consumer surplus is higher under complete information—but also that expected industry profits are lower. Therefore it is conceivable that Hansen's (1988) result about expected welfare is not overturned by the introduction of (partially) common costs. However, we do have the following result.

Result 1. ***(Total surplus with common costs)*** *Assume a duopoly market and a uniform signal distribution ($(n, x) = (2, 1)$). Also suppose that costs are common ($\alpha = \frac{1}{2}$). Then expected total surplus is higher with complete information than with incomplete information:* $\mathbb{E}\left[W^{CI}\right] = \mathbb{E}\left[W^{II}\right]$ *for* $\epsilon = 0$ *and* $\mathbb{E}\left[W^{CI}\right] > \mathbb{E}\left[W^{II}\right]$ *for all* $\epsilon > 0$.

Proof. See the Supplementary Material (Lagerlöf, 2016).

4 Conclusions

This paper has studied the question whether cost uncertainty softens competition in a homogenous-good Bertrand model. Previous literature has studied this question in a framework with independent cost draws and found that the answer is "no." In contrast, the present paper showed that when the cost draws are sufficiently highly correlated and the price elasticity of demand sufficiently low, then cost uncertainty indeed softens competition. In particular, under those conditions the expected price is higher and both expected consumer surplus and total surplus are lower under incomplete information.

The paper also presented a simple and tractable model of price competition with privately known but correlated cost draws. In spite of the fact that all firms use pure strategies, the equilibrium of the model gives rise to price dispersion, since each firm posts a price that depends on its privately observed signal.[20] This model may be useful in applications. For example, it would be interesting, using an infinite-horizon repeated-game version of this framework, to study the effect of cost correlation on the firms ability to collude.

Appendix

Preliminaries

The following (straightforward but useful) results are proven in the Supplementary Material (Lagerlöf, 2016). The expected value of one draw s from the distribution in (3) is given by

$$\mathbb{E}[s] = \int_0^1 sx(1-s)^{x-1}\,ds = \frac{1}{1+x}. \tag{13}$$

The expected value of the lowest draw, $s_{(1)}$, given by

$$\mathbb{E}\left[s_{(1)}\right] = \int_0^1 \left[1-s_{(1)}\right]^{nx} ds_{(1)} = \frac{1}{1+nx}. \tag{14}$$

The expected value of the second lowest draw, $s_{(2)}$, given by

$$\mathbb{E}\left[s_{(2)}\right] = \frac{(2n-1)\,x+1}{\left[x(n-1)+1\right](xn+1)}. \tag{15}$$

Moreover, we can rewrite the intercept of the equilibrium price strategy, A, as follows:

$$A = \frac{1-\alpha}{1+\epsilon+(n-1)\,x} + \frac{\alpha}{1+x} = \frac{1+x+\alpha\left[\epsilon+(n-2)\,x\right]}{\left[1+\epsilon+(n-1)\,x\right](1+x)}. \tag{16}$$

Proof of Proposition 1

Firm i's realized profits, π_i, equal

$$\pi_i = \begin{cases} (1-p_i)^{\epsilon}(p_i-c_i) & \text{if winning} \\ 0 & \text{if not winning.} \end{cases}$$

We can compute the firm's expected profits, $\mathbb{E}\pi_i$, by conditioning on winning and not winning:

$$\begin{aligned} \mathbb{E}\pi_i &= \mathbb{E}\left[\pi_i \mid \text{winning}\right]\Pr\left[\text{winning}\right] + \mathbb{E}\left[\pi_i \mid \text{not winning}\right]\Pr\left[\text{not winning}\right] \\ &= (1-p_i)^{\epsilon}\left(p_i - \mathbb{E}\left[c_i \mid \text{winning}\right]\right)\Pr\left[\text{winning}\right]. \end{aligned}$$

[20] Varian (1980) is a well-known paper modeling price dispersion as the outcome of a mixed-strategy equilibrium.

By using (5) and (6), this can be written as:

$$\mathbb{E}\pi_i = (1-p_i)^\epsilon \left[p_i - (1-\alpha)s_i - \alpha \frac{1+x\chi(p_i)}{1+x}\right][1-F(\chi(p_i))]^{n-1}. \tag{17}$$

The first-order condition associated with firm i's profit-maximization problem is:

$$\begin{aligned}
\frac{\partial \mathbb{E}\pi}{\partial p_i} &= \left\{-\epsilon(1-p_i)^{\epsilon-1}\left[p_i-(1-\alpha)s_i-\alpha\frac{1+x\chi(p_i)}{1+x}\right]+(1-p_i)^\epsilon\right\}[1-F(\chi(p_i))]^{n-1} - \\
&\quad \left\{\frac{\alpha x[1-F(\chi(p_i))]^{n-1}}{1+x}+(n-1)[1-F(\chi(p_i))]^{n-2}f(\chi(p_i))\left[p_i-(1-\alpha)s_i-\alpha\frac{1+x\chi(p_i)}{1+x}\right]\right\} \\
&\quad \times (1-p_i)^\epsilon \chi'(p_i) \\
&= 0.
\end{aligned} \tag{18}$$

Now impose symmetry, set $\chi(p_i)=s$ and $\chi'(p_i)=1/(\partial p/\partial s)$, and simplify:

$$\begin{aligned}
&\left\{-\epsilon(1-p)^{\epsilon-1}\left[p-(1-\alpha)s-\alpha\frac{1+xs}{1+x}\right]+(1-p)^\epsilon\right\}[1-F(s)]^{n-1} \\
=\; &\left\{\frac{\alpha x[1-F(s)]^{n-1}}{1+x}+(n-1)[1-F(s)]^{n-2}f(s)\left[p-(1-\alpha)s-\alpha\frac{1+xs}{1+x}\right]\right\}\frac{(1-p)^\epsilon}{\partial p/\partial s}
\end{aligned}$$

or, using the short-hand notation $h(s) \overset{\text{def}}{=} f(s)/[1-F(s)]$ and rearranging,

$$\begin{aligned}
\frac{\partial p}{\partial s} &= \frac{(1-p)^\epsilon\left\{\frac{\alpha x[1-F(s)]^{n-1}}{1+x}+(n-1)[1-F(s)]^{n-2}f(s)\left[p-(1-\alpha)s-\alpha\frac{1+xs}{1+x}\right]\right\}}{\left\{-\epsilon(1-p)^{\epsilon-1}\left[p-(1-\alpha)s-\alpha\frac{1+xs}{1+x}\right]+(1-p)^\epsilon\right\}[1-F(s)]^{n-1}} \\
&= \frac{(1-p)\left\{\frac{\alpha x}{1+x}+(n-1)h(s)\left[p-(1-\alpha)s-\alpha\frac{1+xs}{1+x}\right]\right\}}{-\epsilon\left[p-(1-\alpha)s-\alpha\frac{1+xs}{1+x}\right]+(1-p)}.
\end{aligned}$$

Next use the fact that $h(s)=x/(1-s)$ and note that $(1-\alpha)s+\alpha\frac{1+xs}{1+x}=s+\frac{\alpha(1-s)}{1+x}$ to obtain:

$$\begin{aligned}
\frac{\partial p}{\partial s} &= \frac{(1-p)\left\{\frac{\alpha x}{1+x}+\frac{x(n-1)}{1-s}\left[-(s-p)-\frac{\alpha(1-s)}{1+x}\right]\right\}}{-\epsilon\left[-(s-p)-\frac{\alpha(1-s)}{1+x}\right]+(1-p)} \\
&= \frac{(1-p)\left\{\alpha x-\frac{x(n-1)}{1-s}[(1+x)(s-p)+\alpha(1-s)]\right\}}{(1+x)\epsilon(s-p)+\epsilon\alpha(1-s)+(1+x)(1-p)}.
\end{aligned}$$

If there is an equilibrium price that is affine in the signal, so that $p=A+(1-A)s$ (with $A\in(0,1)$), then we have $1-p=(1-A)(1-s)$, $s-p=-A(1-s)$ and $\partial p/\partial s=1-A$. Plugging these terms into the above differential equation yields

$$1-A=\frac{(1-A)(1-s)\left\{\alpha x-\frac{x(n-1)}{1-s}[-(1+x)A(1-s)+\alpha(1-s)]\right\}}{-(1+x)\epsilon A(1-s)+\epsilon\alpha(1-s)+(1+x)(1-A)(1-s)}$$

or

$$1=\frac{\alpha x-x(n-1)[-(1+x)A+\alpha]}{-(1+x)\epsilon A+\epsilon\alpha+(1+x)(1-A)}.$$

Solving this equation for A yields

$$-(1+x)\epsilon A+\epsilon\alpha+(1+x)(1-A)=\alpha x-x(n-1)[-(1+x)A+\alpha]$$

or

$$\begin{aligned}
A &= \frac{\epsilon\alpha+1+x-\alpha x+x(n-1)\alpha}{x(1+x)(n-1)+(1+x)\epsilon+1+x}=\frac{\epsilon\alpha+x(n-2)\alpha+1+x}{(1+x)[x(n-1)+\epsilon+1]} \\
&= \frac{[\epsilon+x(n-1)+1]\alpha+(1+x)(1-\alpha)}{(1+x)[x(n-1)+\epsilon+1]}=\frac{\alpha}{1+x}+\frac{1-\alpha}{x(n-1)+\epsilon+1},
\end{aligned}$$

which is identical to the expression stated in Proposition 1.

It remains to verify that firm i's expected profits, as stated in (17), are strictly quasi-concave in p_i. To this end, note that we can rewrite the left-hand side of (18) as

$$\frac{\partial \mathbb{E}\pi}{\partial p_i}=\Xi(p_i)H(p_i)\chi'(p_i),$$

where $H(p_i) \stackrel{\text{def}}{=} (1-p_i)^\epsilon [1-F(\chi(p_i))]^{n-1}$ and

$$\begin{aligned}\Xi(p_i) &\stackrel{\text{def}}{=} \frac{-\epsilon\left[p_i-(1-\alpha)s_i-\alpha\frac{1+x\chi(p_i)}{1+x}\right]+1-p_i}{1-p_i} \\ &\quad -\left\{\frac{\alpha x}{1+x}+(n-1)h(\chi(p_i))\left[p_i-(1-\alpha)s_i-\alpha\frac{1+x\chi(p_i)}{1+x}\right]\right\}. \end{aligned} \tag{19}$$

Note that the term $\chi'(p_i)$ is actually independent of p_i, since the inverse of the equilibrium price is linear in the price (in particular, $\chi(p_i)=(p_i-A)/(1-A)$). Differentiating again yields

$$\frac{\partial^2 \mathbb{E}\pi}{\partial p_i^2} = \left[\Xi'(p_i)H(p_i)+\Xi(p_i)H'(p_i)\right]\chi'(p_i).$$

We need to show that, evaluated at the equilibrium price $p_i=p^{II}$ (so where the first-order condition is satisfied), the above expression is strictly negative. However, the second term in square brackets is zero when evaluated at the price where the first-order condition holds ($\Xi(p^{II})=0$). Moreover, $\chi'(p_i)=\frac{1}{1-A}>0$ and $H(p_i)>0$. It therefore suffices to show that $\Xi'(p^{II})<0$. First differentiate the expression in (19):

$$\begin{aligned}\Xi'(p_i) &= \frac{\left[-\epsilon\left(1-\alpha\frac{x\chi'(p_i)}{1+x}\right)-1\right](1-p_i)+\left\{-\epsilon\left[p_i-(1-\alpha)s_i-\alpha\frac{1+x\chi(p_i)}{1+x}\right]+1-p_i\right\}}{(1-p_i)^2} \\ &\quad -(n-1)h'(\chi(p_i))\chi'(p_i)\left[p_i-(1-\alpha)s_i-\alpha\frac{1+x\chi(p_i)}{1+x}\right]-(n-1)h(\chi(p_i))\left[1-\alpha\frac{x\chi'(p_i)}{1+x}\right].\end{aligned}$$

Next impose symmetry, use the relationships $1-p^{II}=(1-A)(1-s)$, $s-p^{II}=-A(1-s)$, and $(1-\alpha)s+\alpha\frac{1+xs}{1+x}=s+\frac{\alpha(1-s)}{1+x}$:

$$\begin{aligned}\Xi'\left(p^{II}\right) &= \frac{\left[-\epsilon\left(1-\alpha\frac{x\chi'(p^{II})}{1+x}\right)-1\right]\left(1-p^{II}\right)+\left\{-\epsilon\left[p^{II}-s-\frac{\alpha(1-s)}{1+x}\right]+1-p^{II}\right\}}{\left(1-p^{II}\right)^2} \\ &\quad -(n-1)h'\left(\chi\left(p^{II}\right)\right)\chi'\left(p^{II}\right)\left[p^{II}-s-\frac{\alpha(1-s)}{1+x}\right]-(n-1)h\left(\chi\left(p^{II}\right)\right)\left[1-\alpha\frac{x\chi'(p^{II})}{1+x}\right] \\ &= \frac{\left[-\epsilon\left(1-\alpha\frac{x\frac{1}{1-A}}{1+x}\right)-1\right](1-A)(1-s)+\left\{-\epsilon\left[A(1-s)-\frac{\alpha(1-s)}{1+x}\right]+(1-A)(1-s)\right\}}{(1-A)^2(1-s)^2} \\ &\quad -(n-1)\frac{x}{(1-s)^2}\frac{1}{1-A}\left[A(1-s)-\frac{\alpha(1-s)}{1+x}\right]-(n-1)\frac{x}{1-s}\left[1-\alpha\frac{x\frac{1}{1-A}}{1+x}\right],\end{aligned}$$

where the last equality also uses the expressions for h, h', and χ' stated in the text above. The inequality $\Xi'\left(p^{II}\right)<0$ is thus equivalent to

$$\frac{\left[-\epsilon\left(1-\alpha\frac{x\frac{1}{1-A}}{1+x}\right)-1\right](1-A)+\left[-\epsilon\left(A-\frac{\alpha}{1+x}\right)+(1-A)\right]}{(1-A)^2} < (n-1)x\left[\frac{1}{1-A}\left(A-\frac{\alpha}{1+x}\right)+\left(1-\alpha\frac{x\frac{1}{1-A}}{1+x}\right)\right]$$

or

$$-\epsilon\left[(1-A)-\alpha\frac{x}{1+x}\right]-\epsilon\left(A-\frac{\alpha}{1+x}\right) < (n-1)x\left[(1-A)\left(A-\frac{\alpha}{1+x}\right)+(1-A)\left((1-A)-\alpha\frac{x}{1+x}\right)\right]$$

or

$$-\epsilon(1-\alpha) < (n-1)x(1-A)(1-\alpha),$$

which always holds under the assumptions made about the parameters and given that $A\in(0,1)$. □

Proof of Corollary 1

The winning firm will, for some given signal realizations, make a loss if and only if

$$p^{II}\left[s_{(1)}\right] < c_{(1)} \Leftrightarrow A+(1-A)s_{(1)} < (1-\alpha)s_{(1)}+\frac{\alpha}{n-1}\sum_{j=2}^{n}s_{(j)} \Leftrightarrow A+(\alpha-A)s_{(1)} < \frac{\alpha}{n-1}\sum_{j=2}^{n}s_{(j)}. \tag{20}$$

The last inequality holds for some $\left(s_{(2)}, \ldots, s_{(n)}\right) \in [0,1]^{n-1}$ if, and only if, it holds for $s_{(2)} = \cdots = s_{(n)} = 1$. Plugging this into (20) yields $A + (\alpha - A)\, s_{(1)} < \alpha \Leftrightarrow (\alpha - A)\left[1 - s_{(1)}\right] > 0$, which is satisfied for some signal realization if, and only if, $\alpha > A$. By using the expression for A stated in (16) and carrying out some straightforward algebra, this inequality can equivalently be written as (8).

Next consider the claim about the probability. With $n = 2$, (20) simplifies to

$$p^{II}\left[s_{(1)}\right] < c_{(1)} \Leftrightarrow \frac{A}{\alpha} + \left(\frac{\alpha - A}{\alpha}\right) s_{(1)} < s_{(2)},$$

where A is understood to be evaluated at $n = 2$. Also note that the joint density of $s_{(1)}$ and $s_{(2)}$ is given by:[21]

$$g_{1,2}\left[s_{(1)}, s_{(2)}\right] \quad = \quad n\,(n-1)\, f\left(s_{(1)}\right) f\left(s_{(2)}\right)\left[1 - F\left(s_{(2)}\right)\right]^{n-2} = 2x^2\left[1 - s_{(1)}\right]^{x-1}\left[1 - s_{(2)}\right]^{x-1}. \tag{21}$$

The probability that the winner makes a loss can thus be written as

$$\begin{aligned}
&\int_0^1 \int_{\frac{A}{\alpha} + \left(\frac{\alpha - A}{\alpha}\right) s_{(1)}}^1 g_{1,2}\left[s_{(1)}, s_{(2)}\right] ds_{(2)} ds_{(1)} = 2x^2 \int_0^1 \left[1 - s_{(1)}\right]^{x-1}\left[\int_{\frac{A}{\alpha} + \left(\frac{\alpha - A}{\alpha}\right) s_{(1)}}^1 \left[1 - s_{(2)}\right]^{x-1} ds_{(2)}\right] ds_{(1)} \\
= \quad & 2x \int_0^1 \left[1 - s_{(1)}\right]^{x-1}\left[-\left[1 - s_{(2)}\right]^x\right]_{\frac{A}{\alpha} + \left(\frac{\alpha - A}{\alpha}\right) s_{(1)}}^1 ds_{(1)} = 2x \int_0^1 \left[1 - s_{(1)}\right]^{x-1}\left[1 - \frac{A}{\alpha} - \left(\frac{\alpha - A}{\alpha}\right) s_{(1)}\right]^x ds_{(1)} \\
= \quad & 2x\left(\frac{\alpha - A}{\alpha}\right)^x \int_0^1 \left[1 - s_{(1)}\right]^{2x-1} ds_{(1)} = \left(\frac{\alpha - A}{\alpha}\right)^x = \left[\frac{1}{\alpha}\left(\alpha - \frac{1 + x + \alpha\epsilon}{(1 + x + \epsilon)(1 + x)}\right)\right]^x,
\end{aligned}$$

which can be rewritten as the expression in (9). The claim that this expression is strictly increasing in α and ϵ can be verified by differentiation. □

Proof of Lemma 1

The marginal cost of the second most efficient firm is (weakly) lower than the most efficient firm's optimal monopoly price if, and only if,

$$c_{(2)} \leq \frac{1 + \epsilon c_{(1)}}{1 + \epsilon} \Leftrightarrow (1 + \epsilon)\, c_{(2)} \leq 1 + \epsilon c_{(1)} \Leftrightarrow \epsilon\left[c_{(2)} - c_{(1)}\right] \leq 1 - c_{(2)}. \tag{22}$$

Note that we can write

$$\begin{aligned}
c_{(2)} - c_{(1)} \quad &= \quad \left[(1 - \alpha)\, s_{(2)} + \frac{\alpha s_{(1)}}{n-1} + \frac{\alpha \sum_{j=3}^n s_{(j)}}{n-1}\right] - \left[(1 - \alpha)\, s_{(1)} + \frac{\alpha s_{(2)}}{n-1} + \frac{\alpha \sum_{j=3}^n s_{(j)}}{n-1}\right] \\
&= \quad \left[(1 - \alpha) - \frac{\alpha}{n-1}\right]\left[s_{(2)} - s_{(1)}\right] = \left[\frac{(1 - \alpha)\, n - 1}{n-1}\right]\left[s_{(2)} - s_{(1)}\right].
\end{aligned}$$

Plugging this back into (22) yields

$$\begin{gathered}
\epsilon\left[\frac{(1 - \alpha)\, n - 1}{n-1}\right]\left[s_{(2)} - s_{(1)}\right] \leq 1 - (1 - \alpha)\, s_{(2)} - \frac{\alpha}{n-1} s_{(1)} - \frac{\alpha}{n-1} \sum_{j=3}^n s_{(j)} \Leftrightarrow \\
\epsilon\left[(1 - \alpha)\, n - 1\right]\left[s_{(2)} - s_{(1)}\right] \leq n - 1 - (n-1)(1 - \alpha)\, s_{(2)} - \alpha s_{(1)} - \alpha \sum_{j=3}^n s_{(j)} \Leftrightarrow \\
\left\{\epsilon\left[(1 - \alpha)\, n - 1\right] - \alpha\right\}\left[s_{(2)} - s_{(1)}\right] \leq n - 1 - (n-1)(1 - \alpha)\, s_{(2)} - \alpha \sum_{j=2}^n s_{(j)}.
\end{gathered}$$

The right-hand side is non-negative for all signal realizations. The left-hand side is non-positive if

$$\epsilon\left[(1 - \alpha)\, n - 1\right] \leq \alpha \Leftrightarrow \alpha \geq \frac{(n-1)\,\epsilon}{1 + n\epsilon}. \tag{23}$$

Therefore, if (23) holds, then the marginal cost of the second most efficient firm never exceeds the most efficient firm's optimal monopoly price. □

[21] See, for example, Gumbel (1958/2004, p. 53).

Proof of Proposition 3

First consider the claim that $\mathbb{E}\left[p^{II}\right] \geq \mathbb{E}\left[p^{CI}\right]$, with a strict inequality if and only if $\alpha > 0$. Given that $\epsilon = 0$, there cannot be a drastic innovation. Therefore the price under complete information is given by

$$\begin{aligned} p^{CI} &= c_{(2)} = (1-\alpha)\, s_{(2)} + \frac{\alpha}{n-1}\left[s_{(1)} + s_{(3)} + \cdots + s_{(n)}\right] \\ &= \left[\frac{(n-1)(1-\alpha)-\alpha}{n-1}\right] s_{(2)} + \frac{\alpha}{n-1}\left[s_{(1)} + s_{(2)} + \cdots + s_{(n)}\right]. \end{aligned}$$

Taking expectations yield

$$\begin{aligned} \mathbb{E}\left[p^{CI}\right] &= \left[\frac{(n-1)(1-\alpha)-\alpha}{n-1}\right] \mathbb{E}\left[s_{(2)}\right] + \frac{n\alpha}{n-1}\mathbb{E}\left[s\right] \\ &= \left[\frac{(n-1)(1-\alpha)-\alpha}{n-1}\right] \frac{(2n-1)\,x+1}{\left[x\,(n-1)+1\right](xn+1)} + \frac{n\alpha}{(n-1)(1+x)}, \end{aligned}$$

where the last line used (13) and (15). We also have

$$\mathbb{E}\left[p^{II}\right] = A + (1-A)\,\mathbb{E}\left[s_{(1)}\right] = A + \frac{1-A}{1+nx},$$

where the last equality uses (14) and where A in this case (with $\epsilon = 0$) simplifies to

$$A = \frac{1-\alpha}{1+(n-1)\,x} + \frac{\alpha}{1+x}.$$

Using the above expressions we can write

$$\mathbb{E}\left[p^{II}\right] \geq \mathbb{E}\left[p^{CI}\right] \Leftrightarrow A + \frac{1-A}{1+nx} \geq \left[\frac{(n-1)(1-\alpha)-\alpha}{n-1}\right] \frac{(2n-1)\,x+1}{\left[x\,(n-1)+1\right](xn+1)} + \frac{n\alpha}{(n-1)(x+1)}$$

$$\begin{aligned} \Leftrightarrow \quad & \frac{1-\alpha}{1+(n-1)\,x} + \frac{\alpha}{1+x} + \frac{1+x-\alpha}{(1+x)(xn+1)} - \frac{1-\alpha}{\left[1+(n-1)\,x\right](xn+1)} \\ & - \left[\frac{(n-1)(1-\alpha)-\alpha}{n-1}\right] \frac{(2n-1)\,x+1}{\left[x\,(n-1)+1\right](xn+1)} - \frac{n\alpha}{(n-1)(x+1)} \geq 0, \end{aligned} \tag{24}$$

where I used

$$1-A = 1 - \frac{1-\alpha}{1+(n-1)\,x} - \frac{\alpha}{1+x} = \frac{1+x-\alpha}{1+x} - \frac{1-\alpha}{1+(n-1)\,x}.$$

For $\alpha = 0$, the inequality in (24) becomes

$$\frac{1}{1+(n-1)\,x} + \frac{1+x}{(1+x)(xn+1)} - \frac{1}{\left[1+(n-1)\,x\right](xn+1)} - \frac{(2n-1)\,x+1}{\left[x\,(n-1)+1\right](xn+1)} \geq 0$$

$$\Leftrightarrow \quad \frac{1}{1+(n-1)\,x} + \frac{1}{xn+1} \quad \geq \quad \frac{(2n-1)\,x+2}{\left[x\,(n-1)+1\right](xn+1)},$$

which holds with equality (to see this, write the two left-hand-side terms on a common denominator). It remains to show that the inequality in (24) holds strictly for $\alpha > 0$. The left-hand side of the inequality is strictly increasing in α if

$$\begin{aligned} & -\frac{1}{1+(n-1)\,x} + \frac{1}{1+x} - \frac{1}{(1+x)(xn+1)} + \frac{1}{\left[1+(n-1)\,x\right](xn+1)} \\ & + \left[\frac{(n-1)+1}{n-1}\right] \frac{(2n-1)\,x+1}{\left[x\,(n-1)+1\right](xn+1)} - \frac{n}{(n-1)(x+1)} > 0 \end{aligned}$$

$$\begin{aligned} \Leftrightarrow\ & \frac{(xn+1)-1}{(1+x)(xn+1)} - \frac{(xn+1)-1}{\left[1+(n-1)\,x\right](xn+1)} + \frac{n\left[(2n-1)\,x+1\right]}{(n-1)\left[x\,(n-1)+1\right](xn+1)} - \frac{n}{(n-1)(x+1)} > 0 \\ \Leftrightarrow\ & \frac{x\,(n-1)-(xn+1)}{(1+x)(xn+1)(n-1)} + \frac{(2n-1)\,x+1-(n-1)\,x}{\left[x\,(n-1)+1\right](xn+1)(n-1)} > 0 \Leftrightarrow -1 + \frac{(n-1)\,x+1+x}{x\,(n-1)+1} > 0, \end{aligned}$$

which always holds.

Next consider the claim that $\mathbb{E}\left[S^{II}\right] \leq \mathbb{E}\left[S^{CI}\right]$, with a strict inequality if and only if $\alpha > 0$. But this claim follows immediately from the expected price comparison above and the fact that, with inelastic demand, consumer surplus is given by $S = 1 - p$ (so it is linear and decreasing in the price)

Similarly, realized industry profits given inelastic demand equal $p - c_{(1)}$ (so they are linear and increasing in the price). This means that also the claim about profits follows immediately from the result about expected price comparison above.

Finally, given the expression for consumer surplus and realized profits above, it is clear that welfare (the sum of those terms) is independent of the price. Hence the welfare result must also hold. $\square$

Proof of Proposition 4

We have $\mathbb{E}\left[p^{CI}\right] \leq \mathbb{E}\left[c_{(2)}\right]$. Therefore, a sufficient condition for $\mathbb{E}\left[p^{II}\right] > \mathbb{E}\left[p^{CI}\right]$ to hold is that

$$A + (1-A)\,\mathbb{E}\left[s_{(1)}\right] > \mathbb{E}\left[c_{(2)}\right]. \tag{25}$$

Note that

$$c_{(2)} = (1-\alpha)\, s_{(2)} + \frac{\alpha}{n-1}\sum_{j=1}^{n} s_{(j)} - \frac{\alpha}{n-1} s_{(2)} = \frac{\alpha}{n-1}\sum_{j=1}^{n} s_{(j)} - \frac{n\alpha}{n-1} s_{(2)} + s_{(2)},$$

and hence $\mathbb{E}\left[c_{(2)}\right] = \frac{n\alpha}{n-1}\left[\mathbb{E}[s] - \mathbb{E}\left[s_{(2)}\right]\right] + \mathbb{E}\left[s_{(2)}\right]$. Inequality (25) thus becomes

$$A\left[1 - \mathbb{E}\left[s_{(1)}\right]\right] - \frac{n\alpha\left[\mathbb{E}[s] - \mathbb{E}\left[s_{(2)}\right]\right]}{n-1} - \left[\mathbb{E}\left[s_{(2)}\right] - \mathbb{E}\left[s_{(1)}\right]\right] \overset{\text{def}}{=} \varrho > 0. \tag{26}$$

One can show that this inequality is easier to satisfy for larger values of α, as we have $\frac{\partial \varrho}{\partial \alpha} > 0$ (this will follow from the calculations below). Now rewrite the inequality $\varrho > 0$:

$$\alpha > \frac{\mathbb{E}\left[s_{(2)}\right] - \mathbb{E}\left[s_{(1)}\right] - \frac{1-\mathbb{E}\left[s_{(1)}\right]}{1+\epsilon+(n-1)x}}{\frac{\partial \varrho}{\partial \alpha}} = \frac{\mathbb{E}\left[s_{(2)}\right] - \mathbb{E}\left[s_{(1)}\right] - \frac{1-\mathbb{E}\left[s_{(1)}\right]}{1+\epsilon+(n-1)x}}{\frac{[\epsilon+(n-2)x]\left[1-\mathbb{E}\left[s_{(1)}\right]\right]}{[1+\epsilon+x(n-1)](1+x)} - \frac{n\left[\mathbb{E}[s]-\mathbb{E}\left[s_{(2)}\right]\right]}{n-1}}. \tag{27}$$

First compute the numerator of (27):

$$\begin{aligned}
&\mathbb{E}\left[s_{(2)}\right] - \mathbb{E}\left[s_{(1)}\right] - \frac{1-\mathbb{E}\left[s_{(1)}\right]}{1+\epsilon+(n-1)x} = \frac{(2n-1)x+1}{[1+(n-1)x](1+nx)} - \frac{1}{1+nx} - \frac{nx}{[1+\epsilon+(n-1)x](1+nx)} \\
=\; & \frac{nx}{[1+(n-1)x](1+nx)} - \frac{nx}{[1+\epsilon+(n-1)x](1+nx)} = \frac{nx\epsilon}{[1+(n-1)x][1+\epsilon+(n-1)x](1+nx)}.
\end{aligned} \tag{28}$$

Next consider the denominator of (27). First rewrite the second term of that expression:

$$\begin{aligned}
&\frac{n\left[\mathbb{E}[s] - \mathbb{E}\left[s_{(2)}\right]\right]}{n-1} = \frac{n}{n-1}\left[\frac{1}{1+x} - \frac{1+(2n-1)x}{[1+(n-1)x](1+nx)}\right] \\
=\; & \frac{n}{n-1}\left[\frac{[1+(n-1)x](1+nx) - [1+(n-1)x](1+x) - nx(1+x)}{[1+(n-1)x](1+nx)(1+x)}\right] \\
=\; & \frac{nx}{n-1}\left[\frac{[1+(n-1)x](n-1) - n(1+x)}{[1+(n-1)x](1+nx)(1+x)}\right] = \frac{nx}{n-1}\left[\frac{\left[(n-1)^2 - n\right]x - 1}{[1+(n-1)x](1+nx)(1+x)}\right].
\end{aligned}$$

The whole denominator of (27) can therefore be written as

$$\begin{aligned}
&\frac{[\epsilon+(n-2)x]\left[1-E\left[s_{(1)}\right]\right]}{[1+\epsilon+x(n-1)](1+x)} - \frac{n\left[\mathbb{E}[s] - \mathbb{E}\left[s_{(2)}\right]\right]}{n-1} \\
=\; & \frac{nx\left\{[\epsilon+(n-2)x][1+(n-1)x](n-1) - \left[\left(1-3n+n^2\right)x - 1\right][1+\epsilon+x(n-1)]\right\}}{(n-1)[1+\epsilon+x(n-1)][1+(n-1)x](1+x)(1+nx)}.
\end{aligned} \tag{29}$$

The part of the numerator of (29) that is in curly brackets can be written as:

$$\begin{aligned}
&[\epsilon+(n-2)x][1+(n-1)x](n-1) - \left[\left(1-3n+n^2\right)x - 1\right][1+\epsilon+x(n-1)] \\
&= [(n-2)x][1+(n-1)x](n-1) - \left[\left(1-3n+n^2\right)x-1\right][1+x(n-1)] + \epsilon\left\{[1+(n-1)x](n-1) - \left[\left(1-3n+n^2\right)x - 1\right]\right\} \\
&= \left\{[(n-2)x](n-1) - \left[\left(1-3n+n^2\right)x - 1\right]\right\}[1+x(n-1)] + \epsilon n + \epsilon x\left[(n-1)^2 - \left(1-3n+n^2\right)\right] \\
&= \left\{\left[(n-2)(n-1) - \left(1-3n+n^2\right)\right]x + 1\right\}[1+x(n-1)] + \epsilon n + \epsilon nx \\
&= (x+1)[1+x(n-1)] + \epsilon n(1+x) = (1+x)[1+n\epsilon+x(n-1)],
\end{aligned}$$

which means that (29) simplifies to

$$\frac{nx\left[1+n\epsilon+x\left(n-1\right)\right]}{\left(n-1\right)\left[1+\epsilon+x\left(n-1\right)\right]\left[1+\left(n-1\right)x\right]\left(1+nx\right)}. \tag{30}$$

(Since this expression is strictly positive we have indeed $\frac{\partial \varrho}{\partial \alpha}>0$, which was assumed above.) Now, using (28) and (30), inequality (27) simplifies to

$$\alpha > \frac{\frac{nx\epsilon}{[1+(n-1)x][1+\epsilon+(n-1)x](1+nx)}}{\frac{nx[1+n\epsilon+x(n-1)]}{(n-1)[1+\epsilon+x(n-1)][1+(n-1)x](1+nx)}} = \frac{\frac{\epsilon}{1}}{\frac{1+n\epsilon+x(n-1)}{n-1}} = \Psi\left(\epsilon,n,x\right).$$

Next turn to the claim that $\mathbb{E}\left[S^{II}\right]<\mathbb{E}\left[S^{CI}\right]$. This follows from the facts that: (i) the consumer surplus is strictly decreasing in the price and $\mathbb{E}\left[p^{II}\right]>\mathbb{E}\left[p^{CI}\right]$; and (ii) the consumer surplus is convex in the price.

Finally turn to the claim that $\mathbb{E}\left[\Pi^{II}\right]>\mathbb{E}\left[\Pi^{CI}\right]$ for all $\epsilon\geq 1$. This follows from the facts that: (i) the profit function is (in the relevant range, below the optimal monopoly price) strictly increasing in the price and $\mathbb{E}\left[p^{II}\right]>\mathbb{E}\left[p^{CI}\right]$; and (ii) the consumer surplus is, for $\epsilon\geq 1$, concave in the price.

□

Proof of Proposition 5

We search for a threshold value of α such that $\mathbb{E}\left[p^{II}\right]=\mathbb{E}\left[p^{CI}\right]$, and with $\mathbb{E}\left[p^{II}\right]>\mathbb{E}\left[p^{CI}\right]$ for higher values and $\mathbb{E}\left[p^{II}\right]<\mathbb{E}\left[p^{CI}\right]$ for lower values of α. By Proposition 4, $\mathbb{E}\left[p^{II}\right]>\mathbb{E}\left[p^{CI}\right]$ for all $\alpha>\frac{\epsilon}{2(1+\epsilon)}$. Hence, the threshold value must satisfy $\alpha\leq\frac{\epsilon}{2(1+\epsilon)}$, which in particular means that the condition in Lemma 1 is violated. We thus must take the possibility of a drastic innovation into consideration.

Given that $(n,x)=(2,1)$, we can write the lowest and the highest cost parameter as, respectively, $c_{(1)}=(1-\alpha)\,s_{(1)}+\alpha s_{(2)}$ and $c_{(2)}=(1-\alpha)\,s_{(2)}+\alpha s_{(1)}$. This means that the high-cost firm's marginal cost does not exceed the low-cost firm's optimal monopoly price if, and only if,

$$\begin{aligned} c_{(2)} &\leq \frac{1+\epsilon c_{(1)}}{1+\epsilon}\Leftrightarrow \epsilon\left[c_{(2)}-c_{(1)}\right]\leq 1-c_{(2)}\Leftrightarrow \epsilon\left(1-2\alpha\right)\left[s_{(2)}-s_{(1)}\right]\leq 1-s_{(2)}+\alpha\left[s_{(2)}-s_{(1)}\right] \\ \Leftrightarrow\quad & s_{(2)}\leq\frac{1+\left[\epsilon\left(1-2\alpha\right)-\alpha\right]s_{(1)}}{1+\epsilon\left(1-2\alpha\right)-\alpha}=B+\left(1-B\right)s_{(1)},\qquad \text{where } B\stackrel{\text{def}}{=}\frac{1}{1+\epsilon\left(1-2\alpha\right)-\alpha}. \end{aligned}$$

(One can check that B does not exceed unity, given that the condition in Lemma 1 is violated.) Moreover, note from eq. (21) that, with $x=1$, the joint density of $s_{(1)}$ and $s_{(2)}$ is given by $g_{1,2}\left[s_{(1)},s_{(2)}\right]=2$. We can now compute $\mathbb{E}\left[p^{CI}\right]$ as follows:

$$\begin{aligned} \mathbb{E}\left[p^{CI}\right] &= 2\int_0^1\int_{s_{(1)}}^{B+(1-B)s_{(1)}}\left[\left(1-\alpha\right)s_{(2)}+\alpha s_{(1)}\right]ds_{(2)}ds_{(1)} \\ &\quad +2\int_0^1\int_{B+(1-B)s_{(1)}}^1\frac{\left[1+\epsilon\left(1-\alpha\right)s_{(1)}+\epsilon\alpha s_{(2)}\right]}{1+\epsilon}ds_{(2)}ds_{(1)}. \end{aligned}$$

It is convenient to simplify one component of the above expression at a time. First write

$$\begin{aligned} & 2\int_{s_{(1)}}^{B+(1-B)s_{(1)}}\left[\left(1-\alpha\right)s_{(2)}+\alpha s_{(1)}\right]ds_{(2)}=\left[\left(1-\alpha\right)s_{(2)}^2+2\alpha s_{(1)}s_{(2)}\right]_{s_{(1)}}^{B+(1-B)s_{(1)}} \\ =\;& \left(1-\alpha\right)\left[B+\left(1-B\right)s_{(1)}\right]^2+2\alpha\left[B+\left(1-B\right)s_{(1)}\right]s_{(1)}-\left(1-\alpha\right)s_{(1)}^2-2\alpha s_{(1)}^2 \\ =\;& \left(1-\alpha\right)\left[B^2+2B\left(1-B\right)s_{(1)}+\left(1-B\right)^2s_{(1)}^2\right]+2\alpha Bs_{(1)}+2\alpha\left(1-B\right)s_{(1)}^2-\left(1+\alpha\right)s_{(1)}^2 \\ =\;& \left(1-\alpha\right)B^2+2B\left[\alpha+\left(1-\alpha\right)\left(1-B\right)\right]s_{(1)}+\left(1-B\right)\left[2\alpha+\left(1-\alpha\right)\left(1-B\right)\right]s_{(1)}^2-\left(1+\alpha\right)s_{(1)}^2. \end{aligned}$$

Hence,

$$\begin{aligned} & 2\int_0^1\int_{s_{(1)}}^{B+(1-B)s_{(1)}}\left[\left(1-\alpha\right)s_{(2)}+\alpha s_{(1)}\right]ds_{(2)}ds_{(1)} \\ =\;& \left(1-\alpha\right)B^2+B\left[\alpha+\left(1-\alpha\right)\left(1-B\right)\right]+\frac{1}{3}\left(1-B\right)\left[2\alpha+\left(1-\alpha\right)\left(1-B\right)\right]-\frac{1}{3}\left(1+\alpha\right) \\ =\;& B+\frac{\left(1-B\right)\left[1+\alpha-\left(1-\alpha\right)B\right]-\left(1+\alpha\right)}{3} \\ =\;& B+\frac{-B\left(1+\alpha\right)-\left(1-\alpha\right)B\left(1-B\right)}{3}=\frac{B\left[\left(2-\alpha\right)-\left(1-\alpha\right)\left(1-B\right)\right]}{3}=\frac{B\left[1+\left(1-\alpha\right)B\right]}{3}. \end{aligned} \tag{31}$$

Similarly, we can write

$$2\int_{B+(1-B)s_{(1)}}^{1}\frac{\left[1+\epsilon\left(1-\alpha\right)s_{(1)}+\epsilon\alpha s_{(2)}\right]}{1+\epsilon}ds_{(2)}=\frac{1}{1+\epsilon}\left[2s_{(2)}+2\epsilon\left(1-\alpha\right)s_{(1)}s_{(2)}+\epsilon\alpha s_{(2)}^{2}\right]_{B+(1-B)s_{(1)}}^{1}$$

$$\begin{aligned}
&= \frac{1}{1+\epsilon}\left[2+2\epsilon\left(1-\alpha\right)s_{(1)}+\epsilon\alpha\right] \\
&\quad -\frac{1}{1+\epsilon}\left\{2B+2\left(1-B\right)s_{(1)}+2\epsilon\left(1-\alpha\right)s_{(1)}\left[B+\left(1-B\right)s_{(1)}\right]+\epsilon\alpha\left[B+\left(1-B\right)s_{(1)}\right]^{2}\right\} \\
&= \frac{2\left(1-B\right)+\epsilon\alpha\left(1-B^{2}\right)+2\epsilon\left(1-\alpha\right)\left(1-B\right)s_{(1)}-2\left(1-B\right)\left(1+\epsilon\alpha B\right)s_{(1)}}{1+\epsilon} \\
&\quad -\frac{\epsilon\left(1-B\right)\left[2\left(1-\alpha\right)+\alpha\left(1-B\right)\right]s_{(1)}^{2}}{1+\epsilon} \\
&= \frac{\left(1-B\right)\left[2+\epsilon\alpha\left(1+B\right)\right]-2\left(1-B\right)\left[1-\epsilon+\epsilon\alpha\left(1+B\right)\right]s_{(1)}-\epsilon\left(1-B\right)\left[2-\alpha\left(1+B\right)\right]s_{(1)}^{2}}{1+\epsilon}.
\end{aligned}$$

Hence,

$$\begin{aligned}
&2\int_{0}^{1}\int_{B+(1-B)s_{(1)}}^{1}\frac{\left[1+\epsilon\left(1-\alpha\right)s_{(1)}+\epsilon\alpha s_{(2)}\right]}{1+\epsilon}ds_{(2)}ds_{(1)} \\
&= \frac{\left(1-B\right)\left[2+\epsilon\alpha\left(1+B\right)\right]-\left(1-B\right)\left[1-\epsilon+\epsilon\alpha\left(1+B\right)\right]-\frac{1}{3}\epsilon\left(1-B\right)\left[2-\alpha\left(1+B\right)\right]}{1+\epsilon} \\
&= \frac{2\left(1-B\right)-\left(1-B\right)\left(1-\epsilon\right)-\frac{1}{3}\epsilon\left(1-B\right)\left[2-\alpha\left(1+B\right)\right]}{1+\epsilon}=\frac{\left(1-B\right)\left(1+\epsilon\right)-\frac{1}{3}\epsilon\left(1-B\right)\left[2-\alpha\left(1+B\right)\right]}{1+\epsilon} \\
&= \frac{3\left(1-B\right)+\epsilon\left(1-B\right)\left[1+\alpha\left(1+B\right)\right]}{3\left(1+\epsilon\right)}=\frac{\left(1-B\right)\left[3+\epsilon+\epsilon\alpha\left(1+B\right)\right]}{3\left(1+\epsilon\right)}.
\end{aligned} \tag{32}$$

Adding up the expressions in (31) and (32), we obtain

$$\begin{aligned}
\mathbb{E}\left[p^{CI}\right] &= \frac{B\left[1+\left(1-\alpha\right)B\right]}{3}+\frac{\left(1-B\right)\left[3+\epsilon+\epsilon\alpha\left(1+B\right)\right]}{3\left(1+\epsilon\right)} \\
&= \frac{B\left[\left(1+\epsilon\right)+\left(1-\alpha\right)\left(1+\epsilon\right)B\right]+\left(1-B\right)\left[3+\epsilon+\epsilon\alpha\left(1+B\right)\right]}{3\left(1+\epsilon\right)} \\
&= \frac{3+\epsilon+\epsilon\alpha+\left[\left(1+\epsilon\right)-\left(3+\epsilon+\epsilon\alpha\right)+\epsilon\alpha\right]B+\left[\left(1-\alpha\right)\left(1+\epsilon\right)-\epsilon\alpha\right]B^{2}}{3\left(1+\epsilon\right)} \\
&= \frac{3+\epsilon\left(1+\alpha\right)-2B+\left(1+\epsilon-\alpha-2\epsilon\alpha\right)B^{2}}{3\left(1+\epsilon\right)}.
\end{aligned}$$

Use the definition of B to get

$$\begin{aligned}
\mathbb{E}\left[p^{CI}\right] &= \frac{3+\epsilon\left(1+\alpha\right)-2B+B}{3\left(1+\epsilon\right)}=\frac{3+\epsilon\left(1+\alpha\right)-B}{3\left(1+\epsilon\right)}=\frac{\left[3+\epsilon\left(1+\alpha\right)\right]\left[1+\epsilon\left(1-2\alpha\right)-\alpha\right]-1}{3\left(1+\epsilon\right)\left[1+\epsilon\left(1-2\alpha\right)-\alpha\right]} \\
&= \frac{3+3\epsilon\left(1-2\alpha\right)-3\alpha+\epsilon\left(1+\alpha\right)+\epsilon^{2}\left(1-2\alpha\right)\left(1+\alpha\right)-\epsilon\alpha\left(1+\alpha\right)-1}{3\left(1+\epsilon\right)\left[1+\epsilon\left(1-2\alpha\right)-\alpha\right]} \\
&= \frac{2-3\alpha+\epsilon\left(1-2\alpha\right)\left[3+\epsilon\left(1+\alpha\right)\right]+\epsilon\left(1-\alpha\right)\left(1+\alpha\right)}{3\left(1+\epsilon\right)\left[1+\epsilon\left(1-2\alpha\right)-\alpha\right]} \\
&= \frac{2-3\alpha+\epsilon\left[3+\epsilon\left(1+\alpha\right)-6\alpha-2\epsilon\left(1+\alpha\right)\alpha\right]+\epsilon\left(1-\alpha^{2}\right)}{3\left(1+\epsilon\right)\left[1+\epsilon\left(1-2\alpha\right)-\alpha\right]} \\
&= \frac{2-3\alpha+\epsilon\left[3+\epsilon-\left(6+\epsilon\right)\alpha-2\epsilon\alpha^{2}\right]+\epsilon\left(1-\alpha^{2}\right)}{3\left(1+\epsilon\right)\left[1+\epsilon\left(1-2\alpha\right)-\alpha\right]} \\
&= \frac{2+\left(4+\epsilon\right)\epsilon-\left[3+\epsilon\left(6+\epsilon\right)\right]\alpha-\epsilon\left(1+2\epsilon\right)\alpha^{2}}{3\left(1+\epsilon\right)\left[1+\epsilon\left(1-2\alpha\right)-\alpha\right]}.
\end{aligned} \tag{33}$$

We also have (using (14) and (16))

$$\begin{aligned}
\mathbb{E}\left[p^{II}\right] &= A+\left(1-A\right)E\left[s_{(1)}\right]=A+\frac{\left(1-A\right)}{1+2}=\frac{3A+\left(1-A\right)}{3}=\frac{1+2A}{3}=\frac{1+2\frac{2+\epsilon\alpha}{2(2+\epsilon)}}{3} \\
&= \frac{\left(2+\epsilon\right)+2+\epsilon\alpha}{3\left(2+\epsilon\right)}=\frac{4+\epsilon+\epsilon\alpha}{3\left(2+\epsilon\right)}.
\end{aligned} \tag{34}$$

Using the expressions in (33) and (34), we can now solve for the value of α that makes the expected prices equal to each other:

$$\mathbb{E}\left[p^{CI}\right]=\mathbb{E}\left[p^{II}\right]\Leftrightarrow\frac{2+\left(4+\epsilon\right)\epsilon-\left[3+\epsilon\left(6+\epsilon\right)\right]\alpha-\epsilon\left(1+2\epsilon\right)\alpha^{2}}{3\left(1+\epsilon\right)\left[1+\epsilon-\alpha\left(1+2\epsilon\right)\right]}=\frac{4+\epsilon+\epsilon\alpha}{3\left(2+\epsilon\right)}\Leftrightarrow$$

$$(2+\epsilon)[2+(4+\epsilon)\epsilon]-(2+\epsilon)[3+\epsilon(6+\epsilon)]\alpha-\epsilon(2+\epsilon)(1+2\epsilon)\alpha^2 = (1+\epsilon)[1+\epsilon-\alpha(1+2\epsilon)][4+\epsilon+\epsilon\alpha]$$
$$= (1+\epsilon)\left\{(1+\epsilon)(4+\epsilon)+[(1+\epsilon)\epsilon-(1+2\epsilon)(4+\epsilon)]\alpha-(1+2\epsilon)\epsilon\alpha^2\right\} \Leftrightarrow$$

$$\begin{aligned} & (2+\epsilon)[2+(4+\epsilon)\epsilon]-(1+\epsilon)^2(4+\epsilon)-\left[3(2+\epsilon)+\epsilon(6+\epsilon)(2+\epsilon)+(1+\epsilon)^2\epsilon-(1+\epsilon)(1+2\epsilon)(4+\epsilon)\right]\alpha \\ = \; & (1+2\epsilon)\epsilon[(2+\epsilon)-(1+\epsilon)]\alpha^2 \end{aligned}$$

$$\Leftrightarrow \; \epsilon = (1+2\epsilon)\epsilon\alpha^2+\left(3\epsilon-\epsilon^2+2\right)\alpha \Leftrightarrow \alpha^2+\frac{3\epsilon-\epsilon^2+2}{(1+2\epsilon)\epsilon}\alpha = \frac{1}{1+2\epsilon} \Leftrightarrow$$

$$\left[\alpha+\frac{2+3\epsilon-\epsilon^2}{2(1+2\epsilon)\epsilon}\right]^2 = \left[\frac{2+3\epsilon-\epsilon^2}{2(1+2\epsilon)\epsilon}\right]^2+\frac{1}{1+2\epsilon} = \frac{\left(2+3\epsilon-\epsilon^2\right)^2+4(1+2\epsilon)\epsilon^2}{4(1+2\epsilon)^2\epsilon^2} = \frac{(1+\epsilon)\left(4+8\epsilon+\epsilon^2+\epsilon^3\right)}{4(1+2\epsilon)^2\epsilon^2}.$$

Taking the square root (ignoring the irrelevant negative root), we obtain

$$\begin{aligned} \alpha &= -\frac{2+3\epsilon-\epsilon^2}{2(1+2\epsilon)\epsilon}+\sqrt{\frac{(1+\epsilon)(4+8\epsilon+\epsilon^2+\epsilon^3)}{4(1+2\epsilon)^2\epsilon^2}} = \frac{\sqrt{(1+\epsilon)(4+8\epsilon+\epsilon^2+\epsilon^3)}-\left(2+3\epsilon-\epsilon^2\right)}{2(1+2\epsilon)\epsilon} \\ &= \frac{(1+\epsilon)\left(4+8\epsilon+\epsilon^2+\epsilon^3\right)-\left(2+3\epsilon-\epsilon^2\right)^2}{2(1+2\epsilon)\epsilon\left[\sqrt{(1+\epsilon)(4+8\epsilon+\epsilon^2+\epsilon^3)}+(2+3\epsilon-\epsilon^2)\right]} \\ &= \frac{4\epsilon^2(1+2\epsilon)}{2(1+2\epsilon)\epsilon\left[\sqrt{(1+\epsilon)(4+8\epsilon+\epsilon^2+\epsilon^3)}+(2+3\epsilon-\epsilon^2)\right]} = \frac{2\epsilon}{2+3\epsilon-\epsilon^2+\sqrt{(1+\epsilon)(4+8\epsilon+\epsilon^2+\epsilon^3)}}. \end{aligned}$$

Since there is only one root to the equation $\mathbb{E}\left[p^{II}\right] = \mathbb{E}\left[p^{CI}\right]$ in the range where α is positive, we must have $\mathbb{E}\left[p^{II}\right] > \mathbb{E}\left[p^{CI}\right]$ whenever α is larger than the critical level derived above. Similarly, we must have $\mathbb{E}\left[p^{II}\right] < \mathbb{E}\left[p^{CI}\right]$ whenever α is smaller than the critical level. □

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